INVESTIGATING SCHOOL MATHEMATICS

TEACHERS' EDITION

LET'S DO

LET'S TALK

LET'S USE

	Topic	[illegible]	[illegible]ne	Book Two
		Units P and R	Units a, b, c, and d	Units e, f, g, and h
A	Sets, Logical Reasoning, and Patterns	Concept of sets: P13–14 Comparisons of sizes: P1–8, P11–12, P15 Similarities and differences: P17–32 Patterns: P9–10, P16 Maze: P32	Comparing sets: a1–9 Equivalent sets: a5–8 Cardinal number of a set: a13–14 Empty set: a19 Union of sets and addition: b1–13, b25–32, b41 Sets and subtraction: b15–23, b33–37, b45 Sets of ten: c1–4 Skip counting: d54, d62 Logic: a59–60, d42 Patterns: c14	Cardinal number of a set: e1–7 Maze: e40, g22 Sets and addition: e43 Sets and subtraction: e47 Counting sequences: e15, 62; f36; g6, 11, 13; h38 Informal logic: e52, 57; f55; g12, 34, 37; h20, 50 Patterns: f10 Attribute pieces: g23 Puzzle problems: g34, g62, h8 Multiplication and sets: g50–52, g55, g57
B	Numeration and Place Value	More than and less than concept: P33–37, P43–47 Matching, one-to-one correspondence: P37–42 Numerals and numbers 0 to 10: R1–7, R11–12, R17–32 Recognition of number of a set: R8–10, R13–16, R37–38	Introducing ten: c1–4 Grouping by tens: c5–13 Counting to 99: c15–26, d44 One more than: a9, a27, 29, 31, 33, 35 Greater than and less than: a41–49, d55–61 Numeral writing: a15–23, a27–36	Numeral writing: e3 2- and 3-digit numerals: e7–11, f13–18, g1–13 Counting to 99: e13–17, h62 Greater than and less than: e17–23, g12 Roman numerals: e32–39 Expanded notation: f49, g16–17 Regrouping: h33, h41–44, h52–56 Fraction numerals: h21–29
C	Addition and Subtraction of Whole Numbers	Addition combinations 1 to 10 (intuitive): R32–36, R39–44 Subtraction related to sums of 10 or less (intuitive): R45–46 Order of numbers 1 to 10: R48	Addition and subtraction facts, sums through 9: b1–51, c35–45 Zero principle for addition: b9 Inverse relation (+ and −): c57–60, d49–52 Missing addends: c53–56, d45–53 Sums of ten: d1–11 Addition and subtraction facts, sums greater than 10: d13–23, d35–43 Commutative (order) principle: d41, d51	Addition and subtraction facts, sums of 10 or less: e41–51, e59 Missing addends: e53–61 Inverse relation (+ and −): e57–58, h13–14 Basic principles: f1–9 Addition and subtraction facts, sums to 18: f11–35, g35–47, h1–19 Addition and subtraction without regrouping: f47–61, g15–21 Addition and subtraction with regrouping: h31–61
D	Multiplication and Division of Whole Numbers			Multiplication concepts: g49–61
E	Fractional Numbers		Introduction to fractions, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$: d25–33	Halves, thirds, and fourths: h21–29
F	Problem Solving and Applications		Solving money picture stories: c47–51 Discussions: a2, 14, 26, 42; b2, 16, 26, 40; c16, 36, 46, 54; d2, 14, 26, 46, 56 Picture story for zero: a20	Money problems: e25–31, g14 Addition and subtraction problems: e60; f22, 32, 34, 54, 60; g20, 46, 60; h16, 18, 45, 58, 60 Multiplication problems: g59–60 Addition and subtraction problems with regrouping: h45, h58, h60 Discussions: e2, 26, 42; f48
G	Estimation			Introductory concepts leading to regrouping: h31, h39
H	Number Theory		Skip counting sequences: d54 Skip counting even numbers: d62	Odd and even numbers: e12 Skip counting: e62, f36, h38 Nomograph addition: f24 Square numbers: f62
I	Measurement		Concept of length: b53–57 Centimetres: b57–61 Length of paths: b62, d34 Readiness for area concepts: c62 Readiness for volume concepts: d24	Time: e33–39 Centimetres: f37–40 Perimeter: f41–42 Area: f43–44 Litres: f46 Volume: g48
J	Geometry, Number Line, and Graphs	Recognition of simple shapes: P17, P27, P28, P30, P48	Number line: a47; b11, 21, 29, 35, 43, 47; d5, 19–22, 48, 54 Recognition of basic shapes: a12, a50–54 Open and closed curves: a55 Segments and paths: a57 Same size figures: a59–61 Symmetry: d25	Number line: e6, 46, 49; f12, 16, 21, 29; g54, 58; h35 Triangles, squares, rectangles, circles: g23–26 Segments: g27–28 Congruence: g28–30 Similarity: g31–32 Graphing: h48
K	Special Topics		Telling time: c27–33 Money: c45–51 Let's have fun: a12, 24, 40, 50, 62; b14, 24, 38, 52, 62; c14, 26, 34, 44, 52, 62; d12, 24, 34, 44, 54, 62	Time: e33–39 Roman numerals: e32–39 Money: e25–31, g14 Magic squares: h8 Let's have fun: e12, 24, 32, 40, 52, 62; f10, 24, 36, 46, 62; g14, 22, 34, 48, 62; h8, 20, 30, 38, 48, 62

[Continued on inside back cover

Teachers' Edition to accompany

Investigating School Mathematics

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CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
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Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the unfolding and discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics, the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

About the Teacher's Edition

The complete pupils' text is reproduced in this teachers' edition in order that you may have before you at all times, in one book, both the page being studied by the pupils and the pertinent manual material.

Throughout the manual, page numbers for the student text are given letter-numeral designation, while pages for the teacher text are referred to by numeral only.

Contents of Learning Units E, F, G, H

*Enrichment Module

Design Features of Book 2

The Book 2 program is organized into four 64-page learning units. Each learning unit is divided into five or six single-concept modules, ranging from 8 to 16 pages each. The four learning units are labelled E, F, G, and H, while the modules within these units are color coded in the upper right-hand corner of each sheet. The first module of each learning unit is color coded yellow, the second is orange, the third is red, the fourth is light green, the fifth is dark green, and the sixth is blue. Three of the modules in the Book 2 program have a special tinted top margin. These modules are considered as optional modules for the Book 2 program. The last module in Learning Unit G is optional and the last two modules in Learning Unit H are optional.

Each lesson throughout Book 2 is presented on the front and back of a sheet, rather than on facing pages. Thus, the sheets may either be torn out and used one at a time or used in the book itself.

Punchout materials are available as separate items for some of the units. Some of these materials are for use with one particular module, such as the clockface and hands to be used with the module on telling time. Other punchouts can be used with many modules. For example, centimetre strips are available as punchouts and suggested for use in more than one module.

Pre-book activities are suggestions to help develop a concept more completely. Materials required for these activities are given in the teacher's notes.

The first lesson in each module is made up of an investigation and a discussion page. The investigation page is labelled "Let's do" and the discussion page "Let's talk." The investigation phase of the first lesson is designed to provide the child with an opportunity to explore the germ of the concept developed throughout the module. This part of the lesson should be child-centred insofar as possible. That is, the children should be given considerable freedom to explore and investigate the particular concept involved. Following the investigation phase of the lesson, the reversed side of the sheet provides the children with an opportunity to further develop the ideas and prepare the children for the various utilization phases of the module which follow. Following the investigation-discussion lesson in the module, other lessons are provided to give the child an opportunity to practice, develop, and extend the concepts and skills of the module. Each lesson has detailed teaching commentary notes to provide objectives, pre-book activities for further developing the concepts and follow-up suggestions. The last lesson in each module is made up of two parts. The front side of the sheet is labelled "Show you know" and this can be used as a module review and, possibly, as an evaluation instrument to determine the child's understanding of the concepts and ideas presented in the module. The back page of each of the final lessons is labelled "Let's have fun," and is intended as an interesting mathematical side trip to be treated with a light touch and in the spirit of having fun with mathematics. The final lesson in each of the 64-page learning units is labelled "Looking back" and is intended as a cumulative review for that learning unit.

The investigation phase of the first lesson is marked by a blue band on the left side of the page, and the discussion phase of the lesson is marked by a green band on the right side of the page. Each key lesson in Book 2 contains an art demonstration at the top of the page to assist you in introducing the activity required on the page itself. It also serves as a guide to the children to remind them of the particular task to be done on the page.

Mathematics of the Book 2 Program

Most of the mathematical concepts developed in the Book 2 program are extensions of concepts previously introduced in the Book 1 program. In each case, however, sufficient material is supplied for a full development of a particular concept. The following list indicates the major concepts introduced in the Book 1 program and extended in the Book 2 program.

- Classification and Sorting
- Sets and Subsets
- Union and Intersection of Sets
- Operations: Addition and Subtraction
- Numeration and Place value
- Equality and Equations
- Inequality, Order, and Counting
- Basic Principles of Addition
- Introductory Geometric Concepts
- Basic Concepts of Measurement
- Concept of Fractional Number

The following list introduces the major new concepts introduced in the Book 2 program.

- Inverse Relation Between Addition and Subtraction
- Subtraction as Related to Comparison of Sets
- Multiplication
- Two-digit Addition Without Regrouping
- Two-digit Addition With Regrouping
- Two-digit Subtraction Without Regrouping
- Two-digit Subtraction With Regrouping

While this list of mathematical concepts may seem formidable for the Book 2 program, it should be understood that many of the concepts are introduced only on an intuitive level. For example, the concepts of subset and union and intersection of sets are introduced through work with addition and subtraction. The language of subset and union of sets need never arise.

Unit E reviews and extends some of the basic concepts introduced in Book 1. The beginning module introduces numbers, numerals, and two-digit place value. These ideas are developed in sufficient depth to provide the child with an understanding of the concepts necessary to work with sums and differences through 18. This module is also preparation for the next module in Unit E which involves counting, order, and inequalities. Each of these modules also serves as preparation for a later module where three-digit place-value concepts will be developed. Following the two introductory modules there are modules involving work with money and telling time. These modules are placed early in the Book 2 program to provide the teacher with introductory work which should be carried on throughout the school year; that is, work with money, coins, and coin collections and work with telling time should be ongoing activities that are developed throughout the school year. The last two modules of Unit E review concepts of addition and subtraction with development of skills for sums and differences up to 10. The last module, in addition to work with sums and differences, focusses upon missing addend concepts as they relate to finding differences.

Learning Unit F continues with review material from Book 1 focussing primarily upon development of power skills to sums and differences up to 18. The beginning module of Unit F develops the basic commutative and associative principles for addition. These principles are developed as readiness for power skills involving sums and differences to 18. These power skills involve finding sums and differences using sets, centimetre strips, the number line, and regrouping. Unit F contains the measurement module, and, again, this module is placed early in the Book 2 program to form a basis for ongoing measurement activities which should continue throughout the school year. The final module of Unit F involves a topic new to the children: two-digit addition and subtraction without regrouping. This topic is relatively simple but involves concepts of place value. The actual skill involved in finding these two-digit sums and differences is much simpler than is the understanding of the concept of place value and the idea of adding in the ones' place and adding in the tens' place.

Learning Unit G begins with a development of three-digit place value. This development is followed by a module introducing three-digit addition and subtraction without regrouping. Again, the technique involved in this three-digit operational work is quite simple as compared with the understanding of adding in the ones', tens', and hundreds' places. Unit G contains the geometry module which develops and extends certain ideas from the Book 1 program. Following the geometry module, a module is provided which initiates the development of speed skills and mastery of the addition and subtraction facts to 18. This mastery should be considered an ongoing objective and will almost certainly not be accomplished for all children during the study of this particular module. The final module in Unit G introduces multiplication, a mathematical concept new to the children. While this module is considered an optional or extension module as denoted by the colored strip at the top of each page, you will find that most children can have successful experiences in this introductory work with multiplication. However, it is possible to omit this module, depending upon the needs and abilities of your children. Four basic methods are used to develop the concept of multiplication: sets, centimetre strips, number-line jumps, and repeated addition. Each of these methods is considered a power-skill method for finding the product of two numbers. The children are not expected to master basic multiplication facts at this level.

Learning Unit H continues the development of speed skills for sums and differences to 18. Following this, introductory ideas are developed for fractional numbers. The final two modules in Unit H involve addition with regrouping and subtraction with regrouping. Again, these two final modules are considered as optional or extension modules.

General Suggestions

Telling time is introduced in the light green module of Learning Unit E. It is not intended that this unit be all-inclusive with regard to the teaching of telling time. Rather the module should be considered as a stimulus and impetus to assist the children in learning this important skill. The children should be working with basic concepts of telling time from the beginning of the school year as they discuss such things as the time for recess, lunch, reading period, and so on; that is, time telling should be an ongoing process which is practiced on a daily basis throughout the school year.

Work with money is introduced in the red module of Unit E. Again it is intended that this module provide an impetus to work with money which has likely occurred from time to time throughout the school year. Most children already will have had opportunities to work with and count various amounts of money.

Special topics which do not occur on the student worksheets but which should arise in a natural setting throughout the school year are work with the calendar and the language of our ordinal numbers. Again, these topics should be presented in an ongoing situation, rather than in a single lesson or module. For example, you should continually use language such as first, second, and third in various meaningful situations. The children will pick

up these words naturally without undue stress. Also work with the calendar can occur as you discuss such things as special holidays, Saturdays and Sundays, Christmas, Vacation, and so on.

Provision for Individual Differences

Since 7-to-8-year-old children are limited in their ability to work independently, provision for individual differences is particularly difficult; however, the investigations, discussions, and pre-book activities provide you with an unusual opportunity to give the more able children a chance to discover more, discuss more, and do more. In addition to these investigations and activities which are primarily teacher directed, there are accompanying enrichment materials available for *Investigating School Mathematics*, Book 2. Also, for more able children, you may want to make greater use of the follow-up activities suggested in this manual as well as the additional material which can be found in the three modules on tinted or colored paper. This should not indicate that these extension modules are not useful for all of the children. It merely indicates that extra care must be taken in working with average and below average children in these learning modules. There are many excellent outside sources for additional enrichment material. Among the best is the book titled *Workjobs* by Mary D. Lorton, Addison-Wesley Publishing Company. This book presents activity-centred learning for early childhood education.

Teaching Strategies for Book 2

All the pages in Book 2 student text are reproduced in this Teachers' Edition in full color, with annotated answers. Unit notes for each module of Book 2 are provided to orient you to the contents and objectives of the module. Detailed page lesson notes and suggested activities appear next to each pair of pages (or lesson) in each module. Mathematics sections are included in both the module and page lesson notes to clarify the concepts presented and to provide a full explanation whenever a mathematical topic new to the children is introduced.

The general teaching strategy for the Book 2 program is based on a five-point plan, as follows: Preparation, Investigation, Discussion, Utilization, and Extension.

The actual physical design of each module as described in the section on design features of Book 2 reflects this five-point teaching strategy. The preparation, investigation, and discussion phase of the teaching strategy occurs at the beginning of each module. The preparation may be found in the teachers' guide and the investigation and discussion are a part of the actual design of the module and are labelled "Let's do" and "Let's talk" and color coded with blue and green bands. When the child is working on the investigation phase of the teaching strategy, maximum effort should be made to encourage him to work on his own. That is, the child should be given an opportunity to investigate, explore, and discover the mathematical concepts of the module. The teacher should, insofar as possible, act as a resource person for this phase of the lesson. The preparation for an investigation should be very short, no longer than is required to prepare the child to begin the investigation independently. Following the investigation, the teacher has an opportunity through the use of the "Let's talk" page to develop and extend the mathematical concepts which were investigated. By using the various pictorials on the "Let's talk" page, the teacher has an opportunity to draw out the ideas and thoughts from the investigation and at the same time develop those concepts which are needed for the remaining lessons. The remaining lessons of the module constitute the utilization phase of the teaching strategy. In the teachers' manual, each of these utilization lessons is accompanied by a preparation or a pre-book activity as well as follow-up material for use after the children have completed the work on a given sheet. The follow-up material and the supplementary materials, as well as the last lesson in the module, may be thought of as the extension phase of the teaching strategy. Actually, each utilization lesson may be thought of as employing the entire five-point teaching strategy in itself. The preparation and investigation are accomplished through pre-book activity found in the teachers' manual. The discussion part of the teaching strategy can be accomplished as the teacher and children communicate the ideas of the suggested pre-book activity. The utilization phase is the work on the text page and the extension is provided by the teacher in the form of additional or enrichment follow-ups.

A part of the overall philosophy of *Investigating School Mathematics* is to encourage each child to become actively involved in his own learning experience. This is most often and best accomplished through the use of a variety of manipulative aids such as counters, geometric shapes, and measuring devices. In addition to these manipulative materials, each child should be provided with a set of the punchout centimetre strips. These strips are available from ADDISON-WESLEY (CANADA) LTD. and can be used as aids in developing various basic mathematical concepts such as the order of the numbers, addition, and subtraction.

Although some of the techniques for teaching modern mathematics are highlighted throughout this manual, teaching children is your specialty. A manual cannot and should not attempt to dictate the day-to-day handling of your class and the individuals in it. The directions given in the teachers' manual should be used flexibly according to the needs and abilities of your children. Do *not* allow the lesson notes to stifle your own effective teaching methods and creative efforts. Consider follow-up activities as suggestions to inspire your own resourcefulness.

Evaluation of Progress

The broad objectives of any mathematics program are to promote the understanding of the mathematical concepts presented and to develop adequate skill in using these concepts. Certainly the development of meaningful skills in mathematics cannot be completely separated from an understanding of the mathematical concepts.

The problem of evaluating children in a modern mathematics program is quite complex when one attempts to analyze the various types of skills that are developed. A clear understanding of these skills can best be accomplished by categorizing them into two basic classes: power skills and speed skills. The role of understanding is most important in power skills. When sufficient emphasis has been placed upon understanding, the child is then prepared to attack new problems or skills independently. At every stage in the mathematics program, the child is encouraged to seek out and develop concepts for himself. This is what we describe as the power-skill stage. It is the understanding stage. Briefly, we can describe the power-skill as the ability to attack a problem and through some technique, however awkward or however time consuming, find the correct answer. Since power skill has to do with understanding a concept in order to find the correct answer and speed skill has to do with techniques and efficiency in finding correct answers, it becomes clear that most evaluation instruments test speed skills only. For this reason, any sound evaluation of progress in a modern mathematics program must be based on daily observation. In this way, you will be able to observe those children who are able to attack new problems and techniques with the power of understanding. It is very difficult to construct written test items which will reveal understanding and the child's ability in the area of power skills. Your job of evaluating speed skills is much simpler. For the most part, speed skills are those skills which we have always evaluated through traditional types of testing programs. Therefore, any evaluation of achievement should include both your daily observation of a child's creativity and understanding along with the mere routine results of speed-skill tests.

About Resources for Active Learning

With the investigative approach it is important to know what materials are available for activity-oriented classrooms. The lists in the module instructions and in most of the lessons offer you some suggestions. If one or two of these resources are available, hopefully you will be able to use them or adapt the ideas.

In the module introductions there are three kinds of resources named. The "General Activities" would be useful as ongoing activities throughout the module, as a review of concepts, and as practice for basic skills. The "Manipulative Devices" and "Commercial Games" can be used to support the lessons in the Module and throughout that Unit. The "Resource(s)" listed with a lesson are more clearly related to the "Objective" of that lesson. Choose one or two and try them out in a variety of situations.

At the time of this writing, those high quality resources which directly complement the active-learning approach were included. Become aware of those materials that were marketed since that time. Check with your principal, (mathematics) supervisor, or resource teacher for the more recent material that supports this type of learning environment. Several suppliers are noted to help you get started in the search.

Tips on technique: start gradually, choose discreetly, be flexible, experiment, relax, and then have fun and learn with the children.

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Suppliers of Resources for Active Learning

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CCM School Materials, Inc., Chicago, Illinois
Childcraft Education Corp., New York, New York
Creative Publications, Palo Alto, California
Creative Teaching Associates, Fresno, California
Cuisenaire Company of America, Inc., New Rochelle, New York
Educational Supply Co., Toronto, Ontario
Edmund Scientific Co., Barrington, New Jersey
Educational Teaching Aids, Chicago, Illinois
Gamco Industries Inc., Big Spring, Texas
J. L. Hammett Co., Braintree, Massachusetts
Jack Hood School Supplies Co., Ltd., Stratford, Ontario
Herder and Herder, Inc., (Methuen Publications, Agincourt Ontario)
Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario
Ideal School Supply Co., Oak Lawn, Illinois
Lakeshore, San Leandro, California
Learning Research Associates, Inc., (The Book Society of Canada Ltd., Agincourt, Ontario)
Mafex Associates, Inc., Johnstown, Pennsylvania
Math Media Inc., Danbury, Connecticut
Metric Aids Ltd., Toronto, Ontario
Milton Bradley, Springfield, Massachusetts
Moyer-Vico Ltd., Weston, Ontario
Responsive Environments Corp., Englewood Cliffs, New Jersey
Sargent-Welch Scientific Co. of Canada Ltd., Weston, Ontario
Scott, Foresman and Co. (Gage Educational Publishing Ltd., Agincourt, Ontario)
Selective Educational Equipment, Newton, Massachusetts
Sigma Division, Scott Scientific Inc., Fort Collins, Colorado
Teaching Resources, Boston, Massachusetts
TUF, Rowayton, Connecticut
Webster Division, McGraw-Hill Book Co. (McGraw-Hill Ryerson, Ltd., Scarborough, Ontario)
Wff'N Proof, New Haven, Connecticut

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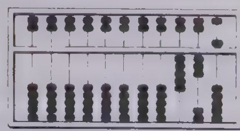
Glossary

abacus A device used for calculating, usually involving sliding beads or counters along a wire.



addend Any one of a set of numbers to be added. In the equation $4+5=9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number that is the result of combining the two numbers is called the sum of the addends.

algorithm Generally used in elementary mathematics to mean one of the various procedures used for computing sums, differences, products, quotients, square roots, etc.

angle Two rays from a single point.

approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

associative principle *See* grouping principle.

average (arithmetic mean) The arithmetic mean of a set of numbers is the quotient resulting when the sum of the number in the set is divided by the number of addends.

bisect To divide in half or to find the midpoint.

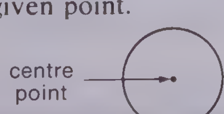
borrow A commonly used term for the regrouping process involved in certain types of subtraction.

Example:
$$\begin{array}{r} \overset{3}{\cancel{4}}\ \overset{13}{\cancel{3}} \\ -1\ 7 \\ \hline 2\ 6 \end{array}$$

carry A commonly used term for the regrouping that is involved in addition.

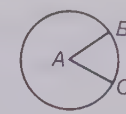
Example:
$$\begin{array}{r} \overset{1}{5}7 \\ +26 \\ \hline 83 \end{array}$$

centre point A given point in the interior of a circle, such that all the points on the circle are the same distance from this given point.

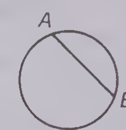


centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

central angle In the figure below, angle BAC illustrates a central angle with respect to a given circle with centre A.



chord A line segment that has its endpoints on a given circle.



circle A set of points, all of which are a specified distance from a given point called the centre or centre point.

circumference The distance around a circle.

circumscribed circle A circle is circumscribed about a polygon when each vertex of the polygon is a point of the circle. In the figure, the circle is circumscribed about the triangle.

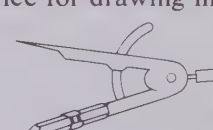
closed figure Intuitively, a figure is closed if you can begin at any point and trace the figure, returning to the initial point.

common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

common multiple A number is a common multiple of two numbers if it is a multiple of each of the numbers.

commutative principle *See* order principle.

compass A device for drawing models of a circle.



composite number Any whole number greater than 1 that is not prime.

congruent angles Two angles are congruent if they are the "same size."

congruent segments Two segments are congruent if they are the "same size."

construction Used in this program relative to drawing models of particular geometric figures, using ruler and compass only.

count To name numbers in regular succession.

cube A rectangular prism (box) such that all faces are squares.

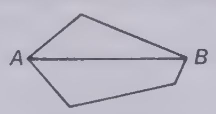
decimal Any base-ten numeral that uses place value to represent a rational number.

decimal point The dot that is used in the decimal symbol.

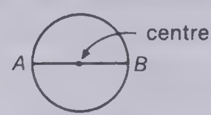
decimetre One tenth of a metre. Ten centimetres.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment AB.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

dimensions The lengths of the various sides or parts of a particular geometric figure.

disjoint sets Disjoint sets have no common elements. That is, when S and T are two sets such that $S \cap T$ is empty, we say that the sets are disjoint with respect to each other.

distributive principle *See* multiplication-addition principle.

division An operation related to multiplication, as illustrated:

$$3 \times 4 = 12 \begin{cases} 12 \div 3 = 4 \\ 12 \div 4 = 3 \end{cases}$$

dividend In the problem $33 \div 7$, 33 is called the dividend.

$$\text{Example:}\quad \begin{array}{r} 4 \\ 7\overline{)33} \\ 28 \\ \hline 5 \end{array} \quad \text{dividend}$$

divisor In the problem $33 \div 7$, 7 is called the divisor.

$$\text{Example:}\quad \begin{array}{r} 3 \\ 7\overline{)33} \\ 28 \\ \hline 5 \end{array}$$

divisor

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

empty set The set that has no objects in it.

equality (equals, or =) A mathematical relation of being exactly the same. The statement $4 + 5 = 6 + 3$ claims that the number $4 + 5$ is exactly the same as the number $6 + 3$.

equation A mathematical sentence involving the use of the equality symbol. Examples: $5 + 4 = 9$; $7 + \square = 8$; $\mathbf{n} + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:

$$\frac{3}{4} \times \frac{6}{8} \qquad \begin{array}{l} \rightarrow \ 4 \times 6 \\ \rightarrow \ 3 \times 8 \end{array}$$

equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, · · ·).

exponent In the symbol 10^3, the "3" is an exponent. It indicates that 10 is used as a factor three times. Thus:

$$10^3 = 10 \times 10 \times 10 = 1000$$
$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

factor tree The example shows two factor trees for the number 30.

```
2 × 3 × 5        2 × 3 × 5
 \ /  /          |   \ /
 6 × 5           2 ×  15
  \ /              \ /
  30               30
```

formula A general fact or rule expressed using symbols. Example: $A = \ell \times w$ (A = area of rectangle, ℓ = length of rectangle, w = width of rectangle).

fraction A symbol for a rational number.

Example: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{2}$.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than (>) One of the two basic inequality relations. Examples: $8 > 5$, $28 > 25$, $80 > 50$.

greatest common factor The greatest number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

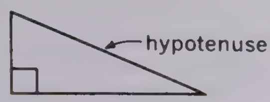
Example: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 5) = (3 \times 4) \times 5$.

hexagon A six-sided polygon.

higher terms A first fraction is in higher terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is greater than the denominator of the second fraction.

Example: $\frac{9}{12}$ is in higher terms than $\frac{6}{8}$.

hypotenuse The side opposite the right angle in a right triangle.

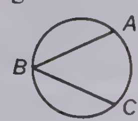


improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}$, $\frac{6}{6}$, $\frac{12}{3}$.

inequality ($>$, $\neq$, $<$) In arithmetic, a relation indicating that the two numbers are not the same, or that one is greater (or less) than the other.

inscribed angle Angle ABC in the figure illustrates an angle inscribed in a given circle.



integers, the set of The whole numbers together with their negatives:

$\{\cdot\cdot\cdot, {}^-3, {}^-2, {}^-1, 0, 1, 2, 3, \cdot\cdot\cdot\}$.

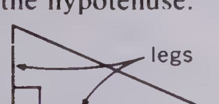
intersection of two sets The set containing those objects and only those objects that are in both of two sets. Example: Set $A = \{c, d, e, f\}$; Set $B = \{e, f, g\}$. The intersection of two sets is the set $\{e, f\}$. We write: $A \cap B = \{e, f\}$.

inverse A number and its negative are additive inverses of each other. A number and its reciprocal are multiplicative inverses of each other. Addition and subtraction are referred to as inverse operations, as are multiplication and division.

least common denominator The least common multiple of two denominators. For $\frac{1}{4}$ and $\frac{5}{6}$, the least common denominator is 12.

least common multiple The smallest number that is a multiple of each of two numbers. For 4 and 6, the least common multiple is 12.

legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

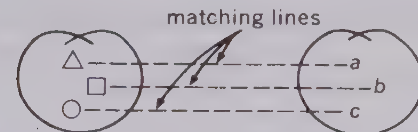
less than ($<$) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

lower terms A first fraction is in lower terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is less than the denominator of the second fraction.

Example: $\frac{6}{8}$ is in lower terms than $\frac{9}{12}$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

matching lines Lines used to indicate or denote the correspondence between the objects in two sets.



measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described above.

metre A unit of length in the Metric System. 100 centimetres.

minus ($-$) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

mixed numeral A symbol given for a fractional number greater than 1 that is a combination of a whole-number symbol and a fraction symbol.

Examples: $2\frac{1}{2}$, $3\frac{2}{3}$, $5\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

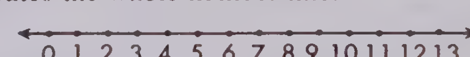
multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$.

negative integer An integer other than those integers which are the whole numbers.

number line A line on which specified points are given number labels or names. The following example illustrates the whole-number line.



number pair Any pair of numbers. In this program, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not an even number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set, and each element of the second set is matched with exactly one element of the first set.

opposite Referring to the relation between two integers whose sum is zero. Example: 2 is the opposite of $^{-}2$; $^{-}8$ is the opposite of 8.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Example: $4 + 5 = 5 + 4$, $2 \times 3 = 3 \times 2$.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with opposite sides parallel.

parentheses (( )) Symbols used to indicate grouping or order of performing operations. Examples:

$$(5 \times 4) - 2 = 18; \quad 5 \times (4 - 2) = 10.$$

pentagon A five-sided polygon.

percent (%) Per 100; for each 100; $\frac{1}{100}$.

perfect number A number that is half the sum of its factors. Examples: 6, 28, and 496.

perimeter The sum of the lengths of the sides of a given polygon.

perpendicular Two lines that intersect in right angles are perpendicular to each other.

pi (π) The ratio of the circumference to the diameter of a circle; approximately 3.14.

placeholder In this program, this term is used to indicate the small box in which you write the solutions to equations.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. The system permits a given digit to stand for different numbers, depending upon its location or position within a given numeral. The number a given digit stands for in a symbol is determined by its position in the numeral and by the base being used in the particular system. In the ordinary base-ten system, for example, a numeral 2 in the third place from the right would stand for 200. Other examples are as follows: in the numeral 3257, the 5 stands for 5 tens or 50; in the numeral 36289, the numeral 6 stands for 6000 (or $6 \times 10 \times 10 \times 10$).

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

positive integer Any whole number other than zero.

power The following examples illustrate powers of 10: 10^2, 10^5, 10^3, 10^7. These are powers of 7: 7^2, 7^4, 7^3, 7^5.

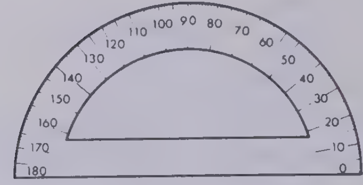
prime factor A factor that is a prime number.

prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.

proper subset Each subset of A except A itself is called a proper subset of A.

protractor A device used for measuring angles.



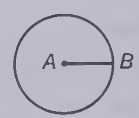
Pythagorean Theorem For any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.

quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radian A unit for measuring angles. A radian is approximately 57.3 degrees.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used for certain types of comparisons.

ray A point on a line and all the points on one side of that point that are on that line.

reciprocal Two numbers are reciprocals of one another if their product is 1. Example: $\frac{4}{7}$ and $\frac{7}{4}$.

rectangle A quadrilateral that has four right angles.

regrouping A procedure commonly used in manipulating place-value symbols in adding or subtracting.

remainder

Example:
$$\begin{array}{r} 6 \\ 7\overline{)47} \\ \underline{42} \\ 5 \end{array} \longleftarrow \text{remainder}$$

repeated addition Finding the sum of a set of numbers, each of which is the same.

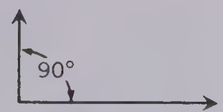
Example: $5 + 5 + 5 + 5$.

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

repeating decimals A decimal that repeats a given digit or group of digits over and over without end.

Example: .333 · · ·

right angle An angle that has the measure of 90 degrees.



A right angle

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

rounding Giving an approximation for a number.

scale drawing A drawing constructed so that the ratio of all the dimensions in the drawing to those of the actual object is the same.

scientific notation A number is said to be written in scientific notation if it is indicated as a number between 1 and 10 times a power of 10. Example: Speed of light in kilometres per second,

$$300\,000 = 3.0 \times 10^5.$$

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group, collection, family, or aggregate of objects. At the heart of the concept of set is man's ability to think of a collection of objects as a single entity.

similar triangles Two triangles are similar to each other if their sides can be matched so that the ratio of the length of each pair of sides is the same.

skip count To count by multiples of a given number. Example: Counting by fives—0, 5, 10, 15, 20, · · ·.

solution The number or numbers that result from solving an equation or a given problem.

solve To find the number or numbers that, when substituted for the variable or placeholder, make the given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

surface area The sum of the area of each face of a figure.

times (×) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

triangle A three-sided polygon.

union of two sets The set consisting of those objects which are in one or the other or both of two sets. Example: Set $A = \{c, d, e, f\}$; Set $B = \{e, f, g\}$. The union of the two sets is the set $\{c, d, e, f, g\}$. We write: $A \cup B = \{c, d, e, f, g\}$.

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.

volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \cdots\}.$$

Numbers, Numerals, and Two-digit Place Value

Pages e-1 to e-12

General Objectives

To strengthen understanding of number concepts
To review reading and writing of numerals
To review the place value of two-digit numbers
To introduce the use of the number line
To introduce odd and even numbers

This module provides review material for some beginning number concepts, such as the number of a set, order, and the use of the number line. In addition to a review of numeral writing, two lessons are provided to review two-digit place value. For many children, this simple review material will be very useful in establishing confidence and getting school started on a successful note.

Mathematics

Since this module reviews many of the basic number concepts of the Book 1 program, the number of mathematical topics is considerable. The module begins by examining briefly the concept of sets and the number of a set. This is followed by a brief look at order of cardinal numbers and the last part of the module develops two-digit place value. The beginning work of the module contains the following mathematical concepts at an informal level.

Sets
Subsets
One-to-one correspondence
Equivalent and non-equivalent sets
Cardinal numbers
Order

In addition to these basic mathematical concepts, the idea of developing a numeration system by grouping by tens is introduced. Even though the number of topics in the module is large, the children should feel fairly comfortable with these mathematical ideas since much of the work in the Book 1 program centred around the development of these basic ideas. Since this unit deals both with the concept of number and with the numerals we write for these numbers, the number-numeral language presents itself immediately. The number itself is an abstract concept associated with collections of equivalent sets. The numerals are merely names or symbols for these abstract concepts. Maintaining the correct usage of the words number and numeral may at times be quite difficult and even undesirable. That is, occasions will arise when it will not be clear whether you should say number or numeral. Also there will be times when correct usage will be awkward and undesirable. You may ask a child to go to the chalkboard to write 2, saying, "Go to the chalkboard and write the number 2." This is incorrect usage, but it would be clumsy to say, "Go to the board and write the numeral for the number 2." Do not get bogged down in language. Children understand that when you say write the number 2, you mean write the symbol for the number 2. We have a rule of thumb for the number-numeral terminology: Whenever you are sure that you are referring to the symbol itself and not to the number, say numeral. In other cases say number. But keep in mind that it is better to abuse the language slightly than to make a major issue of it and confuse the children. In development of two-digit place value, we first introduce the children to the number ten and its symbol, "ten." This is done primarily to provide a logical sequence in the flow of ideas from that of a one-digit number to development of the concept of using two digits as a symbol for a number; that is, the general concept of using two digits for a number is developed prior to the children's writing the symbol 10 for the number ten. For example, the children may see a set of two tens and four and be able to write 24 for this number in the set prior to seeing one set of ten and writing 10 for the number.

Teaching Yellow Module, Unit E

Approximate Time: 6 to 8 days

Materials

cards or tagboard about 15 centimetres wide and 25 centimetres long
clothespins
cord or rope about 3 metres long
counters, at least 9 for each child
crayons
felt objects for set demonstrations
flannelboard
objects to be used in set demonstrations-pencils, erasers, sticks
overhead projector
sets of objects which are grouped into tens

Vocabulary

equivalent sets	number	one-to-one matching
even number	number line	place value
greater	numeral	set
less	odd number	two-digit numerals

Since the material in this module largely reviews concepts presented in Book 1, the vocabulary lists only the most important words or phrases needed to describe the content of the module. The materials for this unit will be used primarily for sets to review the ideas of cardinal number. It is important for the children to participate actively in working with both concrete and semi-abstract materials. They should also be given many opportunities to recognize and compare sets, especially those in patterned arrangements for numbers greater than five. The children should practice writing numerals as part of their daily work according to need.

Evaluation of Progress

Evaluation of the concepts and skills of this module is fairly simple and straightforward. Key items to test for are: the number in a set of ten or less, the order of the numbers 0 to 9, number-line labelling, numeral writing, and the meaning of a two-digit numeral.

Resources for Active Learning

General Activities:

Place Value-Base ten:

CHIP TRADING ACTIVITIES–SET II, Cards 1–3, Scott Scientific

Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 70–74, Wiley

Place Value-Grouping in other bases:

CHIP TRADING ACTIVITIES–SET I, Cards 6–16, Scott Scientific

MATHEX: Numeration No. 2, pp. 29–32, Encyclopaedia Britannica Publications Ltd.

MATHS MINI-LAB, Cards 4–13, Selective Educational Equipment

Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 70–74, Wiley

Manipulative Devices:

Cuisenaire Rods (Cuisenaire Co.)

Grid Kit (Scott Scientific)

Multi-base arithmetic blocks (Educational Teaching Aids: Herder and Herder – Methuen Publications)

Number-Blox (Creative Publications)

Unifix material (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games:

Dominoes games (school supplier)

Investigation
Page e-1

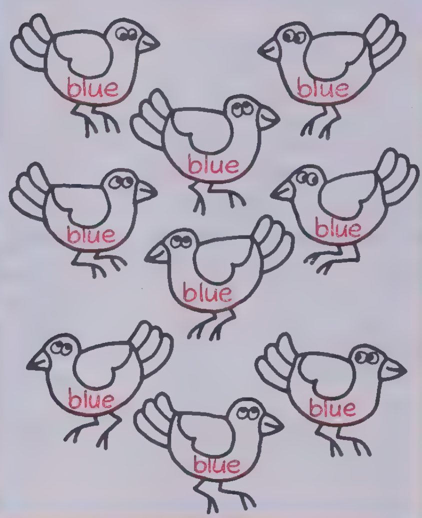
An assumption of this investigation is that the children are able to count to 12. The investigation itself deals with simple counting. Children must count the set of birds and the set of fish to determine which color to use to color each set. Read the directions with them pointing out the investigation question in the bottom section of the page. Suggest that they keep the members of the set that they draw very small or they will not fit them in the space provided. (If you prefer, you might distribute sheets of newsprint on which the children can draw their set of 12.)

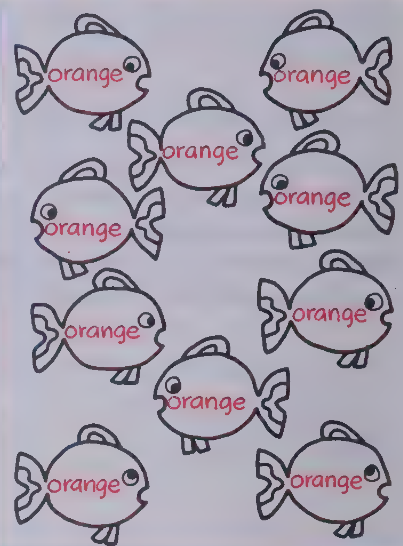
The colored sets on this page may be used to stimulate children to think about certain number concepts. For example, as they work, ask questions such as: "Are there more orange objects than blue?" (Yes) "How can you be sure?" (Draw matching lines between the orange and blue objects.) "How many groups of ten can you find in the set you made?" (1 group) "How many are left over?" (2) Emphasize that 12 may be thought of as 10 and 2. In this way you can introduce the basic number concepts and initiate the idea of place value.

Let's do

Color the set of 9

Color the set of 10





Can you draw and color a set of 12?

Any set of 12 objects can be drawn.

Purposes

To review basic number concepts

To strengthen the idea of grouping by ten

This first investigative lesson provides an introduction to number and place value. Since these concepts will be developed specifically in the following lessons, the activity here should be treated with a light touch.

Preparation

Materials

objects to be used in set demonstrations: pencils; erasers; sticks; crayons, orange, blue, and at least one other

Display a set of three of four objects for the children. Ask them how many objects you are holding. Then ask a child to pick out a set from among materials on a table or desk and show it to the class. Again, others should tell how many are in the set. Continue until children have reviewed the numbers one through ten. You might also write a numeral for the number of each set identified. In addition, you might have a set of five children come to the front of the room; ask others to tell how many there are in this set. Then ask another child to join them and ask how many there are now in the set. Similarly suggest that they count the number of children at a table, or the number of doors in the room, or the number of windows and so on. The intent of this preparation should be simply to review the number of a set and the reading of the numerals 0 through 10. The time spent on this preparation should be kept to a minimum and the questioning should be brisk.

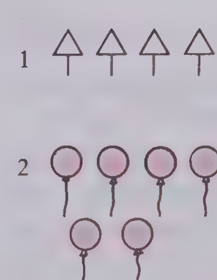
Let's talk

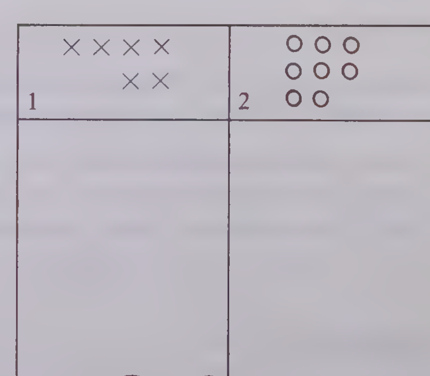
Discussion
Page e-2

The illustration on page e-2 is provided as a basis for discussion. Point out various groups of children and chairs in the picture and ask children to tell how many in each group. Encourage the children to find out how many by counting; they should not be concerned with addition at this time. For example, you might ask: "How many chairs in the front row?" "How many in the back row?" "How many chairs altogether?" "How many children?" "Are there enough chairs for all the children to sit down?" Children should be able to answer you simply by counting. When the illustration has been thoroughly discussed, you might refer again to sets in the classroom. In particular, stress groups of 10 and some left over. For example, ask about 13 children to stand on one side of the room and the remaining children on the other. Have ten chairs placed at the front of the room. Ask the children to go up to the chairs and all who can to be seated. Count how many groups of ten (1) and how many left over (3). You might write 1 ten and 3 or 13 on the chalkboard, but this will be treated in a following lesson. Use activities of this sort to review for the children how we can think of the number of a set in groups of ten and some left over.

Follow-up

Many children will benefit from opportunities to match equivalent sets and compare non-equivalent sets. Cut brightly colored pictures such as toys, lemons, candy, cookies, and the like, from old magazines. Paste these on paper plates, in a variety of patterns. Make several examples of each number from zero to nine. Show one pair of sets and ask the children to tell you which has the greater number. If the sets turn out to be equivalent, stress that the objects may be matched one-to-one. At times you may need to ask, "Which is more?" You might also show the children how to fold their papers three times to make eight sections, and label these 1 to 8. Draw sets of nine or less on the chalkboard. Ask the children to draw a set containing more than each set on the corresponding section of their paper.





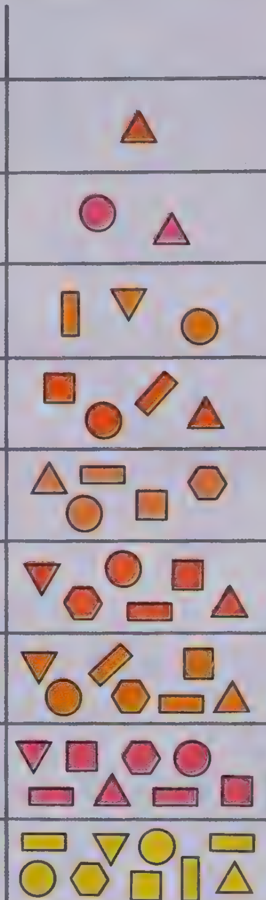
The children's answers will vary.

Teaching
Page e-3

Call the children's attention to the middle section of the first row, and ask how many geometric figures there are in the middle section. When the children answer "zero," point out the word *zero* on the left and the symbol 0 on the right. Ask them to start with the top line, trace over each dashed numeral, and complete the line by writing the numeral in the spaces provided.

Give similar directions for the second row, and direct the children to complete the rest of the page independently.

Write the numerals.



zero	0	0	0	0	0
one	1	1	1	1	1
two	2	2	2	2	2
three	3	3	3	3	3
four	4	4	4	4	4
five	5	5	5	5	5
six	6	6	6	6	6
seven	7	7	7	7	7
eight	8	8	8	8	8
nine	9	9	9	9	9

Objective

Given a set of 9 or less, the child will be able to give the number of the set and write the numeral.

This lesson not only reviews the numbers 0 to 9, but also the writing of the numerals 0 to 9. Some children will benefit from extra practice on worksheets similar to page e-3 or to that shown in the follow-up.

Pre-Book Activity

Materials

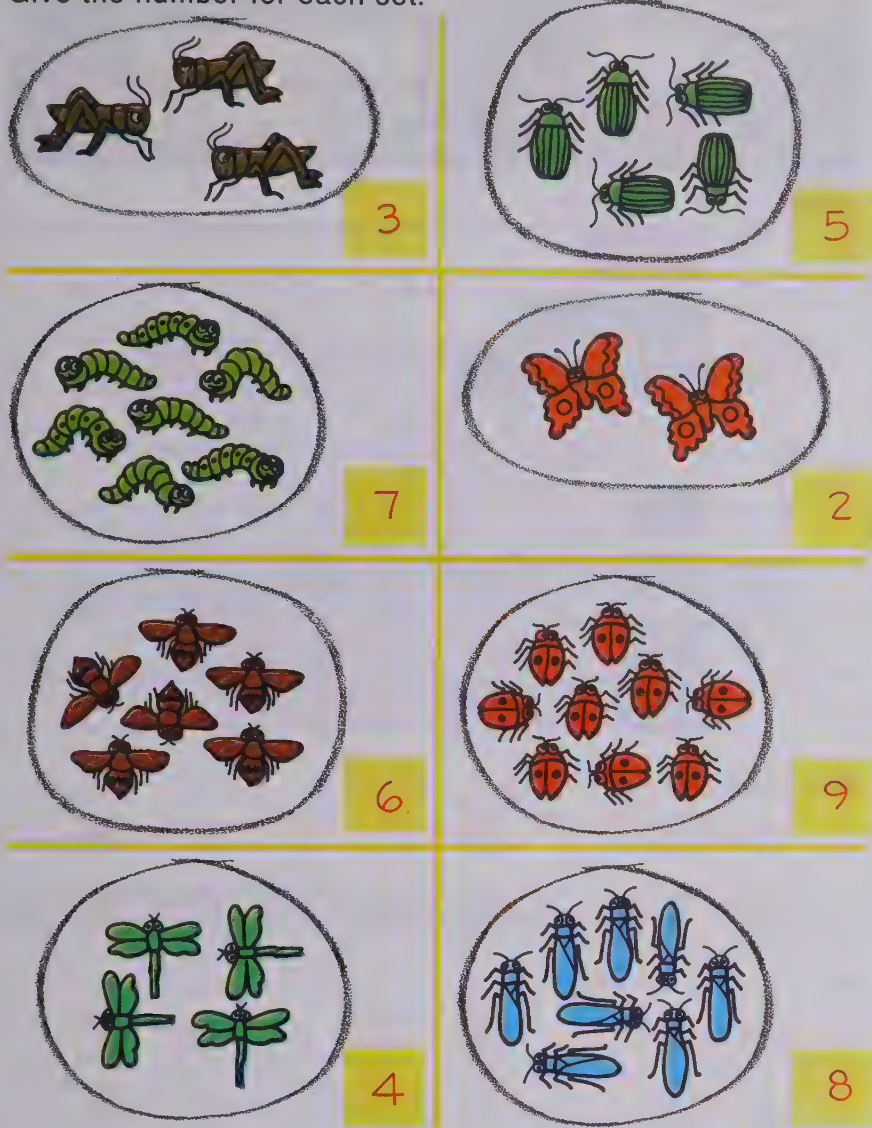
counters, at least 9 per child

After you are sure that each child has at least nine counters, display a set of six objects. You might use the chalkboard, flannelboard, or overhead projector to show your sets. Ask the children to build a set that has as many objects as yours. Ask them to give the number of the set. Write the numeral 6 on the chalkboard and review how to form it with the children. For example, lead the children in writing the numeral in the air. You might then direct them to write the numeral 6 on a piece of ruled writing paper you distribute. Continue similarly with other numbers less than 10. Review both a set for each number and the manner of writing each numeral as shown on page e-3.

Some children may require extra practice with numeral writing. You might give these children an opportunity to trace over large numerals drawn on the chalkboard. Adapt the amount of time you spend on this activity to the needs of the children.

e 4 four

Give the number for each set.



Numbers and numerals, 2–9

Teaching
Page e-4

Read the direction line with the children. Ask how many grasshoppers there are in the first set. When someone answers "three" direct them to write the numeral 3 in the shaded box. Instruct the children to do this for each set. According to children's needs you might also review the number of several sets orally. Include descriptions of sets that contain no objects. Ask, for example: "How many children in this room are ten feet tall?" "How many wild tigers are in the room?" Remind the children that you are using different ways to describe the empty set and that the number of the empty set is zero.

This page may also be used as a basis for discussion of more than and less than concepts. For example, for each pair of sets, you might ask the children to place an X on the set that has more.

Follow-up

Provide the children with prepared work-sheets similar to the following one, or put the material on the chalkboard and give the children ruled writing paper.

Make a set to match each numeral. Practice writing the numeral.		
3 △ △ △ 33	7	5
8	6	4

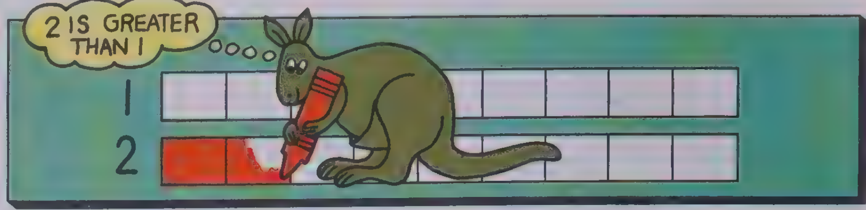
Resources for Active Learning

Nuffield Project: MATHEMATICS BEGINS 1, "One to One . . .," pp. 27–39, Wiley

Teaching
Page e-5

Call the children's attention to the numerals on the left side of the chart. Tell the children that these numerals show how many boxes in each row are to be colored. Direct them to start coloring each row at the left.

Read the questions at the bottom of the page and direct the children to ring the numeral which shows their answer for each pair. When the children have finished the page, discuss how the stair-step pattern of colored boxes shows the order of the numbers 2 through 9. Observe with them that the pattern shows that 7 is greater than 5, that 8 is greater than 7, and so on. Continue similarly to relate the comparison of pairs of numbers in the bottom section with the rows of colored boxes.



Complete the coloring.

2
3
4
5 yellow
6 dk. green
7 black
8 brown
9 blue

Which number is greater?

3	5	7	1	8	9
6	2	0	4	6	5

Which number is less?

2	8	3	7	6	9
5	1	4	0	8	5

Objective

Given two numbers both less than ten, the child will be able to identify the greater number.

Given a number line with ten dots, the child will be able to write the numerals in order from 0 to 9.

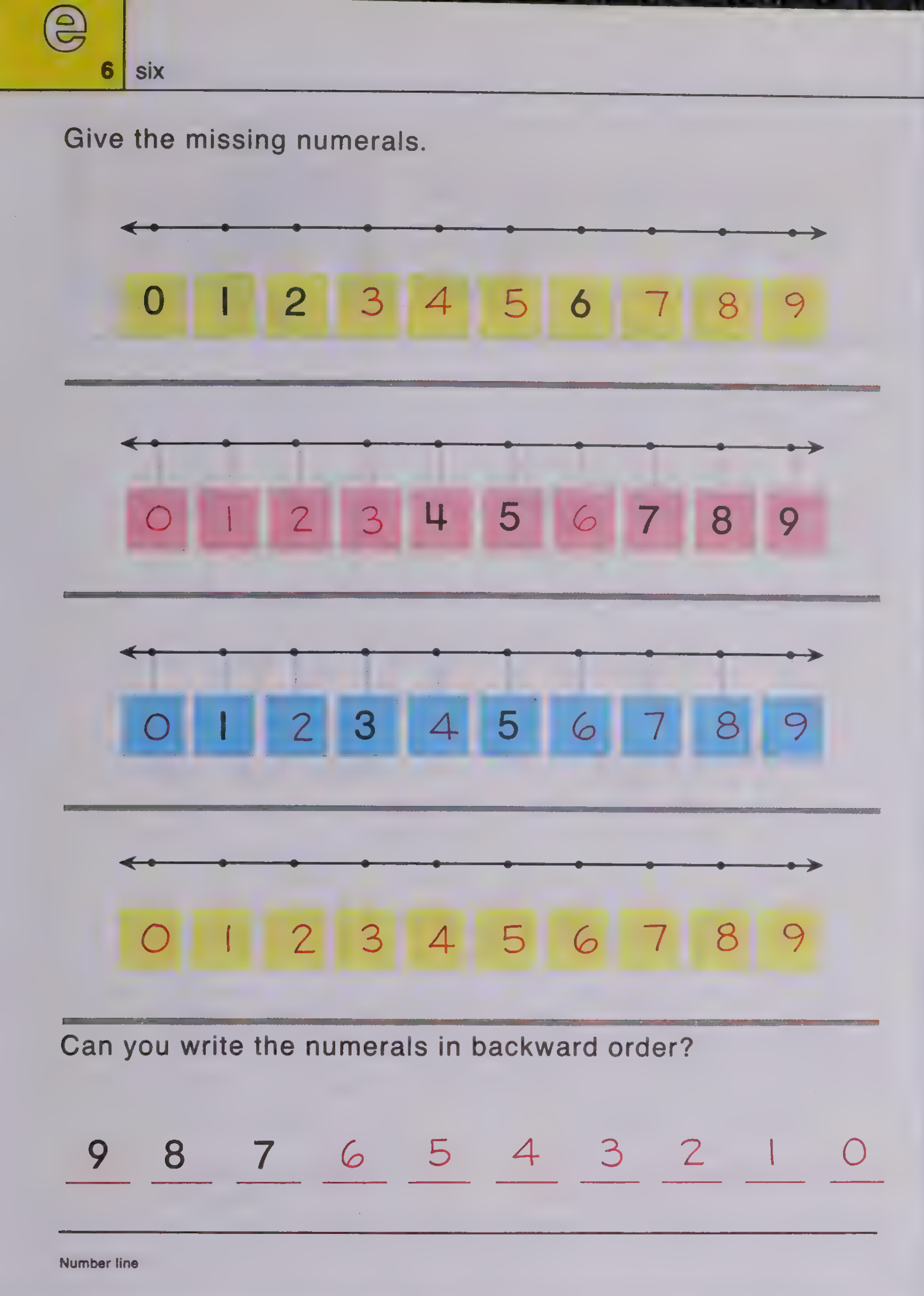
Various activities reviewing comparisons of sets would be suitable to accompany this lesson. The number line is stressed in the pre-book activity as a useful tool in a study of comparing numbers, but you might use comparisons with actual set objects also.

Pre-Book Activity

Materials

clothespins
cord or rope about 3 metres long
numeral cards for 0 to 9, about 15 centimetres wide and 25 centimetres long

Since this lesson introduces the number line on page e-6, you might have the children label a display number line with you. For example, ask two children to hold taut, for all to see, a piece of cord or heavy rope about 3 metres long. Have available clothespins and signs for the numerals from 0 to 9 which may be attached to the line. Hold up the sign for 9 and ask someone to pin it on the line. Discuss the choice of position made by this child by asking questions such as, "Is there someone who would like to put it in a different place?" or "Why is the far right position a good choice?" Then hold up the sign for 1 and ask someone to pin it where they think it should go. Continue for the other numbers, helping


e

6 six

Give the missing numerals.

0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

0 1 2 3 4 5 6 7 8 9

Can you write the numerals in backward order?

9 8 7 6 5 4 3 2 1 0

Number line

Teaching
Page e-6

It would be helpful to use the overhead projector or a chalkboard number line while presenting and discussing this page. However, if you followed the pre-book activity suggestions, children should not have difficulty. Point out the first number line at the top of the page. Read the directions with the children and direct them to fill in the blank spaces with the correct numerals. Instruct them to continue with the other number lines similarly. The last exercise on the page should be considered an extra challenge for those who finish quickly or for those who wish to try it.

children space the signs about 30 centimetres apart. When the number signs have been placed in order and the line is held clearly in view, ask questions such as, "Which is greater 9 or 3? Which is greater, 7 or 6? Which is less, 5 or 3?" and so on. If you prefer, you might use the overhead projector to introduce the number line.

Follow-up

Guide the children in making a number booklet for 0 to 9. Distribute six sheets of large newsprint to each child. Direct them to fold each piece in half and to put one inside the other so they have 12 pages including the top or cover page. On the outside of the booklet, they could print the word NUMBER copying from a model you print on the chalkboard. Then at the top of the first inside right-hand page, they should print the word ZERO, draw a set to show zero and write the numeral 0 below it. On the second right-hand page, they should print ONE, draw a set of one and write the numeral 1 below it, and so on to 9. Remember, to provide them with models for the number words.

Resources for Active Learning

Nuffield Project: MATHEMATICS BEGINS (1), "Class . . . booklets," p. 38, Wiley

Number lines:

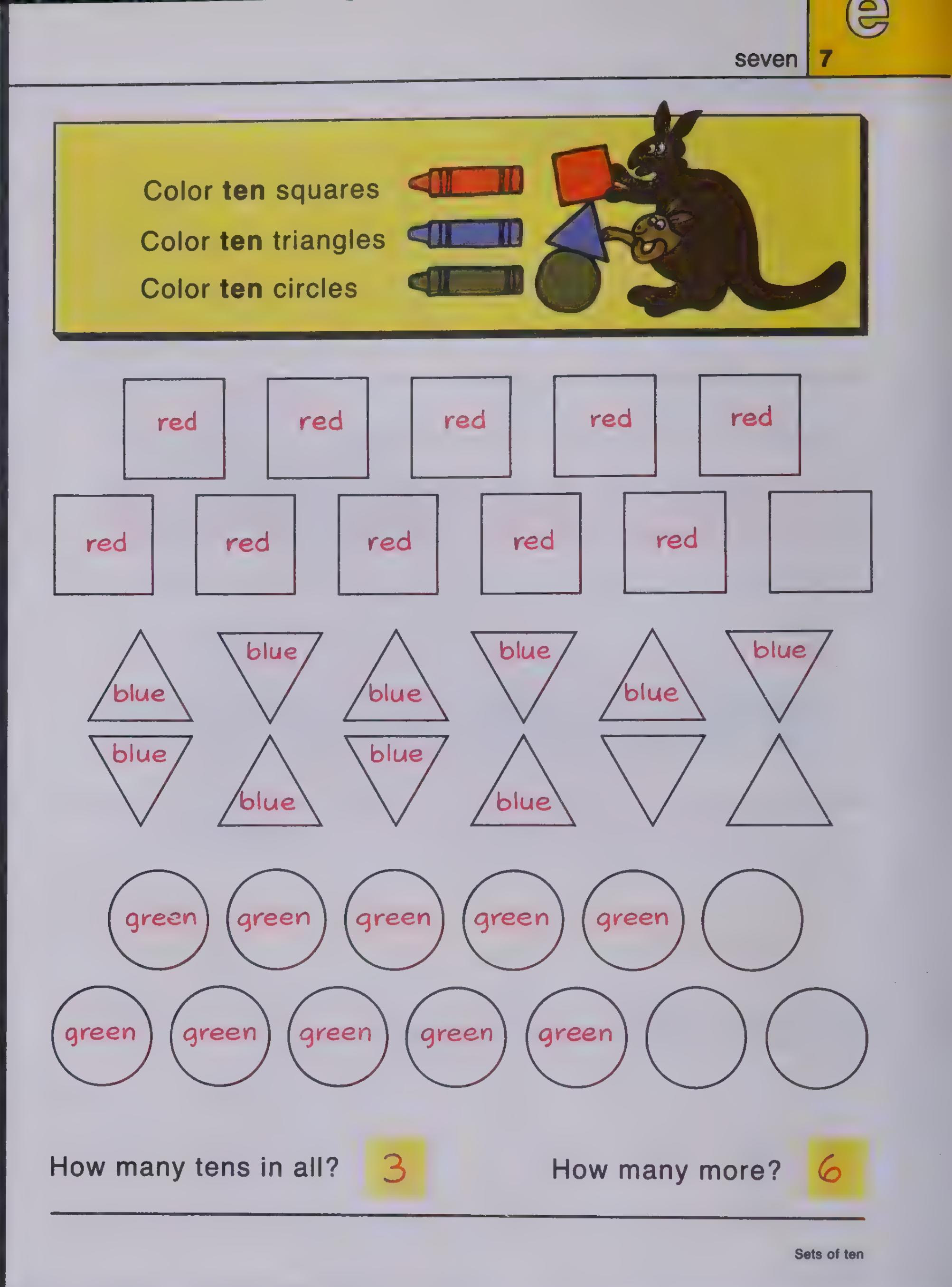
MATHEX: Numeration No. 2, pp. 23–24, Encyclopaedia Britannica Publications Ltd.

MATHS MINI-LAB, Card 52, Selective Educational Equipment

WORKJOBS, pp. 136–137, Addison-Wesley

Teaching
Page e-7

Observe with the children the three different sets of figures. If you wish, use the terms square, triangle, and circle to refer to them, but this vocabulary is not necessary to the lesson. Explain to the children that they should color ten squares red, ten triangles blue, ten circles green. When the children have finished the coloring, ask them to tell how many groups of ten they have colored and how many shapes were not colored. Guide them in completing the questions in the book.


e

seven 7

Color **ten** squares

Color **ten** triangles

Color **ten** circles

red red red red red

red red red red red

blue blue blue blue blue blue

blue blue blue blue

green green green green green

green green green green green

How many tens in all? 3

How many more? 6

Sets of ten

Objective

Given a group of 35 objects or less, the child will be able to arrange them in groups of ten with 9 or fewer left over.

This lesson specifically develops the practice of grouping objects by tens. Concept development is very important at this level. Consider this lesson as part of an on-going objective to develop a concept of place value.

Pre-Book Activity

Materials

sets of objects which can be grouped into tens

Provide each child with 25 to 35 counters such as beans, buttons, sticks, or pipe cleaners. Also provide paper cups for grouping the beans and buttons, and rubber bands for grouping the sticks or pipe cleaners. Ask each child to form a set with his beans or counters. When he has chosen his set, he should put any remaining counters aside. Then ask him to see how many groups of 10 he has in his set. For example, ask him to count to 10 with his counters, then ask if he has any left over. Ask if he can count to 10 again. Use the cups and rubber bands for the sets of 10. When they have grouped their sets into as many tens as they think they can, ask a child to show his set and describe how many objects he has in his set. Then ask if he can write a numeral to show how many. Elicit from the children phrases such as: 2 tens and 5, 1 ten and 4. Have them explain how the numeral 25 represents 2 tens and 5, and how 14 represents 1 ten and 4.

How many? Write the numerals.

2 tens and 4

We write 24.

3 tens and 2

We write 32.

1 tens and 3

We write 13.

2 tens and 3

We write 23.

3 tens and 0

We write 30.

1 tens and 3

We write 13.

1 tens and 1

We write 11.

1 tens and 0

We write 10.

Place value—two-digit numerals

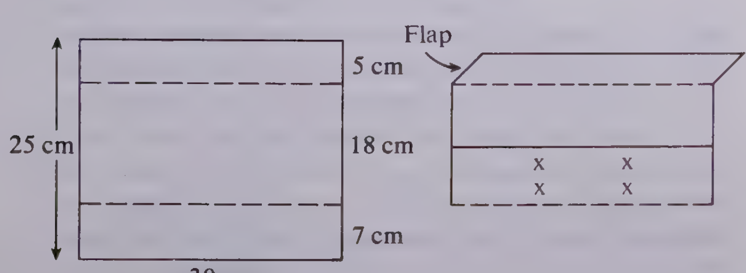
Teaching
Page e-8

For the first exercise, instruct the children to check the sets of ten and the number left over to be sure that the correct answer has been given. Direct them to trace over the dashed numerals, and stress the fact that 24 means 2 tens and 4.

Ask the children to complete the page by themselves. Move around the room and give special attention to those having difficulty. When the children finish, use the exercises as a basis for further discussion. In particular, use the last frame to point out how 10 may be thought of as 1 ten and 0.

Follow-up

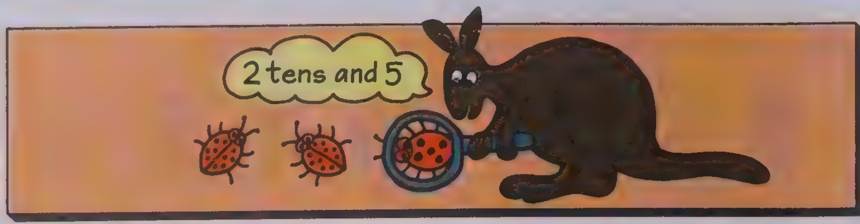
Place-value charts provide an opportunity for children to study the place-value meaning of two-digit numbers. Make place-value charts for each child by folding a piece of tagboard like this:



Fold the lower flap forward and staple the tagboard at the points marked X to make three pockets. Do not label the pockets at this time. Ask the children to mark two sets of small index cards with the numerals 0 to 9. (Using two colors will help separate the "tens" set from the "ones" set.) Direct the children to arrange each set in order from 0 to 9 and to be sure that no numeral was missed. Place one set each in the middle and right-hand pockets. Secure the flap over the pocket with a rubber band to keep the cards in place. Two pockets of these individual place-value charts may be used to help develop understanding of two-digit numerals. All three may be used later for three-digit numerals.

Teaching Page e-9

Use the illustration at the top to introduce the page by discussing how the kangaroo is counting the spots (not including the eyes) on the ladybug. Then call the children's attention to the first set and to the dashed numerals. Emphasize that since the picture shows "3 bugs with ten dots and one bug with 4 single dots, they should complete the phrase 3 tens and 4. Finally, they should write the two-digit numeral 34 in the yellow shaded box. Direct them to complete the page by completing the phrases and writing the numeral for the number of dots in each set in the shaded box. Continue to stress that the first digit indicates the number of tens and the second digit the number less than 10 which is left over.



How many?

3 tens and 4 — 34

4 tens and 2 — 42

1 tens and 4 — 14

4 tens and 0 — 40

1 tens and 5 — 15

1 tens and 0 — 10

4 tens and 6 — 46

2 tens and 2 — 22

Objective

Given illustrated sets of less than 100, the child will be able to describe how many tens and how many left over, and write the corresponding numeral.

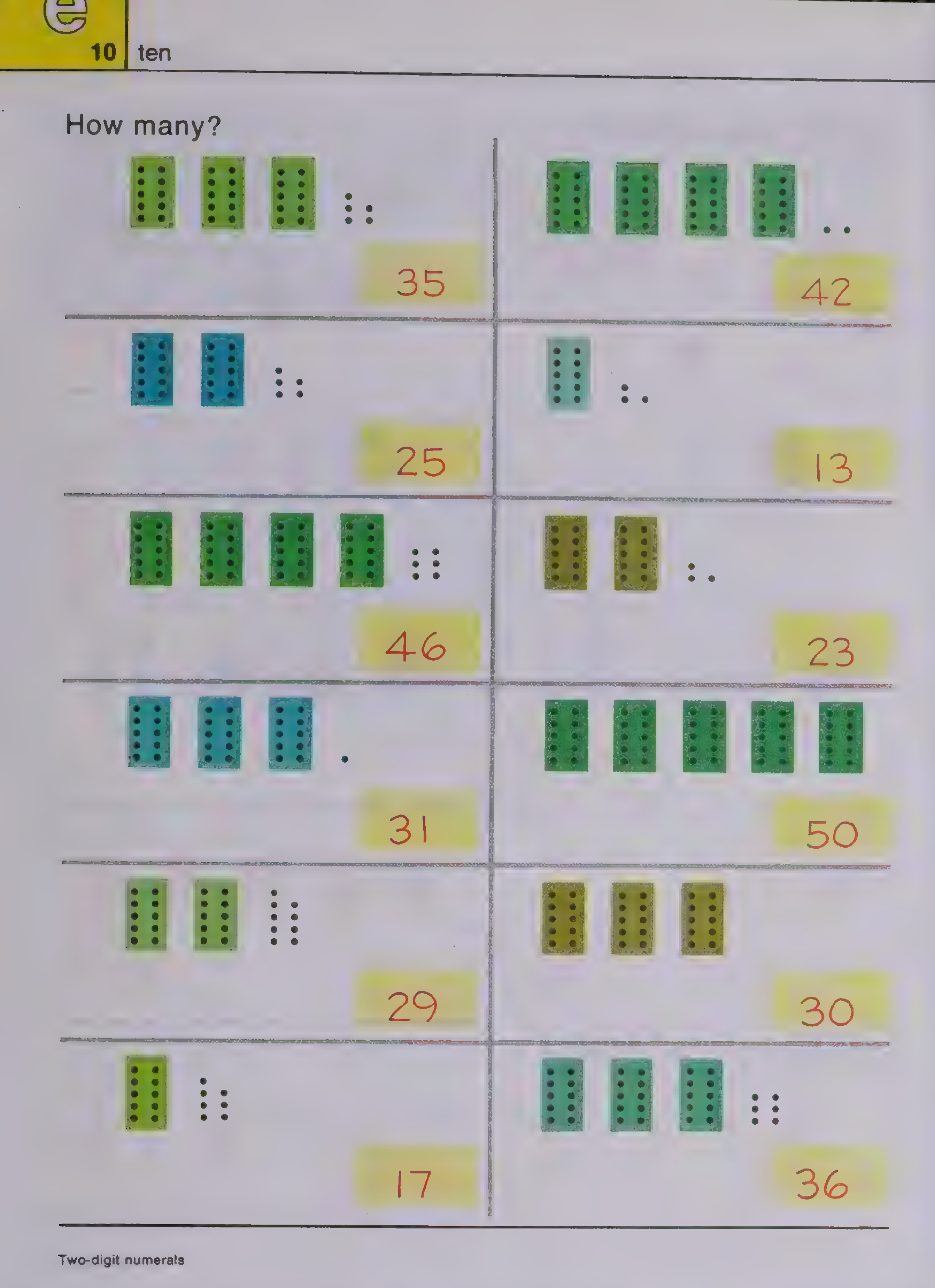
This lesson continues the broad, ongoing objective of developing an understanding of place value. Adapt your time schedule to the development rate of the children.

Pre-Book Activity

If the children need to use concrete materials, you might provide a pre-book activity similar to that of the previous lesson wherein each child has counters to manipulate. However, if you think the children would benefit just as much from a demonstration, provide activities such as the following:

Display a set of 39 pencils and ask the children to group them by tens. When the children point out 3 tens and 9, display the numeral 39. Now place another pencil with this set and ask the children if they need to do any more grouping by tens. Draw out the response that there are now 4 sets of ten. Ask another child to group this last set by placing a rubber band or pipe cleaner around it. Now display the numeral 40 and remark that there are 4 groups of ten and 0. Continue by placing another object with the set and displaying the numeral 41.

You might also have children use the individual place-value charts mentioned in the follow-up on page e-8. For example, display a set of 5 tens and 6, perhaps by showing 5 strips of ten units and 6 single units of felt on the flannelboard. Ask the children to find the correct



Teaching
Page e-10

Call the children's attention to the first set. Emphasize that 35 indicates 3 tens and 5 and have the children check to see that this is the number of the set. Instruct them to write the numeral 35 in the yellow box.

Direct the children to complete the page by writing the numeral for the number of dots in each set in the shaded box. Remind them that in each exercise, the first digit indicates the number of tens, and the second digit the number left over.

tens' numeral and place it in the middle pocket of their chart. Then ask them to find the correct numeral for the ones left over in their second pack of cards and place it in the pocket on the right. Ask the children to hold the charts so you can see what numerals they chose. Vary the sets and ask the children to place the proper numerals in their pocket charts.

Follow-up

To emphasize place value, duplicate an exercise similar to the following one, or write it on the chalkboard.

Match.	
3 tens and 8	90
6 tens and 4	46
4 tens and 6	64
9 tens and 0	38

Resources for Active Learning

Place value games and activities:

CHIP TRADING ACTIVITIES—SET II, Cards 6, 7, Scott Scientific

DEVELOPMENTAL MATH CARDS, C-17, 13, Addison-Wesley

FREEDOM TO LEARN, pp. 116–117, Addison-Wesley

MATHEX: Operations No. 3, pp. 9–16, Encyclopaedia Britannica Publications Ltd.

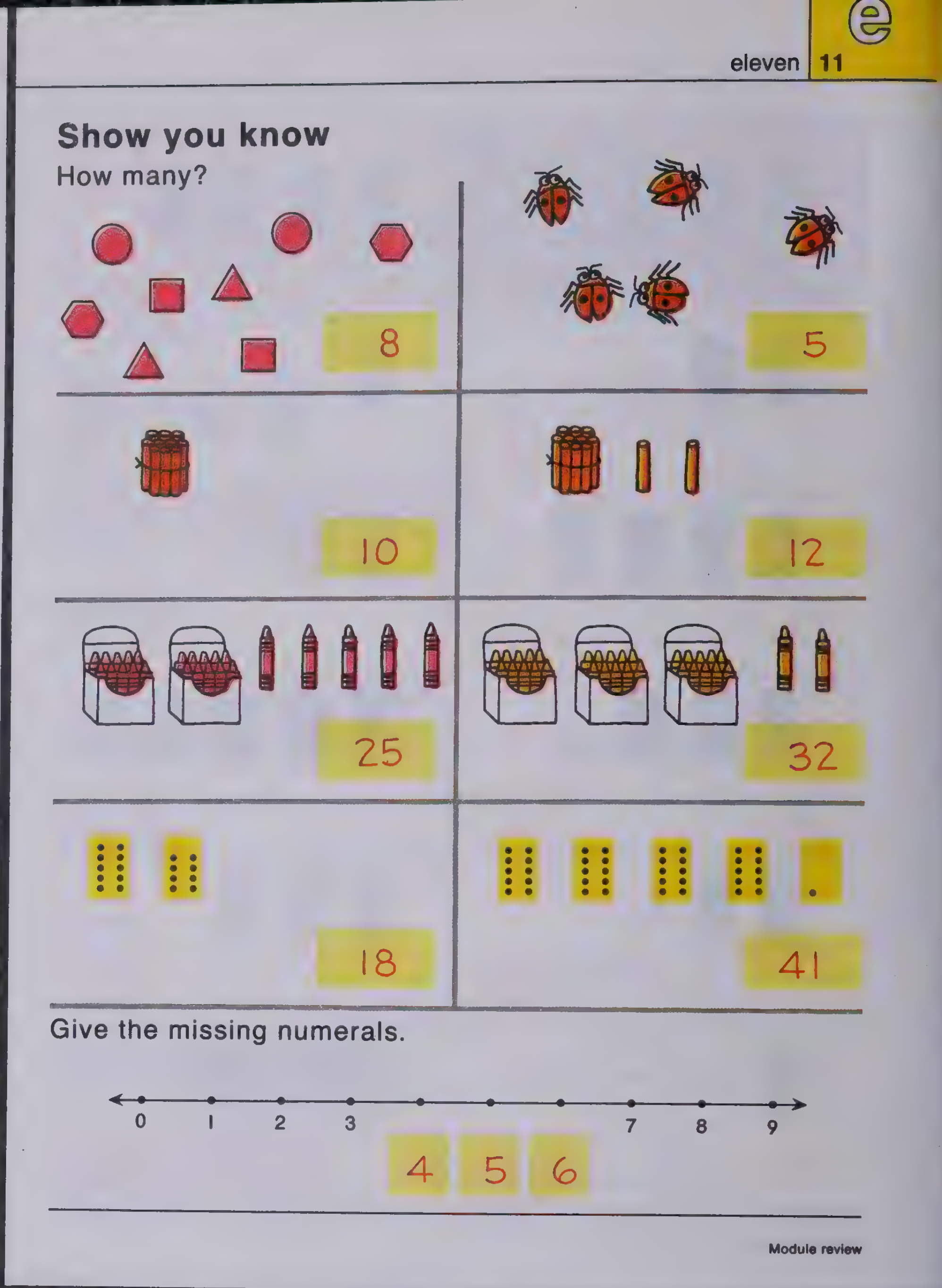
Nuffield Project: COMPUTATION AND STRUCTURE 2, p. 71, Wiley

TEACHING AIDS FOR ELEMENTARY MATHEMATICS, pp. 24–25, Holt, Rinehart and Winston

WORKJOBS, "Number Dots," pp. 146–147, Addison-Wesley

Teaching
Page e-11

Call attention to the first frame and ask the children to figure out how many objects are in the first set. Then explain that they should write the number of the set in the yellow box. You might also mention that in the top frame on the right, it is intended that they count the bugs, not the dots on the bugs' backs. Before you direct them to continue with the remaining frames, point out the number line at the bottom of the page. Explain that they should fill in the blank spaces to show how the numbers from 0 to 9 should be ordered.

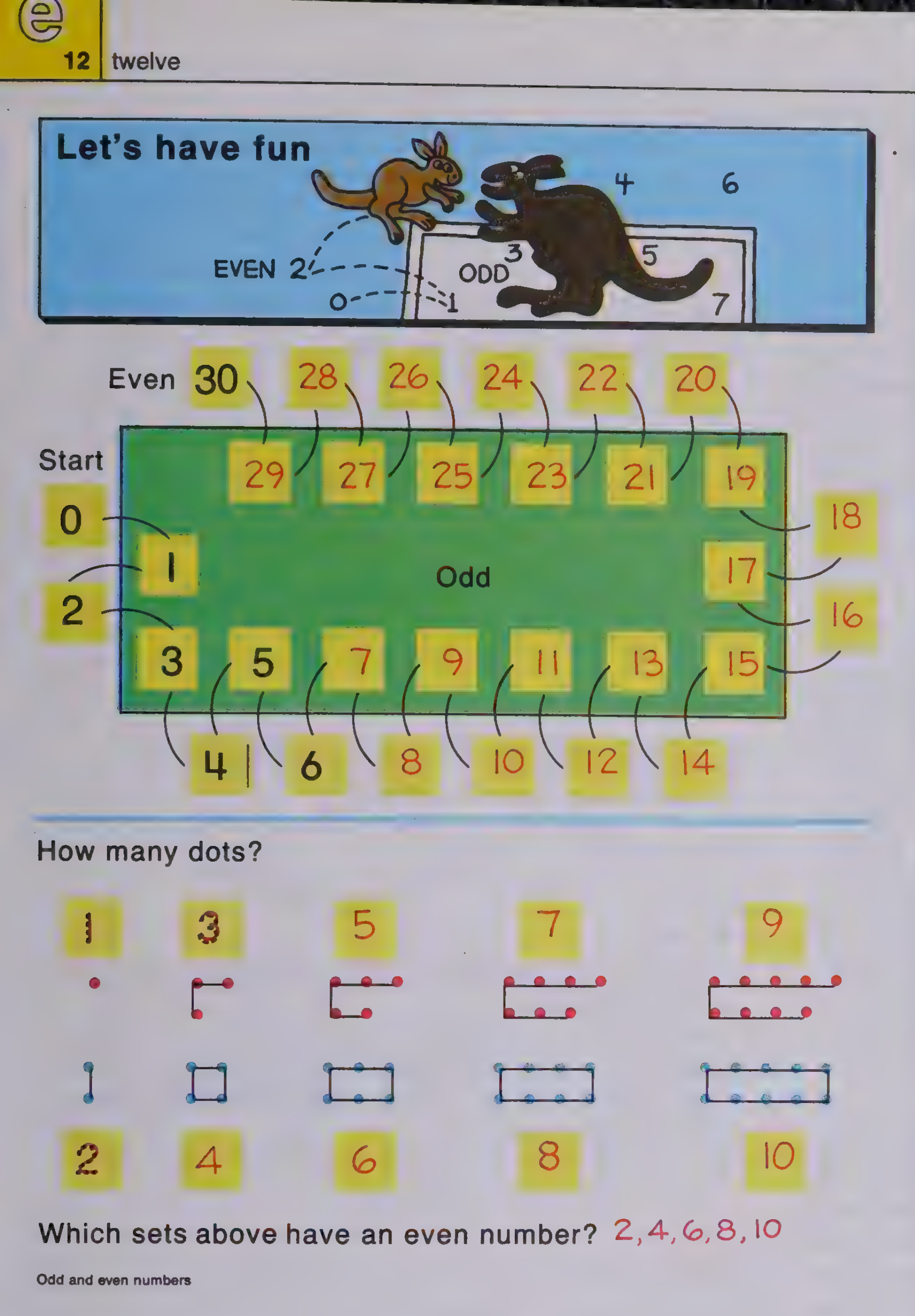


Objectives

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

It would be suitable to use one of the previously suggested activities to review the place value meaning of two-digit numbers. (See page e-7 or e-9.) You might prefer to use an oral game for such a review. For example, play the "What's My Rule?" game in the following manner. Ask the children to name a two-digit numeral (or a number less than 100). When a child has named a number, respond with the phrase which explains its place value meaning. That is, if a child says "47," respond with "4 tens and 7." Challenge the children to discover your rule. When a child thinks he knows the rule, he might fold his arms as a sign to you. Then you can ask him to respond to the next two-digit number by expressing it with the "____ tens and ____" phrase.



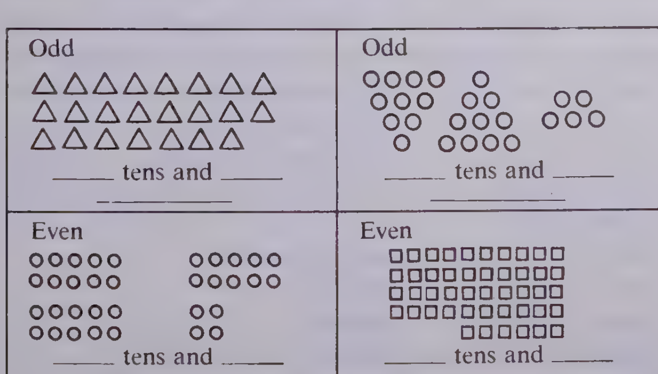
Teaching
Page e-12

Page e-12 is provided as a change of pace page and should be treated with a light touch. Call the children's attention to the demonstration art and talk about the kangaroo jumping in and out of the pen. Then relate that idea to the number "path" activity in the top half of the page. Suggest that they continue to count the number of jumps and fill in the yellow boxes with the numeral for the appropriate counting number. As children finish, discuss the fact that some numbers show up on the outside of the green space and some on the inside. Read together the numbers on the inside (1, 3, 5, 7 . . .) and explain that we call these numbers *odd numbers*. Similarly read the numbers on the outside and explain that we call these numbers *even numbers*.

Direct the children to complete the listings of the odd and even numbers in the second section of the page by counting the number of dots above or below each yellow space. Finally, explain that they should put a check (✓) mark on the sets at the bottom of the page that have an even number of dots. You might suggest that they try to do this by examining the shape in which the dots are arranged, rather than by counting.

Follow-up

Prepare a worksheet which stresses both grouping by tens, and odd and even numbers.



Resources for Active Learning

Number pattern-even and odd numbers:

DEVELOPMENTAL MATH CARDS, D[1]1, Addison-Wesley

ENRICHMENT OF ARITHMETIC, two games, 2/21–22, Webster, McGraw-Hill

MATH ACTIVITIES, Game 2–86, pp. 66–67, Allyn and Bacon

Nuffield Project: COMPUTATION AND STRUCTURE (2), pp. 78–81, Wiley

Counting, Order, and Inequalities

Pages e-13 to e-24

General Objectives

To develop counting skills, particularly in spanning a decade

To develop the concepts of greater than and less than

To strengthen understanding of place value by utilizing the concept of order

To introduce the inequality symbols >, greater than and <, less than

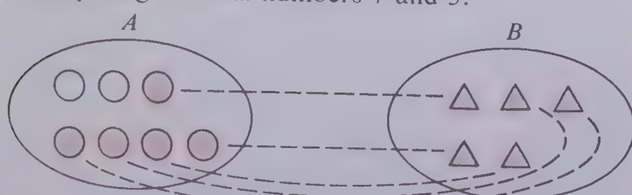
This module begins with a review of counting skills which children used in Book 1; and then introduces them to inequalities. Although the concepts needed for ordering whole numbers are basically intuitive, the understanding and correct use of the inequality symbols must be developed through usage. If children are able to order the numbers 0 through 9, a knowledge of place value should suffice to enable a child to transfer this ability to the order of two-digit numbers. However, skill must be developed before the child can comfortably use the symbols to express the greater than and less than concepts.

Mathematics

This module develops the mathematical concepts involving order of the numbers 0 to 99 and counting. The idea of counting is closely related to that of order since order must be established upon the set of cardinal numbers in order for counting to become meaningful as the tool for assigning the cardinal number to a given set. The idea of one number being greater than another is intuitively obvious for the set of whole numbers. The following is a precise mathematical definition of this idea based on sets.

> If a set from cardinal number A has a proper subset that is equivalent to a set from cardinal number B, then cardinal number A is greater than cardinal number B ($A > B$), and cardinal number B is less than cardinal number A, ($B < A$).

This definition is illustrated in the following diagram by comparing cardinal numbers 7 and 5.



Note that the set of shaded circles is equivalent to set B. Furthermore, the set of shaded circles is a proper subset of set A, since it is a subset of A that does not contain all of A. From the definition 7 is greater than 5 and 5 is less than 7, an intuitive explanation of this is simply that a set of 7 has more objects than a set of 5. Once this definition of order has been imposed upon the set of cardinal numbers, then all of the numbers 0 to 99 can be ordered and the process of counting becomes meaningful in terms of establishing the cardinal number of a given set.

Teaching Orange Module, Unit E

Approximate Time: 6 to 9 days

Materials

abacus
crayons
demonstration inequality symbols (These may be cut out of cardboard.)
demonstration number line
hundred board (a board marked off in 10 rows of 10, with hooks for spools, tags or tickets labelled 1–100)
1 set of strips per child
sets of objects which can be bundled into tens such as pipe cleaners, sticks, counters

Vocabulary

decade	least
greater	lesser
greater than	less than
greatest	

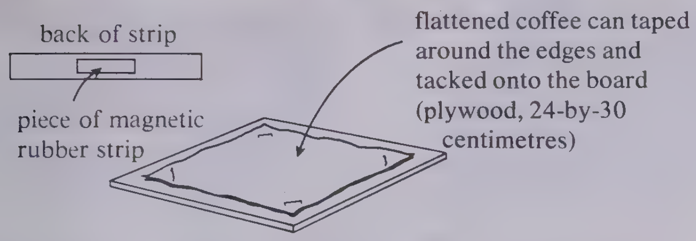
Although oral counting activities are one of the best means for treating concepts of order and inequality, certain materials will be a definite aid to the children. The investigation of the module requires use of the strips which can be purchased to accompany this program. Children should be given as much free time as necessary to familiarize themselves with the strips.

If the white strip is considered the unit, or one, then each of the other strips will have a determined length or number value:

white: 1	dark green: 6
red: 2	black: 7
light green: 3	brown: 8
purple: 4	blue: 9
yellow: 5	orange: 10

Some children may need considerable free play to explore the relationships between the strips before they will be at ease in assigning number values to them.

For greater ease in manipulating the strips, each child would benefit from the opportunity to work with a magnetic strip board. Such a homemade device may be made from a piece of plywood (24-by-30 centimetres) on which has been fastened an opened flattened out coffee can (or comparable piece of metal). Then, on the uncolored back side of each centimetre strip, glue a small piece of magnetic rubber strip. (Such magnetic strips with adhesive backs may be purchased from most school supply companies.) Thus, when the children build trains and explore combinations using the magnetic board, the strips will adhere to the board and remain in position. This board can also be used in back of the individual booklet pages. That is, the text page may be placed on the board and the strips will adhere to the page due to the magnetism of the board underneath.



For further reference on ways in which the strips may be used, see *Mathematical Awareness* by John V. Trivett, (New York: Cuisenaire Company of America, Inc.)

The Hundred Board suggested in the materials list and on page e-16 is an excellent device for use in counting and ordering activities. The abacus is also an excellent device for reinforcing understanding of place value concepts.

Evaluation of Progress

How well the children understand place value may be evaluated by their responses to exercises of the type found in the module review on page e-23. While many children can tell how many tens and ones there are in a given two-digit numeral, the real test of understanding comes when they are asked to compare various two-digit numbers.

Resources for Active Learning

General Activities:

Nuffield Project: COMPUTATION AND STRUCTURE (2), "Counting Toward Addition," pp. 42–57, Wiley

Probability:

MATHEX: Matching and Graphing No. 1, pp. 30–35, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: PROBABILITY AND STATISTICS, Wiley

Manipulative Devices:

Abacus or abacus board (Educational Teaching Aids; school supplier)

Chips with hole in centre (Educational Teaching Aids; Selective Educational Equipment)

Cuisenaire Rods

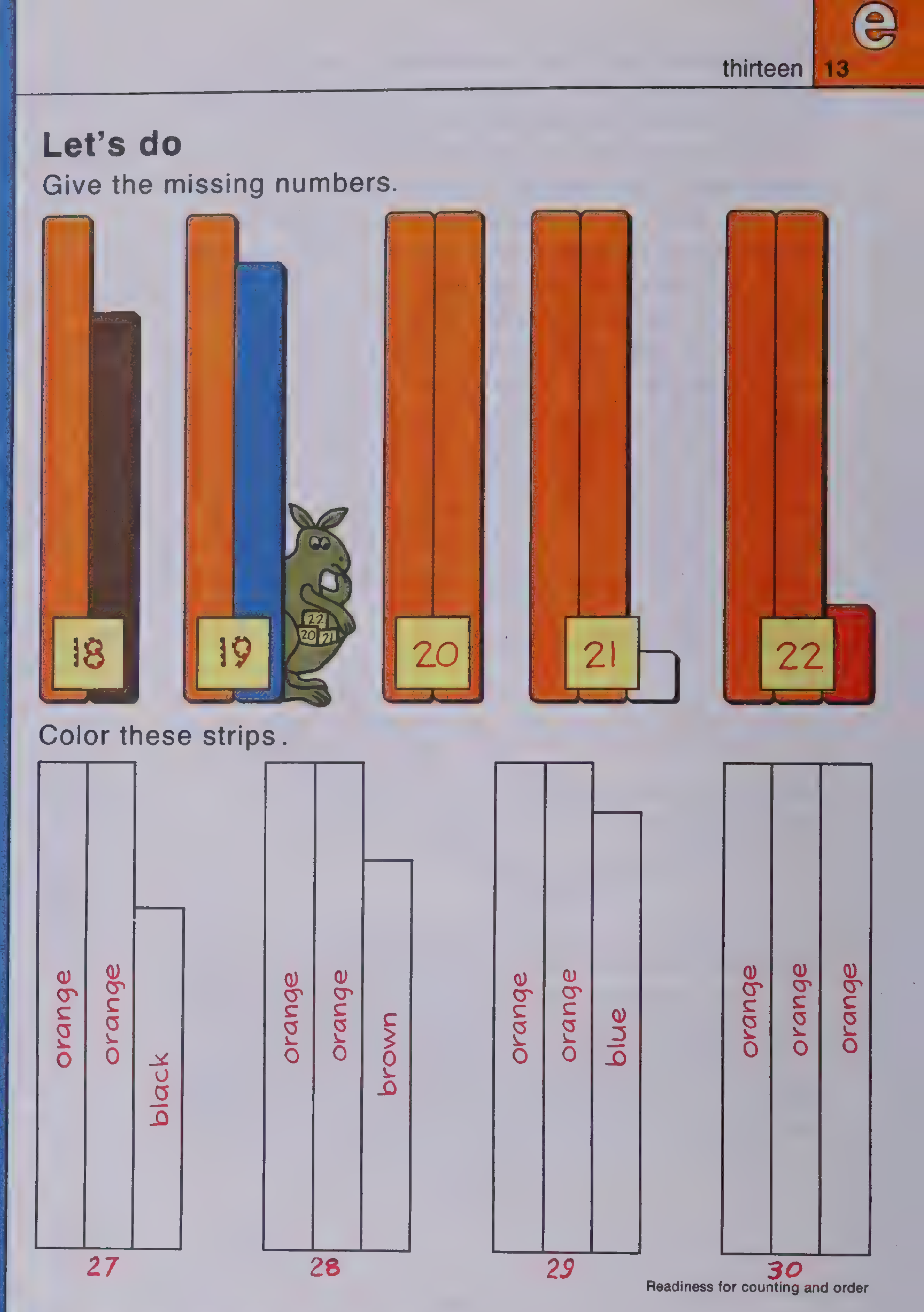
Flip and Build cards (Teaching Resources)

Hundred number board (Hammett; Learning Research Assoc.; Mafex Assoc.)

Hundred peg board and cylinders (Educational Teaching Aids; Responsive Environments Corp.)

Investigation
Page e-13

Ask the children to use the orange strip and one other strip to show the number 11, the number 12, and so on, until children have built the numbers from 11 to 18. Then call their attention to page e-13. Have the children build the numbers by placing their strips on top of the illustrations. Explain that they should write the number represented by each group of strips in the yellow space provided. After children have built and discussed the numbers to 22, point out the blank spaces at the bottom. Explain that in the bottom section spaces for building certain numbers are provided. They should use the orange strips and another strip, or the orange strips and white strips, to place on the outlines and then color the outlines accordingly. Also suggest that they write the numeral for each number under the pictured strips. Notice that this investigation suggests that children use either the orange strips and one other, or the orange strips and the white strips. Point out what occurs if they use orange strips and white strips. That is, ask the children to build the number 28 and then 29 and 30 with these strips. Then elicit from them what they did when they counted from 29 to 30. Some children may immediately choose a third orange strip to use after the nine white strips for 29. Others may count one more white strip and need encouragement to replace these ten white strips with one orange strip.



Purpose

To strengthen understanding of the process of counting from one decade into another

The correct use of the strips can play a vital role in helping children develop concepts of the place value system. It is recommended that the activities suggested here in the teacher's edition be developed extensively to accompany the work on the printed page.

Preparation

Materials

1 set of strips for each child; crayons

Distribute the strips so that each child has a complete set. Encourage the children to arrange the strips on their desks or table in any way they wish. For example, some may make a design or the shape of an animal or tree or house; others may arrange them in order of length, building various kinds of "staircases." Allow free play of this kind depending upon the children's previous experience with the strips. Those children who worked with the strips as a part of the Book 1 program of this series may simply enjoy an opportunity to once again familiarize themselves with the strips. Others may need more free play with the strips before beginning the investigation.

In either case, review with the children the number of each strip if the white strip is thought of as the unit, that is, as "1."

Let's talk

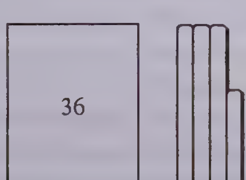
Readiness for counting and order

Discussion
Page e-14

The illustrations on this page are intended to be used as a basis for counting, for discussing more than and less than when no counting is needed, and for comparing the size of two sets by matching or by counting. Call the children's attention to the illustration of the stepping stones at the top of the page. Count together with them. Then ask questions such as, "How many stepping stones are there before the bridge?" "the log?" "How many stepping stones in all?" The point here is to associate counting with determining how many there are in a set. As you discuss the middle section, ask children whether there are more apples than plums. Here the children should be able to tell which set has more simply by observation. In the bottom section, children are again comparing two sets. However, here it is not obvious by sight which set has more. Stress that with these two sets, it is necessary to match or to count in order to tell which set has more. Children might draw lines from a frog to a fish to make a one-to-one matching. When they discover that they have used up the frogs but still have one fish not matched, they should realize that there are more fish than frogs. The frogs and fish may also be compared simply by counting both sets; the actual physical act of drawing lines is not essential.

Follow-up

Distribute a set of strips to each child. Give the children numbers, either orally or written on cards, to build with their strips. For each number the child should use his strips to show its place value meaning. You might want some children to record their choice of strips either by completing a phrase such as "__________ orange and a __________" or by tracing the shape of the strips used. Sample:



3 orange and
a dark green

Resources for Active Learning

MATH ACTIVITIES, "Jump Board," Game 2–28, pp. 35–36, Allyn and Bacon

Mathex: Numeration No. 2, "Activities . . . Greater Than Ten," pp. 8–9, Encyclopaedia Britannica Publications Ltd.

Teaching
Page e-15

Call the children's attention to the sequence in the top row. Ask them to trace over the dashed numerals and read the entire line together and then discuss what to write in the last two boxes. Point out that in the other rows they are to fill in the missing numerals.

Give the children an opportunity to complete the second row and then read the numbers in that row aloud, so that each child can check his paper.

Direct the class to complete the rest of the page.

Write the missing numerals.

0	1	2	3	4	5	6	7	8
5	6	7	8	9	10	11	12	13
12	13	14	15	16	17	18	19	20
16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33
29	30	31	32	33	34	35	36	37
36	37	38	39	40	41	42	43	44
48	49	50	51	52	53	54	55	56
57	58	59	60	61	62	63	64	65

Objective

Given a sequence of numbers, the greatest of which is less than 100, the child will be able to complete the sequence in order.

Continue to supplement the printed page with strip activities and other activities such as those suggested in the follow-up.

Pre-Book Activity

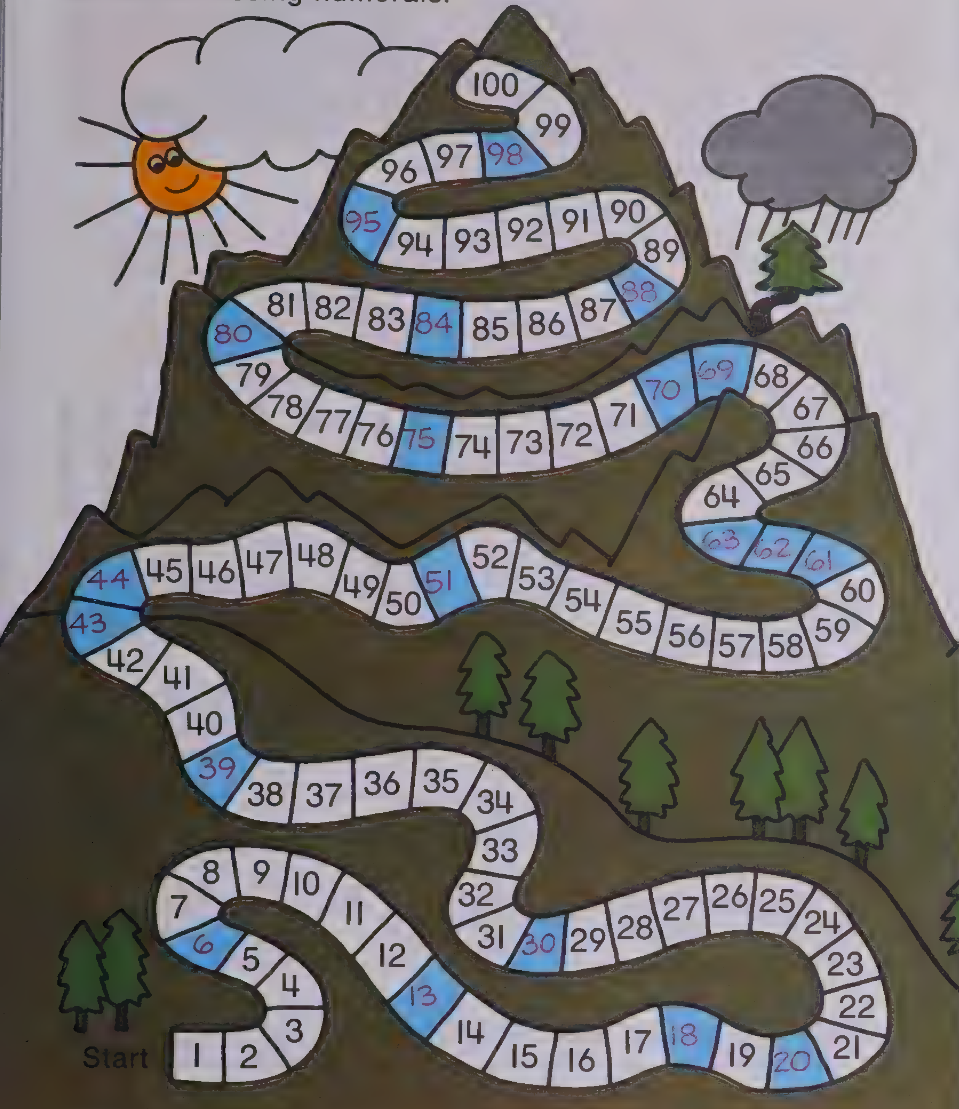
Involve the children in a short oral activity on numbers that come just before or just after a given number. For example, ask what comes just after 55, 68, 39, 70. Next, ask the class what comes just before these numbers.

You might also give the children practice in reading short sequences of two-digit numerals. For example, write 56, 57, 58, 59, 60, 61, 62, 63 on the chalkboard, and ask the children to read this sequence together.

Follow-up

If an abacus is available, use it with the children when they enter the room each morning. For example, as each child comes in, he should flip over one disc on the abacus. The tenth child who comes in will have to push one disc over on the tens' place and return the discs in the ones' place to be ready to continue the counting. Then the eleventh child continues again to push a disc over on the ones' place, and so on. Such an activity not only reviews counting, but also reinforces understanding of place value.

Climb the mountain.
Write the missing numerals.

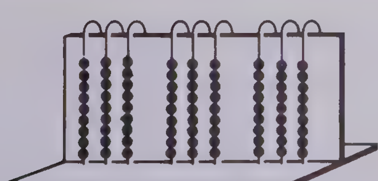


Counting

TEACHING
Page e-16

Ask the children to read the directions. Point out that they are to start at the bottom of the mountain, count by ones and write in the missing numerals.

Continue practice with counting and naming the various decades until most children can count rather routinely to 100. From time to time, you will probably want to use activities such as those suggested in the follow-up since the ability to count to 100 should be considered an ongoing objective.



A Hundred Board can be invaluable in working with sequences. Although many commercial varieties are available, you can make a sturdy one by using a 1-by-1-metre piece of plywood, cup hooks arranged in 10 rows of 10, and key tags, labelled 1 through 100. (Leave one side of each key tag blank to use later in working with multiples.)

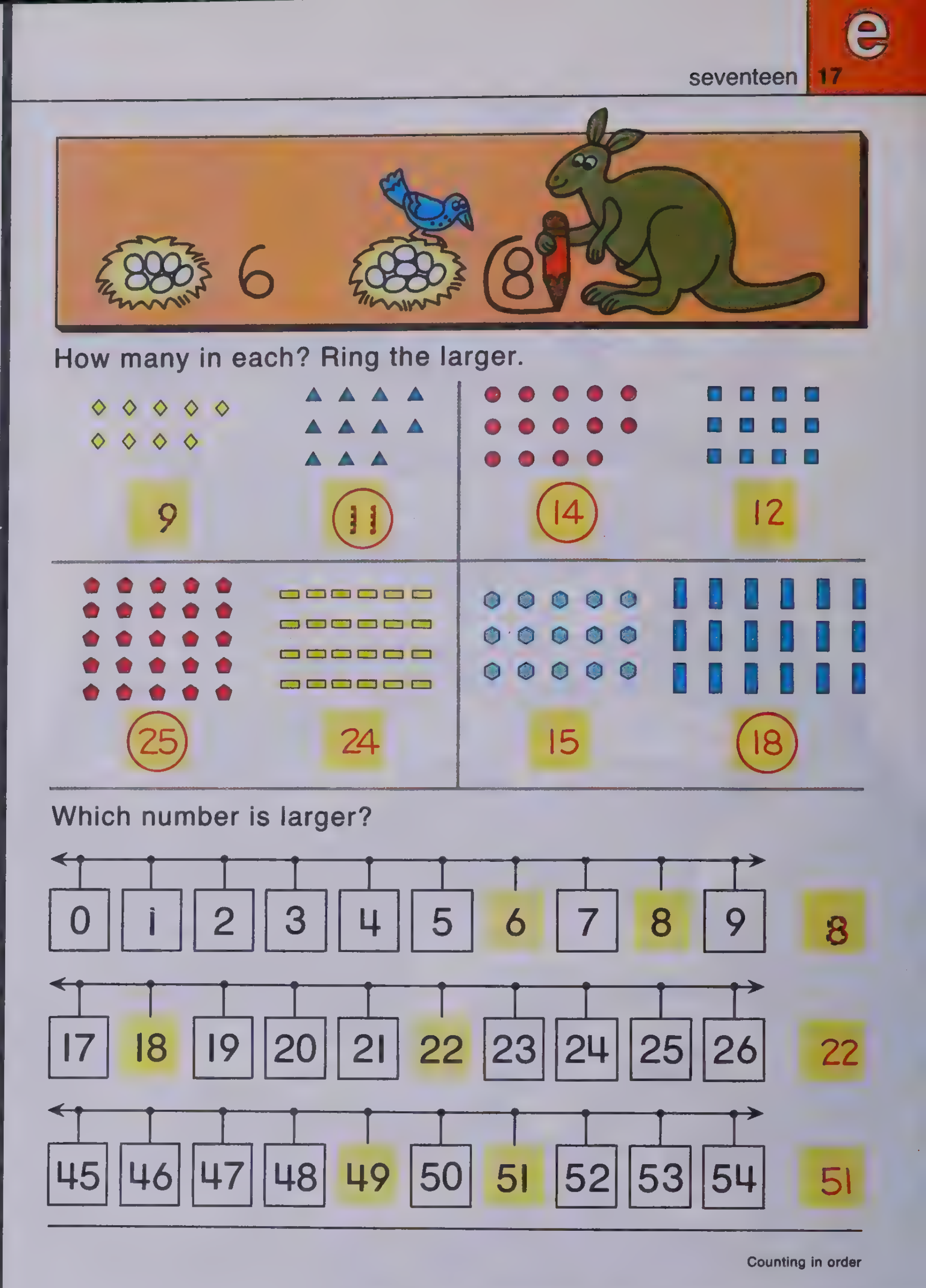
A Hundred Board can also be made of heavy poster or railroad board, 100 brass paper fasteners, and 100 tags made of bright-colored construction paper—approximately A-9 (37-by-52 mm)—punched near one end.

Begin the practice on sequences by exposing a tag labelled 45, for example, and then asking what comes next. Allow the child who answers "forty-six" correctly to turn over the proper tag. Then use a number in a new decade and continue urging children to give the next number in order.

Teaching
Page e-17

Direct the children's attention to the first frame and ask them to figure out which set has more objects or elements. Point out the dashed numerals which give the number of each set. When the children have traced over the 9 and 11, explain that they should put a ring around the number of the larger set. You might choose to work the remaining frames together with the children and continue discussing ways of determining which set is the greater set.

In the bottom section of the page, explain that each number line shows a part of the number line. Two numbers on each line are shaded yellow. The children are to compare these two numbers and answer the question: "Which number is larger?" Work the first example together; be sure they see how position on the number line can aid them in choosing the greater number.



Objective

Given two numbers both of which are less than 100, the child will be able to identify the greater number.

Pre-Book Activity

Display the Hundred Board described in the Follow-Up for page e-16 (or have available a pile of cards numbered from 1 to 100). Ask two children each to turn over a tag on the Hundred Board (or to display a card from the pile). Then ask the remaining children to write down which of the two numbers they think is greater. When all have agreed upon the greater number, continue by having other children choose two tags (or display two cards). If you think the children are capable, you might make this activity more challenging. For example, have one child turn over a tag on the hundred board. Then ask if anyone can choose and turn over a tag which will show a number greater than the first (or less than the first). In order to do this correctly, the children must be familiar with the order of numbers on the board. The same adaptation might be made with the cards if they are piled or spread out in order.

Which number is greater?

5	8	10	7	18	20
6	2	10	8	19	20
7	1	10	9	21	20
7	9	10	11	22	20
3	9	10	12	23	20

Which number is less?

14	17	27	30	25	28
15	17	28	30	27	24
16	17	29	30	26	36
18	17	31	30	35	25
19	17	32	30	28	32

Teaching
Page e-18

Explain to the children that in the top section they should ask of each number pair: "Which number is greater?" and ring the numeral which shows their choice. In the bottom section of the page, they should ask: "Which number is less?" and circle the numeral for the smaller number. Notice that in some columns, the tens do not change from one decade to another, and in some columns the ones' place remains constant. Finally, in the last frame both decades and ones are changed. You might want to point this out to some of the children as they work. As they compare the numbers, children should be encouraged to think of counting, 25 comes before 28 or 36 comes after 30. However, it would be helpful to point out that they might look at tens and ones places of two numbers, 2 tens and 7 is more than 2 tens and 4.

Follow-up

To provide another counting activity, duplicate the following type of exercise.

Fill in the missing numerals.							
27,	___,	29,	___,	31,	___,	33,	___
64,	65,	___,	___,	68,	69,	___,	___
32,	33,	34,	___,	___,	___,	38,	39
___,	___,	17,	___,	___,	20,	___,	___
___,	50,	___,	52,	___,	54,	___,	56
78,	79,	___,	___,	___,	___,	___,	85

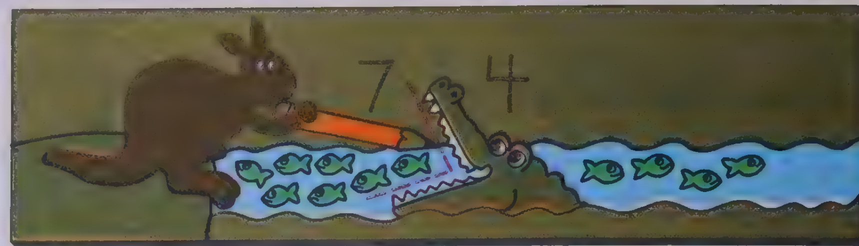
Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "One More," B$^{\underline{4}}$ 12, Addison-Wesley

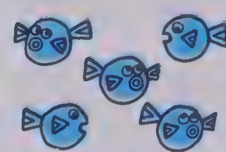
Teaching
Page e-19

Call attention to the demonstration art at the top and continue discussion of Chomper the Crocodile and his great appetite. Emphasize that the symbols are used to make it easy to show which of two numbers is greater or less than the other. Read the phrase: "7 is greater than 4."

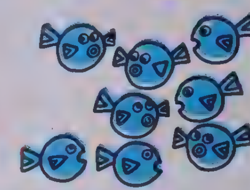
In the first frame have the children trace over the dashed numerals and symbol, and read with them: "5 is less than 8." Instruct them to complete the remaining frames.

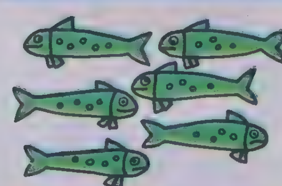


Tell how many fish. Then put the "mouth" in the .

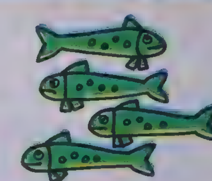


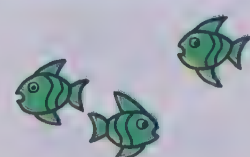
$5 < 8$



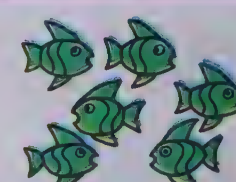


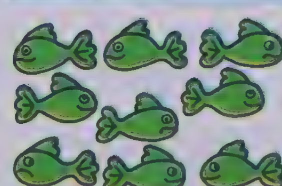
$6 > 4$



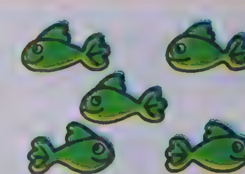


$3 < 6$





$9 > 5$



Objective

Given two numbers both of which are less than or equal to 20, the child will be able to indicate the greater number by using an inequality symbol, < (less than) or > (greater than).

It is important that children's first introduction to the meaning of the inequality symbols be accurate. The pre-book suggestion and demonstration art should be an aid to a careful development.

Pre-Book Activity

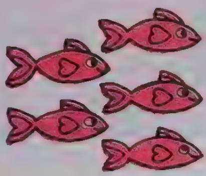
Use cardboard to cut out two open mouths (one left and one right) and have objects available for use in comparing sets. Then, to lead into the development on page e-19, tell the children a story of "Chomper, the Crocodile." For example, you might say that ever since the day Chomper was born he has had a tremendous appetite. Now, he is very big and needs as many fish as he can eat to fill his gigantic stomach. Display two sets of objects which might be thought of as fish, one set should obviously have fewer members than the other. Hold up the cardboard symbols and ask the children to pretend that these belong to Chomper. Then ask for a volunteer to show how Chomper's mouth would be positioned between the two sets, recalling that Chomper wants the greater number of fish. Write the number of the two sets on the chalkboard and place the appropriate inequality symbol between them. Then explain the manner of reading the symbol:

$12 > 10$ twelve is greater than ten

or

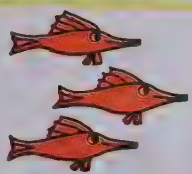
$10 < 12$ ten is less than twelve

How many in each set?



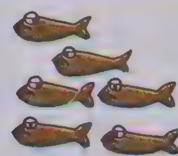
5 is greater than 2

5 > 2



3 is less than 6

3 < 6



Put > or < in each ○.

7 > 2	6 < 9	10 > 4
4 < 5	8 > 7	5 > 1
3 < 10	4 > 3	7 < 8
1 < 2	0 < 6	10 > 9
13 < 14	15 > 10	12 > 0
10 < 20	18 > 17	16 < 17

Teaching
Page e-20

As you discuss the first frame, help children relate the sets to the numerals 5 and 2. As they trace over the numerals with the greater than sign between them, stress that the symbol simply replaces the phrase "is greater than." Develop the second frame similarly, and help children read the statement $3 < 6$ as "3 is less than 6."

Next read the directions for the bottom section of the page for the children. Explain that they should first decide which number is greater and then place either < or > between each pair to complete a true statement. If you wish to check the children's work with them, write the completed statements on the chalkboard or overhead projector. An oral reading of the inequalities will not help any child who has repeatedly reversed the meaning of the two symbols.

Follow-up

To review both inequalities and order, play "Unscramble 100." Remove many tags from a Hundred Board and shake them up in a shoe box. Have two children each choose a tag from the box and show it to the rest of the class. Ask the two children to tell which number is smaller or which is larger and then hang the tags in their proper place on the Hundred Board. Call on the children, two at a time, until each child has had a turn.

Mathematics

For this lesson which requires the use of the inequality symbols, < and >, we present the following definition of less than and greater than.

If a and b are whole numbers, then saying that $a > b$ means that there is a whole number c greater than 0, such that $a + c = b$. $b < a$ means that $a > b$.

For example, saying that 27 is less than 36 means that there is a whole number, 9 in this case which, when added to 27, equals 36.

Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB, Cards 24–27, Responsive Environments Corp.

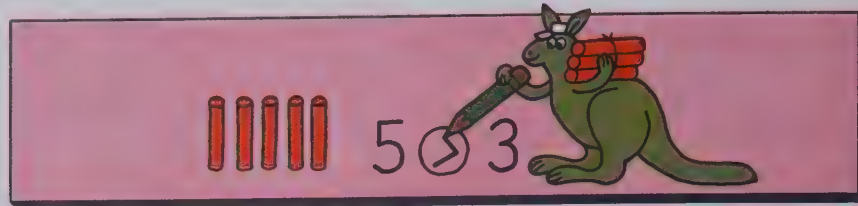
IT'S A TANGRAM WORLD, "Tangram Math," pp. 18–22, Educational Science Consultants

MATHEX: Numeration No. 2, "Order Relations," pp. 18–21, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: MATHEMATICS BEGINS 1, "Recording . . .," pp. 39–41, Wiley

Teaching
Page e-21

Call the children's attention to the sets of 4 and 3. Elicit from them that 4 is greater than 3 and direct them to place the appropriate inequality symbol on the yellow space. Review with them that > means *greater than* and < means *less than*. Then discuss that 4 tens is greater than 3 tens. If possible use actual objects to show these sets. You might stress that in comparing two numerals, children should first look at the tens' place. Only if these tens' places are the same need they be concerned about comparing the ones' place. Finally discuss that 40 is greater than 30 and relate it to 4 > 3 and 4 tens is greater than 3 tens. As they work on page e-21, they should realize that, since all of the numerals have zero in the ones' place they need only compare the tens' places.



Put > or < in each .

4 > 3

4 tens > 3 tens

40 > 30

40 > 10	20 < 50	80 < 90
40 > 20	50 > 20	70 < 90
30 < 40	50 > 40	60 < 90
10 < 40	50 < 60	90 > 60
10 < 50	50 < 70	90 > 70
10 < 60	80 > 50	70 < 80

Less than and greater than

Objective

Given a pair of numbers both of which are less than 100, the child will be able to indicate the greater number by placing between them the correct symbol > or <.

Consider the correct use of the inequality symbols as on-going objective. Observe the children carefully to catch any misuse of the symbols before incorrect habits are formed.

Pre-Book Activity

Materials

objects in bundles of ten and singles

Involve the children in an oral activity such as the "What's My Rule?" game. Ask a child for two numbers less than 100. When he gives a pair of numbers, repeat the larger of the two (your rule). When the children discover your rule, they must not tell it, but they should show they know it by folding their arms across their chests. When called on, a child who knows the rule should ask the teacher for a pair of numbers less than 100 and respond with the larger of the two numbers, showing that he knows the rule. Continue until most children know the rule.

Put > or < in each ○

40 > 30	50 > 20	30 < 80
45 > 35	51 > 21	36 < 86
47 > 37	56 > 26	39 < 89
2 < 6	7 > 4	5 > 1
32 < 36	27 > 24	65 > 61
52 < 56	87 > 84	95 > 91
39 < 40	19 < 20	84 > 78
40 > 39	18 < 19	56 < 62
50 > 49	20 > 19	73 < 80
49 < 50	21 > 19	44 > 39
48 < 51	24 > 19	57 < 60

Teaching
Page e-22

Read the directions at the top of the page. You might want to work through several pairs together with the children. Notice the development: in the top frame the numbers of each pair differ only in the tens' place; in the second frame the numbers of each pair differ only in the ones' place. Use these exercises to help children realize that in comparing numbers they should think about tens' places first and only if these places are the same do they need to compare the ones' places. Direct them to complete the bottom section of the page by themselves.

Follow-up

Many children will need further practice in the use of the inequality symbols for comparing numbers. A duplicating master such as the one in the next column would be helpful for those who found page e-22 difficult. For others simply use examples similar to those on the bottom of page e-22.

Use > or < in each ○.

2 < 5	thus	12 < 15
6 > 3	thus	16 ○ 13
8 < 9	thus	38 ○ 39
5 ○ 4	thus	65 ○ 64
7 ○ 8	thus	47 ○ 48
20 < 50	thus	26 < 56
60 > 30	thus	65 ○ 35
80 ○ 90	thus	81 ○ 91
50 ○ 40	thus	53 ○ 48
70 ○ 80	thus	74 ○ 89

Teaching
Page e-23

Call the children's attention to the sequences in the top row. Point out that the first few numbers of each sequence are given and they are to continue the counting and write the missing numerals. Then explain that between each pair of numbers given in the bottom section of the page they should place the correct inequality symbol. Remind them that > means greater than, and < means less than.

Show you know

Write the missing numerals.

14	15	16	17				
37	38	39					
76	77	78					

Put > or < in each .

9 > 2	12 < 15	20 > 18
3 < 8	16 > 11	20 > 19
0 < 5	11 < 12	20 < 21
6 > 2	13 > 10	20 < 22
7 > 6	9 < 11	38 < 40
8 > 0	19 < 20	38 < 41
1 < 4	19 > 18	42 > 39

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Often it is important to separately evaluate a child's ability to compare numbers from his ability to use the symbol correctly. Oral discussion will aid you in such evaluation.

Pre-Book Activity

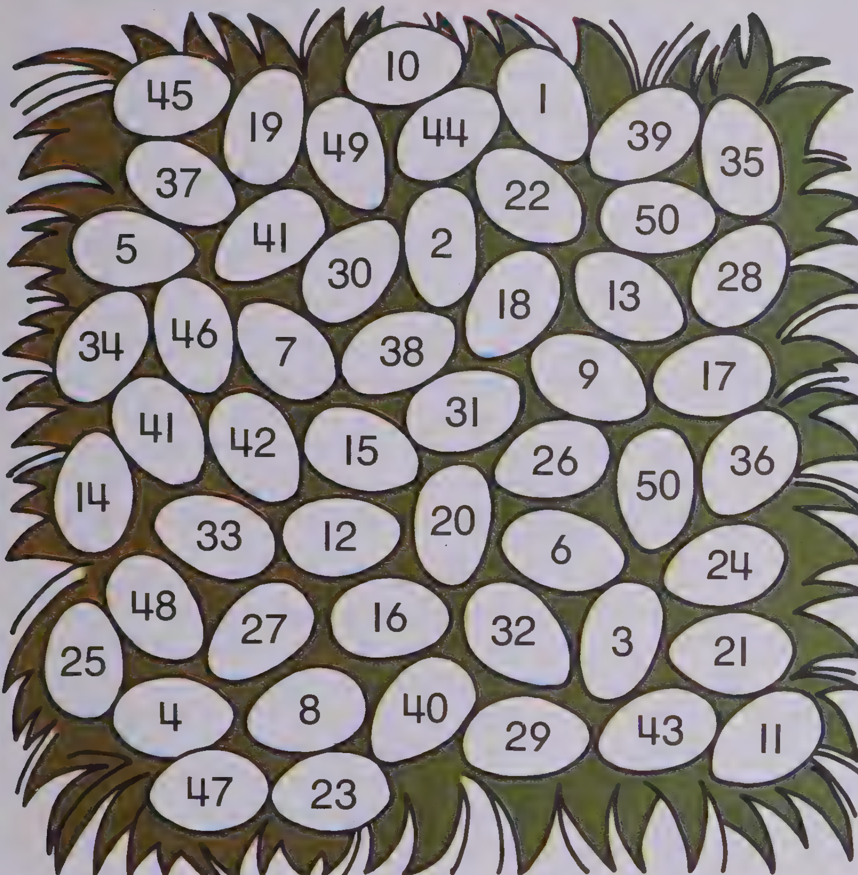
It would be helpful to use a variety of activities to review counting and comparing numbers. For example, write large numerals 0 to 99 on large index cards. Shuffle the cards and hold them so that the numerals are not visible. Ask six children to come up, draw a card, order themselves smallest to largest, and then face the class and display their cards. The class should judge whether the ordering is accurate. Allow these six children to choose six other children to continue the game. Play until everyone gets a turn. Then increase the number of cards to be ordered to eight or ten, thus giving each child at least one more chance to participate and a larger set to be ordered.

Follow-up

Prepare thirty small index cards, half of them stating some number of tens and ones (such as: "4 tens and 6") and the other half showing the corresponding numerals (46). Glue three rows of library-card pockets, (5 in each row) on two sheets of brightly colored poster board. Label one set of pockets with numerals 1 through

Let's have fun

Two of the eggs do not belong in this nest.
What are their numbers? 41 and 50



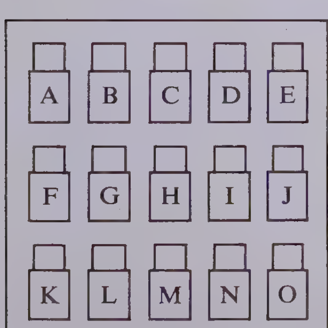
Counting puzzle

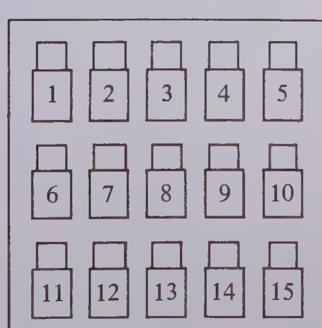
Teaching
Page e-24

Give careful directions to the children for this change of pace page. You might explain that two of the eggs in the box are duck eggs and do not belong. They are to try and find out which two these are. Explain that the eggs which do not belong are eggs which repeat a number that is already on another egg. Give them a chance to discover a systematic way to find the 'misfit' eggs. One way would be to start counting and marking out the eggs starting at 1. When all the children finish, ask them how many eggs there are in all. This might be a tricky question for some; although eggs are numbered from 1 to 50, since there are two 50's and two 41's, there are 52 eggs in all!

15 and the other set with letters A through O.

Shuffle one set of cards and place them face down in the pockets of either sheet. Then shuffle the other pack and place the cards in the remaining pockets.





To play the game, hang the sheets where everyone can see them. Direct a child to choose a pocket on the left sheet and then a pocket on the right. He (or the teacher) removes the pair of cards and shows them to the class. If they match, a point is scored. If they do not match, they are returned to their pockets, face down, and the other team gets a turn. Try to choose two evenly matched teams to make the game fair. The basic game equipment is worth making carefully, since children are highly motivated by this activity, and the game can easily be adapted to review a great many skills and concepts by changing the basic set of cards.

Resources for Active Learning

A CLOUDBURST, Vol. 1, No. 8721, Midwest Publications

Money

Pages e-25 to e-32

General Objectives

To review the value of the penny, nickel, dime, and quarter

To develop skill in finding the value of a collection of coins less than one dollar by counting

To develop skill in making change with amounts of 25¢ or less

To provide activities in purchasing items for less than one dollar

To introduce Roman numerals

The material presented in this module should be considered the minimum to a study which might be continued and extended periodically throughout the entire year. It is not intended that children achieve all of the objectives by the end of the module. Skill in working with coins should be an ongoing objective throughout the school year.

We cannot emphasize enough that the technique stressed throughout this entire module by which children can find the value of a coin collection is *strictly one of counting*. It is in no way intended that addition or subtraction, even of basic facts, be taught at this time. All of the material in this module, that is, both the activities suggested and the printed pages, may be successfully completed by dependence on the counting technique. Later in the year, the use of exercises with coins to reinforce addition and subtraction concepts might be appropriate, but at this time only the counting technique should be used.

Teaching Red Module, Unit E

Approximate Time: 5 to 7 days

Materials

charts of items, toys, etc., priced less than a dollar

materials for a play store: large cartons for shelves, small boxes, labels

a supply of play-money coins

tagboard: 22-by-28-centimetre pieces prepared as described on page e-29, envelopes

Although the most beneficial way to help children develop skill with money is to have them use real coins, this is not always practical or even possible. Therefore, a collection of printed coins to be punched out and used by each child can be purchased separately. It is suggested that you supply each child with an envelope for storing his coins. Also on this same punchout card are the face and hands of a clock to be used with the Light Green Module, Unit E.

One of the most important activities of this module is playing store. To help children become thoroughly involved in this activity, ask them to bring empty containers or used items from home to label and use at the store. It is suggested that the values marked be less than a dollar and that children be expected to count change only to 25¢ or less. However, if the children are very familiar with the use of money, adapt these suggestions to their ability.

Evaluation of Progress

Your best evaluation of a child's understanding and level of skill will come from your observation of his behavior during the activities. However, a child who successfully completes the printed material is sufficiently demonstrating his ability to deal with coin values. Page e-31 reviews the basic concepts covered in the module and may be used as an aid in your evaluation.

Resources for Active Learning

General Activities:

MATHEX: Measurement and Estimation No. 5, "Money," pp. 39–43, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: COMPUTATION AND STRUCTURE 2, "Money," pp. 91–102, Wiley

Manipulative Devices:

Store play materials (CCM School Materials, school supplier)

Commercial Games:

Count Your Change Game (Lakeshore; school supplier)

Spin-a-Coin (Creative Publications; Creative Teaching Assn.)

Investigation
Page e-25

Point out the illustrated coins at the top of the page. Identify each coin by name and give the value for each. Then explain to the children that the picture on the centre of this page is called a graph. And the number of coins in each column can be read from the numbers on the left. Explain that they are to color one bar for each coin, starting at the bottom, to show how many of each coin they have among their punchouts. As soon as children understand what they are to do, encourage them to work independently. If some children are not familiar with the names and values of the coins, give them guidance, but do not over-emphasize. The purpose of this investigation is simply to introduce a topic which might very well be treated as a subject for ongoing study throughout the year. Since the investigation also introduces the child to work with a graph, point out the columns. Ask questions such as, "How many nickels do you have?" "Are there more pennies than there are dimes?" "Which coin do you have most of?"

Let's do

Penny	Nickel	Dime	Quarter

1 cent	5 cents	10 cents	25 cents
1¢	5¢	10¢	25¢

Can you complete the **graph** to show how many "coins" you have?

Purposes

To introduce the coins: penny, nickel, dime and quarter
To provide an experience in working with a bar graph

Preparation

Materials

punchout coins which can be purchased separately to accompany this series, or play-money coins

Distribute an envelope to each child for storing his coins. It might also be helpful for each child to put his initials on his envelope so everyone can keep their own coins after playing store. Then give out either play-money coins or the coins from the punchout cards that can be purchased to accompany this book. Each envelope should contain 4 quarters, 8 nickels, 10 dimes and 16 pennies.

Let's talk

Readiness for work with coins

Discussion
Page e-26

The items on this page are intended to be used as material for a discussion of the value of the coins and which coins may be used to purchase these items. It is in no way intended that addition combinations be introduced here or any where throughout this module; counting techniques only should be used. Use questions such as, "What coins could you use to buy a model airplane?" "If you used a dime to buy a whistle, how much change would you get?" "If you used a quarter to buy a kite, can you count out the change you would get?" Notice that all of these questions may be answered simply by using counting techniques. Demonstrate, or have children who are capable, demonstrate a variety of ways of counting with the punchout coins up to 25¢. Examples might be 10, 20, 25, or 10, 15, 20, 25 and so on. Also show how change may be counted beginning with the price of the item and counting with coins up to 25¢. For example, if the item purchased is a 12¢ kite, the clerk might count "13, 14, 15, 25," using 3 pennies and a dime.

Follow-up

Many children will benefit from a play-store in which they will have the opportunity to deal with coins. Encourage those who know the coin values and those who can count out change to work as store clerks, and help others with the use of coins. If enough children are not familiar with counting change, you might be "store keeper" for a few minutes before every math lesson. Have a child choose an item to purchase with some of his coins (use small values at first). Then demonstrate the transaction while you count aloud giving him change. Thus, if he gives you two dimes to purchase an item priced at 12 cents, as you give him the item and coins, you would say, "That's 13, 14, 15, 20 cents."

Such a store may be constructed from large cartons and items brought in by the children. Empty milk cartons, candy wrappers, and small boxes may be labelled with appropriate price tags.

Resources for Active Learning

Nuffield Project: BEGINNINGS 1, "Imitative Play," pp. 24–29, Wiley

Teaching
Page e-27

Call the children's attention to the first frame. Ask them to identify the coins they see and to tell how they would count them to find the value of the whole collection. Suggest that they begin with the coin of greatest value, the nickel, and count 5, 6, 7, 8, 9 cents. Point out the cent symbol and the yellow box in which they should write the value of the collection. You might want to work other frames together with the children. For example, in the frame containing two nickels and three pennies, point out how to count the two nickels saying 5, 10, 11, 12, 13 cents. Also stress that if they become confused counting the nickels, they can simply think of a nickel as five pennies and count by ones. Help children realize that this is true for other coins as well. Children who have difficulty counting the nickels as 5 or the dimes as 10, should benefit from the follow-up activity.

Find the value of each coin collection.

Coin collections

Objective

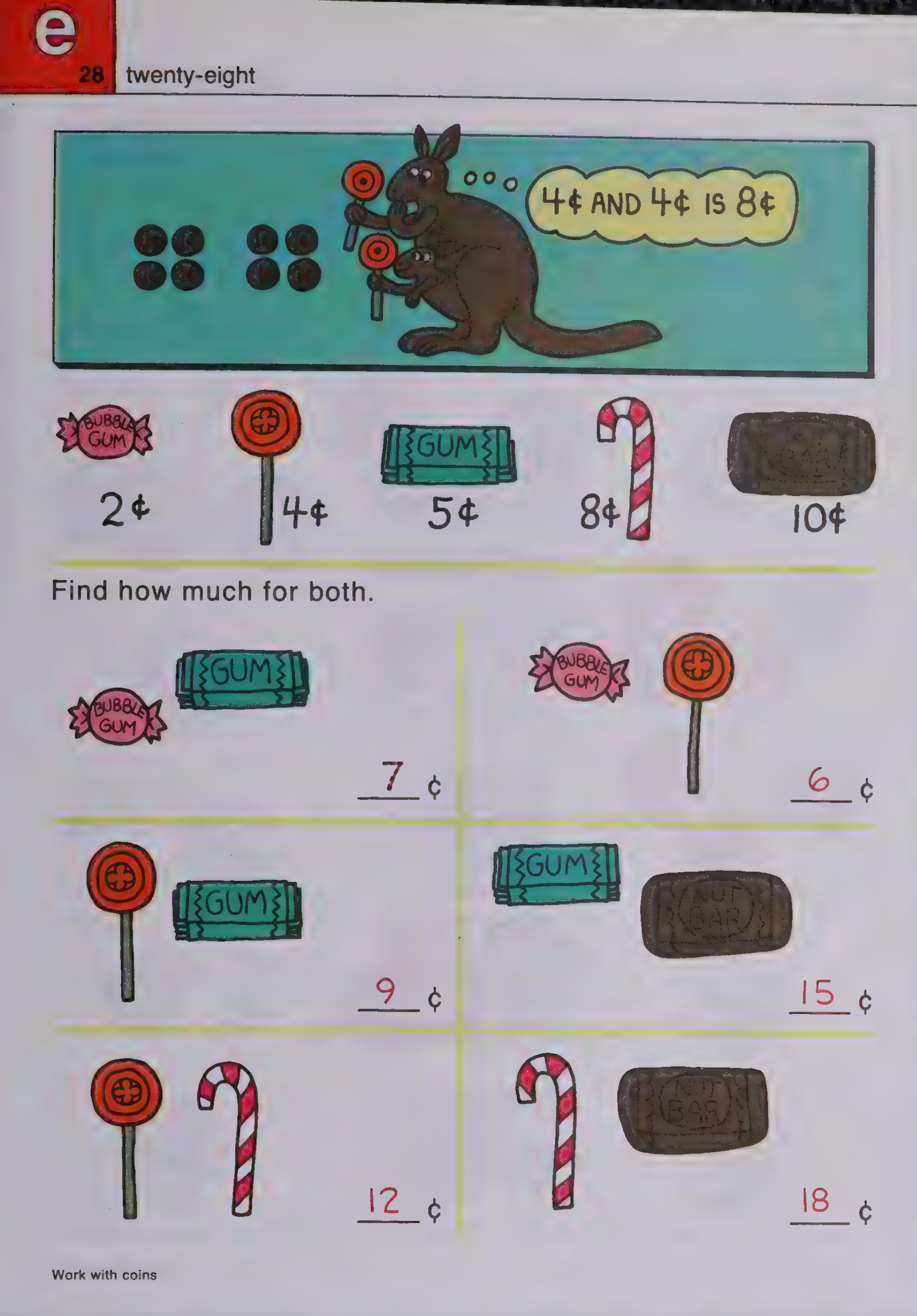
Given a collection of coins the value of which is 35 cents or less, the child will be able to find the value of the coin collection by counting.

Encourage many store activities and much individual work with their play coins. Children will grow in their understanding of coin values from such experiences more than from work on the printed page.

Pre-Book Activity

As suggested in the follow-up for the previous lesson, you might find it helpful to demonstrate the process of counting change. If you think the children have need for such a demonstration, remember to at first limit the amount available to be 'spent' to 25¢ or less. Gradually let volunteers take over the role of storekeeper.

To introduce the children to the counting of coin collections, direct them to form groups of two or three. Then ask them each to take any coins they want from their envelope of play-money coins. They should then combine them with the coins of the other members in their group. Then the group should try to figure out how many coins they have. As you observe them counting their coins, watch whether they are arranging all the dimes together, the nickels together, and so on, and whether they are placing these groups in order of value. For those who are not being so orderly, give hints to help them sort and arrange their collections. As time permits, you might encourage them to leave their collection arranged on their table and move to examine the collection of another group.



e 28 twenty-eight

4¢ AND 4¢ IS 8¢

2¢ 4¢ 5¢ 8¢ 10¢

Find how much for both.

7 ¢

6 ¢

9 ¢

15 ¢

12 ¢

18 ¢

Work with coins

Teaching
Page e-28

Ask children to identify the illustrated items shown at the top and give their cost. Then explain that each frame shows two items and that to find the total cost they should figure out what coins they might use to buy these items. Thus, for example, in the first frame they could use two pennies for bubble gum and a nickel for a pack of gum. They should then count these coins to find how much money both items cost together. Only after they have counted the actual coins should they write in the yellow box the total cost of the items.

Follow-up

Many children will need practice in counting nickels and dimes by fives and tens. Ask the children to lay out only their play-money nickels and count with them 5, 10, 15, 20, 25, 30, 35, 40. Then ask them to lay out their dimes and count by tens. Finally introduce the children to a game you might call "Can You Guess My Coins?" Although children may eventually play this in partners, begin with yourself as leader and challenge the children as a group. Choose a collection of dimes and nickels. Count these coins aloud and challenge the children to guess what coins you have. For example, if you have 4 dimes and 3 nickels, you might count 10, 20, 30, 40, 45, 50, 55 cents. As you count aloud, children might use their coins and choose nickels and dimes to match your counting. Thus for this example, as you say 10, they would separate a dime from their collection of nickels and dimes into another area, as you say 20 they separate another dime, and so on. Then they separate nickels in the same way until they have four dimes and three nickels. Be sure to keep the value of your collection below a dollar. Also begin with just a few coins, gradually building up to more complicated collections.

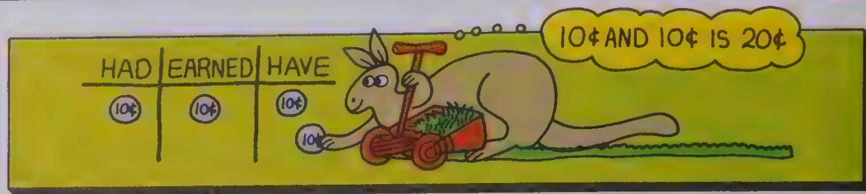
Resources for Active Learning

Franklin Series: LEARNING ABOUT MEASUREMENT, "Money," p. 98, Lyons and Carnahan

WORKJOBS, "The Money Game," pp. 224–225; "The Store," pp. 226–227, Addison-Wesley

Teaching
Page e-29

Read the line of directions with the children. Then point out the three columns headed "Had," "Earned," and "Have." Relate a story such as: "Jill had a nickel; then she earned 7 cents for running an errand. How much does she have now?" Help them begin with the nickel in the "Had" column and count "5," move to the "earned" column and continue "10, 11, 12 cents." Then point out that they should write 12¢ in the "Have" column. Work through another frame if necessary, but encourage children to complete the table independently or with their group when they are able.



Complete the table.

Had	Earned	Have
		12 ¢
		14 ¢
		17 ¢
		33 ¢
		30 ¢

Combining collections of coins

Objectives

Given a first collection of coins and a second collection of coins earned, the child will be able to figure out the total value.

Given an item to buy and coins with a value of 25¢ or less, used for the purchase, the child will be able to figure out the change to be received.

Keep in mind that only a counting technique is needed for the children to figure out the total value of coin collections and the amounts of change treated in this lesson.

Pre-Book Activity

Materials

coins

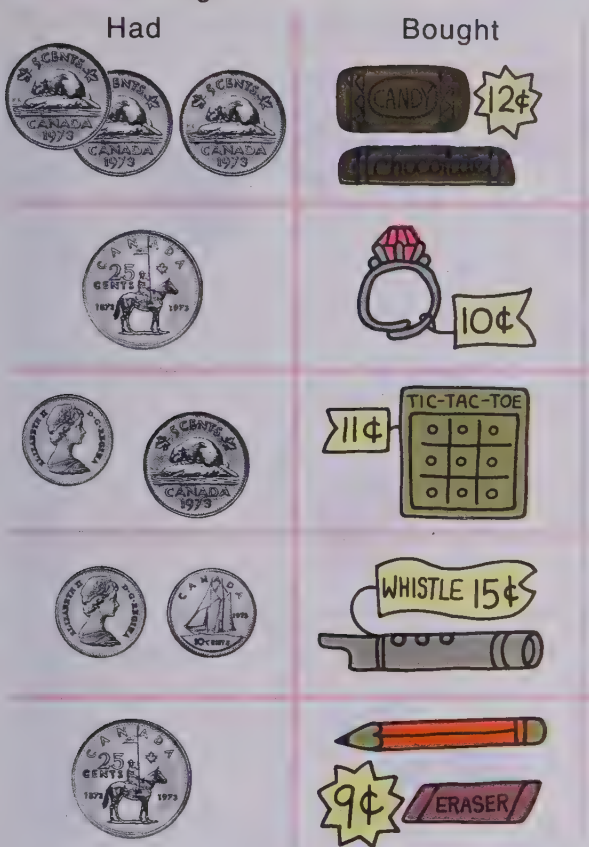
22-by-28-cm pieces of tagboard prepared as described below

Assign the children into six groups or teams. Prepare as many 22-by-28 pieces of tagboard as you need so that you have one for each group. On one side of each piece of tagboard, write (or draw) tasks or jobs such as those listed below. On the other side draw or tape coins to assign a monetary value to each task.

Closing the windows and shades	27¢
Emptying the wastepaper basket	22¢
Sweeping the floor	30¢
Straightening the tables and chairs	20¢
Stacking books	25¢
Raking leaves	17¢

Place these six cards along the chalk tray so that the

Find the change.



Change
3 ¢
15 ¢
4 ¢
5 ¢
16 ¢

Making change

Teaching
Page e-30

If children have been "playing store" and taking turns as storekeeper, they should not need other preparation for this page. Explain the headings of the three columns. Point out that the coins in the "Had" column represent the coins given to the clerk when purchasing the item in the "Bought" column. Work through the first frame to help the children find the change. For example, show how they might think of the price of the item purchased and continue counting with their assorted coins until the original amount as in the "Had" column is reached. Or you might want them to take coins as in the "Had" column, count out the amount purchased and see what's left. Suggest that they place the coins for their change right on the page in the frame for the change column. Ask them to write that amount in the space provided next to the cent symbol. If possible, encourage the children to complete the frames independently. However, if some still have difficulty counting change of 25¢ or less, work the frames together with them.

task side is showing. Meanwhile, direct each group of children to form a collection of coins so that they begin with an amount of 15 or 20 cents. Explain that each of the tasks is a way in which they might earn some money. Each team or group should pick one child to choose a card from the chalktray. As each group picks a card, ask someone in that group to begin with their 15¢ or 20¢ and continue counting to find their new total amount. When each group has done this, help them compare their totals. Point out that not only the amount earned, but the amount they had to begin with determined the greatest total.

Follow-up

To give the children more practice in determining the value of sets of coins, duplicate a work sheet like the one in the next column.

Put a ring around the set that buys more.		
25 10 5	or	10 10 10
5 5 25 25	or	25 10 10 10 10
25 10 10 5	or	25 25 5
25 5 10 25	or	25 25 10
10 10 10 10 10 5	or	25 25

Remember that the value of the set should be determined by counting coins.

Resources for Active Learning

Franklin Series: LEARNING ABOUT MEASUREMENT, "Making Change," pp. 99–101, Lyons and Carnahan

Teaching
Page e-31

Point out to the children that there are different sections on this page which review things they have just studied. Explain that for the top frames they are to try to find the value of each collection. Emphasize that they should count the coins illustrated or that they can use their assorted coins to make matching sets and count these. When they have decided on the value of the collection, they should write the number of cents in the space provided. In the bottom sections of the page, they should carefully read the column headings before they put down an answer. In the first set of frames, they need only begin counting the coins under "Had" and then continue counting the coins in "Earned" column to find the value that should be written in the "Have" column.

The last set of frames is similar to those on page e-30. Give explanations as necessary, but encourage children to do independently as much of this page as possible.

Show you know.

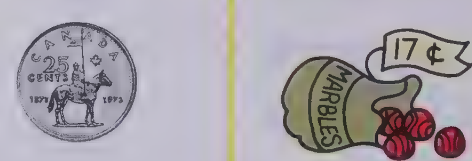
Find the value of each collection.

9 ¢ 12 ¢ 18 ¢ 30 ¢

Give the final amount.

Had	Earned	Have
		18 ¢

Had	Bought	Change
		8 ¢



Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Much of your evaluation of a child's understanding will come from your observation of his activities at the play store. Do not put undue emphasis on his performances on this written page.

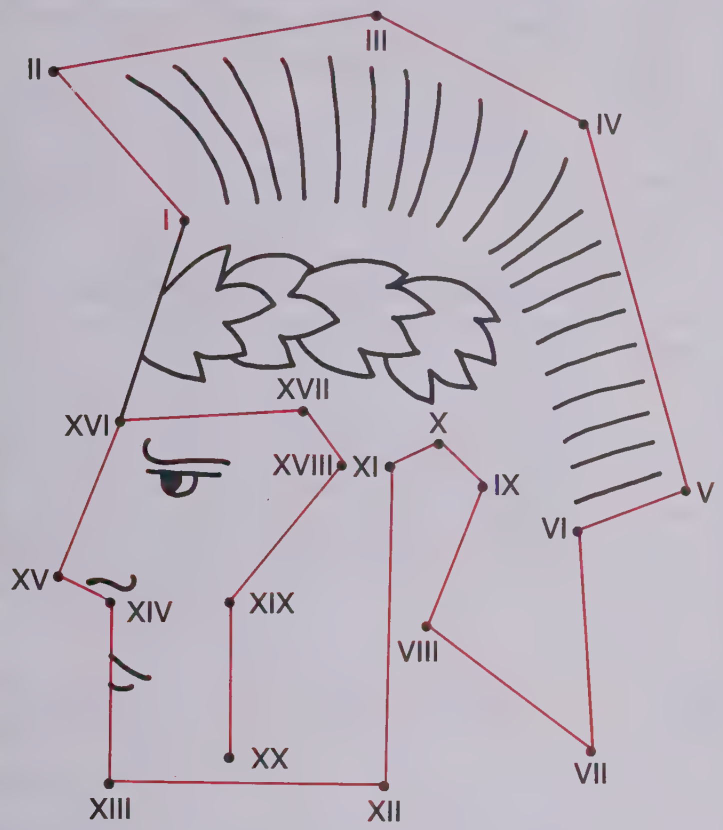
Pre-Book Activity

To help the children review how to count coin collections, have stationed around the room seven or eight collections (labelled 1, 2, 3 and so on) of nickels, dimes, pennies, and quarters in varying amounts (all less than one dollar). (You might tape the coins to cardboard.) Direct the children to form groups of three or four. To begin, assign each group to a coin collection. Explain that they should try to figure out the value of the coins in each collection. Give each child a piece of paper so that he can record the number of each station he studies and the value of the coins in each collection.

Let's have fun

Can you use the code to help you complete the dot picture?
This "code" uses **Roman numerals.**

1	I
2	II
3	III
4	IV
5	V
6	VI
7	VII
8	VIII
9	IX
10	X
11	XI
12	XII
13	XIII
14	XIV
15	XV
16	XVI
17	XVII
18	XVIII
19	XIX
20	XX



Teaching
Page e-32

Read the directions at the top of the page with the children. Point out the two columns of numerals and explain that this is the code they are to follow to complete the dot picture. You might write part of it on the chalkboard to explain how each of the numerals with which they are familiar corresponds to a different kind of numeral, called Roman numeral. They are to complete the dot picture by matching the Roman numerals in the code with their ordinary counting numerals (1, 2, 3, . . .) so that they can connect the Roman numerals in the dot picture in the correct order. Give guidance as necessary. This change of pace page is intended as a simple introduction to Roman numerals and should be treated with a light touch.

Follow-up

Before class prepare a chart showing pictures of five or six small toys priced under a dollar and eight or ten envelopes each containing coins totaling 30 to 99 cents. (The envelopes should be labelled with a letter keyed to the correct value of the coins inside, for your convenience.) Ask the children to choose one toy from the chart. Then give an envelope to each of two different children. Ask each child to find out how much his set of coins is worth and then tell the class the amount in his envelope. Ask the children to find the total amount in the two envelopes and to decide whether or not they can buy the chosen toy.

If you prefer, you might adapt this to a group activity by simply placing a chart of five or six items priced under a dollar at each of the stations used in the pre-book activity. Each group should work at a station to determine which items on the chart could be purchased with the coins at that station.

Resources for Active Learning

Connecting dots:

A CLOUDBURST. Vol. 1, No. 8231, Midwest Publications

ENRICHMENT OF ARITHMETIC, pp. 1/35–38, Webster, McGraw-Hill

Roman Numerals:

Franklin Series: LEARNING ABOUT MEASUREMENT, pp. 78–80, Lyons and Carnahan

MATHEX: Numeration No. 2, pp. 43–47, Encyclopaedia Britannica Publications Ltd.

Telling Time

Pages e-33 to e-40

General Objectives

To emphasize telling time to the hour and the half hour

To introduce the written notations for indicating time

To familiarize children with the vocabulary of telling time

To introduce the children to telling time in intervals of five minutes

To familiarize children with the use of Roman numerals

To continue, as an ongoing objective, to develop the children's ability to tell time

This module on telling time initiates an ongoing objective. Many children will already be familiar with the clock and not have difficulty in telling time to the hour and the half hour. However, some may have difficulty using the usual notation for writing time (such as 3:15), and telling time in intervals of five minutes past the hour. Children will not and should not be expected to master these new concepts by the time they have completed the module. This module should be considered an initiation for the study of telling time and work in this area should continue throughout the year.

Teaching Light Green Module, Unit E

Approximate Time: 4 to 6 days (or extended)

Materials

demonstration clock

cardboard clock, face and hands (available as separate punchout item)

Although this module is included here to help the children develop the skill of telling time, it is recommended that this skill be developed continuously throughout the year. Many of the activities which are suggested as follow-ups may be used periodically. Also at different times of the day, you might simply refer to the clock and ask children to tell what time it is, or you might write a message on the blackboard such as, "At 2:15, get your coats." or, "At 11:25 stand up." If this is done periodically throughout the year, children should develop the ability not only to tell time from the clock, but also to interpret the written notation. Punchout clocks with movable hands are available from Addison-Wesley, so that each child will be able to work with a clock and show times on his clock.

Resources for Active Learning

General Activities:

Time:

ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 2, pp. 48–59; Book 2, Unit 2, pp. 48–65, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Time," pp. 70–76, Herder and Herder

FREEDOM TO LEARN, pp. 134–137, Addison-Wesley

MATHEX: Measurement and Estimation No. 5, pp. 35–36, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: BEGINNINGS 1, pp. 92–96; COMPUTATION AND STRUCTURE 2, pp. 82–90, Wiley

Time—the calendar:

ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 2, pp. 62–65, Addison-Wesley

Franklin Series: LEARNING ABOUT MEASUREMENT, pp. 81–84, Lyons and Carnahan

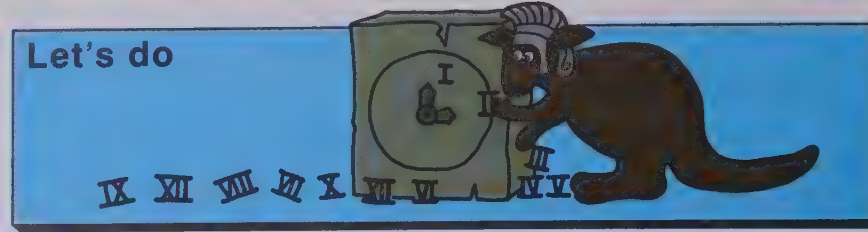
Manipulative Devices:

Egg timers (Creative Publications; dime store)

(Racemaster) stopwatch (Edmund Scientific; Math Media)

Investigation
Page e-33

As you introduce page e-33, explain that the clock shown has two sets of numerals. Explain that the main point of the investigation is to put Roman numerals on the clock face. Point out the code at the bottom. They should find the ordinary numeral and write the matching Roman numeral next to it on the clock. This investigation should serve simply as an introduction to an ongoing development of the skill of telling time. The Roman numerals need not be stressed although you might want to explain that they were used by the ancient Romans and today can still be seen in a few places such as on the cornerstone of a building and on some clocks.



Can you put numerals on your clock face?

Roman numerals

I	II	III	IV	V	VI	VII	VIII	IX	X	XI	XII
1	2	3	4	5	6	7	8	9	10	11	12

Purpose

To familiarize the children with telling time
To review reading time from a clock
To present Roman numerals as they are used in telling time

It is recommended that the punchout clocks available with this series be used in a variety of activities other than those suggested for this module.

Preparation

Materials

paper fasteners
cardboard clock face with movable hands

The individual clock faces are not essential to the investigation on page e-33, but may be used as preparation. Suggest the children set their clock at the time shown in the demonstration art on page e-33 and try to tell what time it is. Also use the cardboard clock for practice in showing time to the hour. For example, say: "Fix your clock so that it shows 6 o'clock," "9 o'clock," and so on.

Let's talk

Can you tell what you might be doing at these times?

Discussion
Page e-34

The clocks on this page are intended to provide you with a basis for a discussion of daily events which occur at various times. Include Saturdays and Sundays in your discussion as well as week days. For example, ask the children what time the clock in the first frame shows. When they respond, "Eight o'clock," ask if they can tell from the clock whether it's eight in the morning or eight o'clock in the evening. Point out that the clock could mean either time. Then suggest that they think of eight o'clock in the morning and ask them to explain some things which they might be doing at eight o'clock in the morning. Encourage much discussion, particularly to be sure that each child realizes that the clock itself does not tell them whether the time shown refers to morning or to evening.

Follow-up

Suggest that the children decide on an event during the day (or night) which they would like to pantomine. They might do this alone or with one or two others. Explain that they should decide on the time (hour) of this event and set a clock at this hour for all to see. Then they should perform the pantomime and the other children should try to guess whether the event takes place in the morning or evening (before noon or after) and what action is being pantomimed.

Resources for Active Learning

For activities involving Roman numerals refer to Red Module, page e-32 of this book.

Time – the clock:

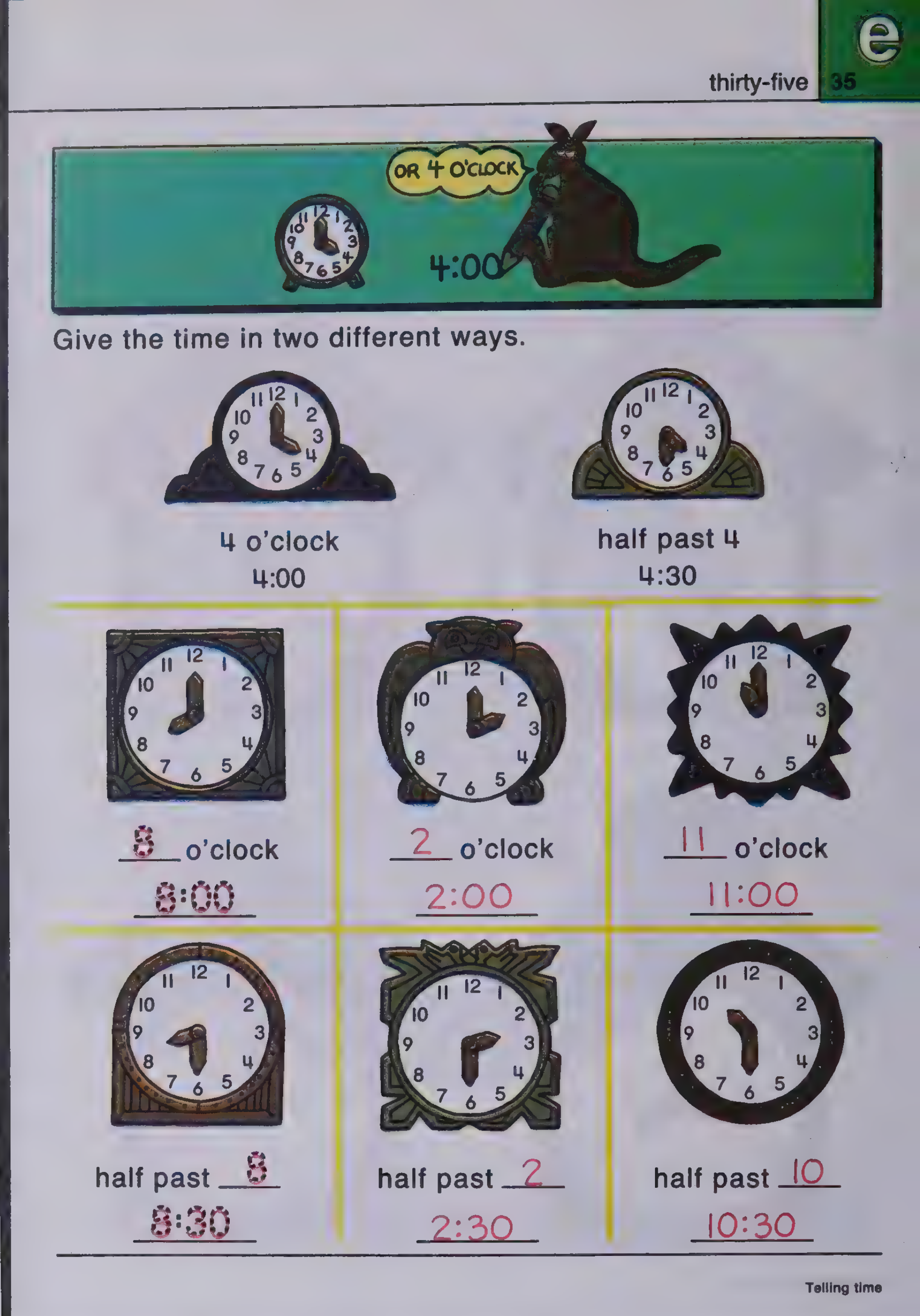
ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 2, pp. 60–61; Book 2, Unit 2, pp. 66–68, Addison-Wesley

Franklin Series: LEARNING ABOUT MEASUREMENT, pp. 74–80, Lyons and Carnahan

WORKJOBS, "Time," pp. 228–229, Addison-Wesley

Teaching
Page e-35

Point out the two ways of writing time to the hour and the half hour which are illustrated in the first row. Ask the class what time is shown on the clock in the first frame in the next row. When a child responds, "Eight o'clock," ask for volunteers to write 8:00 in two ways on the board. Instruct all the children to write 8 in the first blank and 8:00 beneath it. Point out that the rest of this row is to be completed in the same way. Before having children complete this page independently, review with them the notation for writing half past the hour. Ask a child: "What is the hour shown in the clock in the first frame in the last row?" We say that it is half past eight and we write 8:30. Direct all the children to trace over the dashed 8 after half past and then trace over the dashed 8:30 on the line beneath it. Tell the children to complete the row in this manner.



Objective

Given a clock showing an hour or half hour, the child will be able to read the clock, tell the time, and write it in time notation.

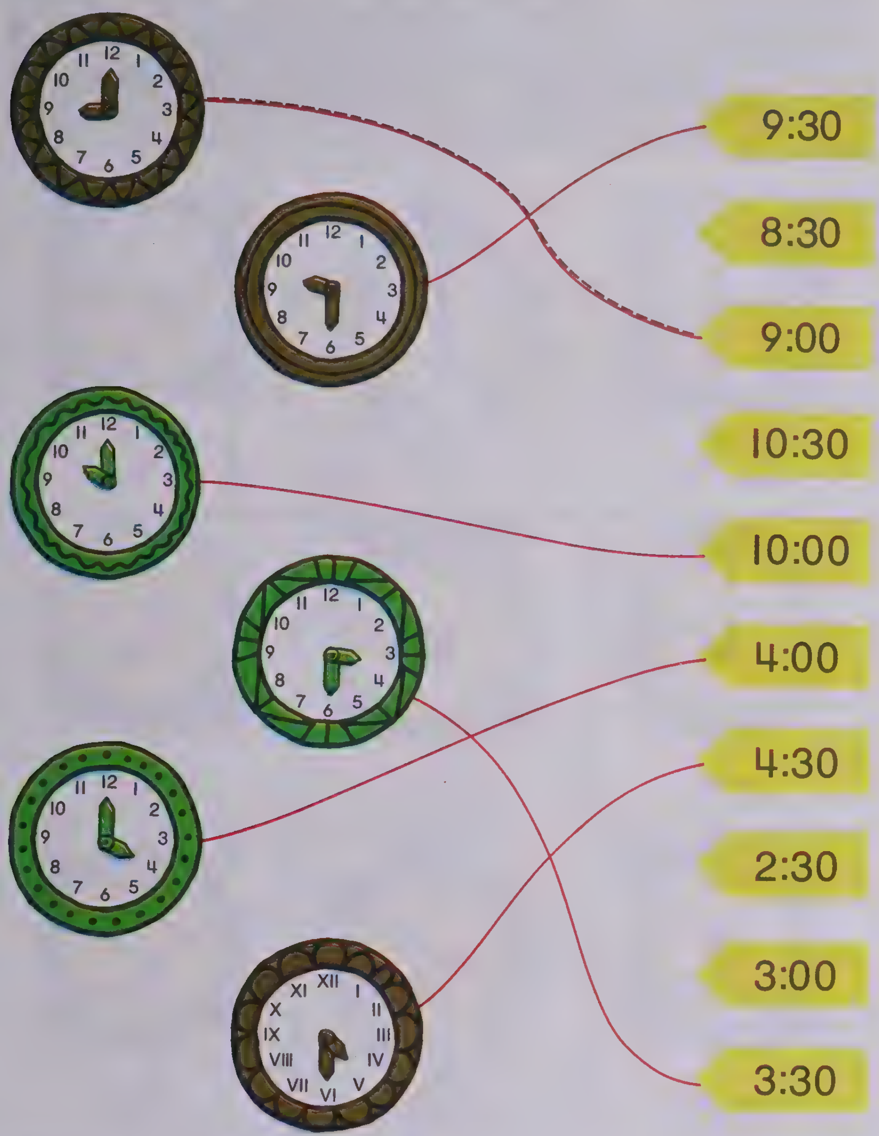
Pre-Book Activity

Materials

demonstration clock
individual clocks

Give the children various hours to set on their clocks. You might, for example, give them the hour, such as four o'clock, and have them show this on their clock; or you might write a schedule of daily events on the chalkboard and possible times, such as: rising-7:00, school-8:30, recess-10:00, and so on. Then ask the children to show on their clock the time for rising or the time for recess. Include in your list times that are half past the hour. When you mention one of these times, display a large demonstration clock. Show that 30 minutes after the hour can be called half past the hour because the minute hand has gone half way around the clock while the hour hand has moved half way past a given hour. Give the children plenty of examples to see this relationship. Use both "30 minutes" and "half past" when speaking of half past the hour. Show the phrase "o'clock" and point out that it means "according to the clock." When we say four o'clock, we mean that it is four according to the clock. Also show the symbolic form, such as 8:00 for eight o'clock and 8:30 for eight-thirty. Write times on the chalkboard using this notation and ask the children to show this time on their clocks.

Match each clock with the correct time.



Telling time

Teaching
Page e-36

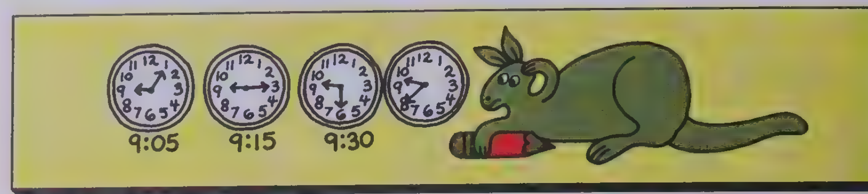
Read the directions at the top of the page with the children. Ask the children to determine what time is shown on the first clock at the top of the page. When they respond nine o'clock, direct them to find 9:00 in the column of times and then trace over the matching line. Instruct the children to complete the page by matching the time shown on each clock with the times given on the right.

Follow-up

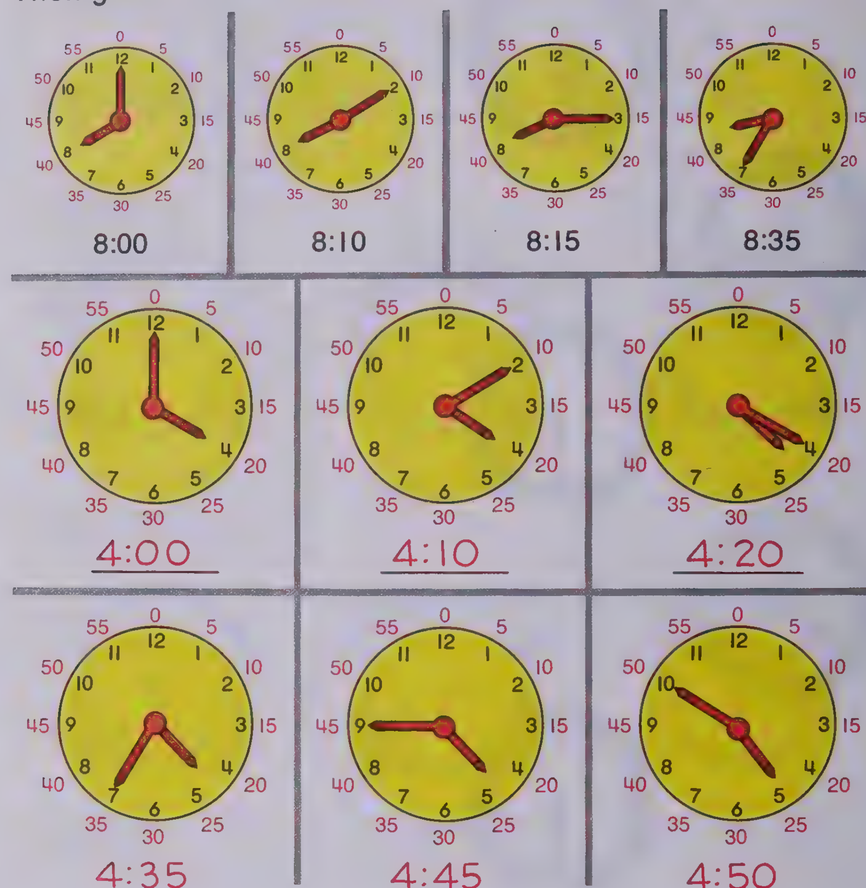
If you let the children keep their clocks at their desks, they may be further motivated to learn to tell time carefully. For example, suggest that they show on their clock the time for certain activities, such as art-1:00, gym-11:00, or recess-2:00. Then suggest that they watch the classroom clock and raise their hands when their clocks match the classroom clock. You might also simply list various times showing the hour, and half past the hour with the symbolic notation and ask the children to show these times on their clocks. You might turn this into a relay game. Divide the class into two teams. Have two lists on the chalkboard, one for each team. Point to the first time in each list. The first child in each team should show this time on his clock. If these are shown correctly then the second child should show the second time on the list. If the first child did not show the correct time, ask the second child to show this first time correctly on his clock. The first team to finish showing the times in their list will win.

Teaching
Page e-37

Read the directions at the top of the page with the children. Ask the children what they think the small red numerals around the outside of the clock mean. Point out that these numerals are simply guides showing how to count around the clock by fives. Also use the examples at the top to show how time notation may be used for times other than the hour and half past the hour. Work through the first two clocks which show 4:00 and 4:10, together with the children. Remind the children that the numeral showing the hour helps them count by fives. Then ask: "What time is shown on the first clock in the second row?" When a child responds, "four o'clock," tell all the children to write 4:00 in the space below the clock. Then ask: "What time does the next clock show?" When someone answers ten minutes after four, or 4:10, direct the children to write 4:10. According to the ability of the children, you might want to work through the remaining frames with them; however, if you think they are capable, encourage them to complete the page independently.



Study the examples.
Then give the time.



Objective

Given a clock showing 5, 10, 15, 20 . . . 55 minutes past the hour, the child will be able to tell the time and write it in time notation.

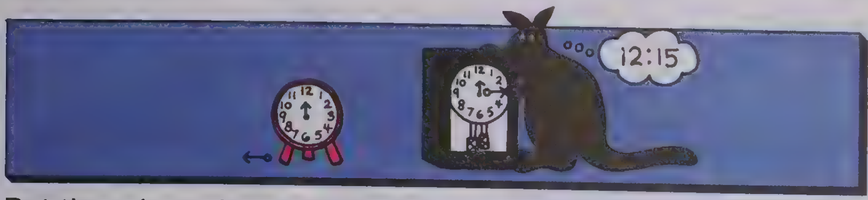
Pre-Book Activity

Materials

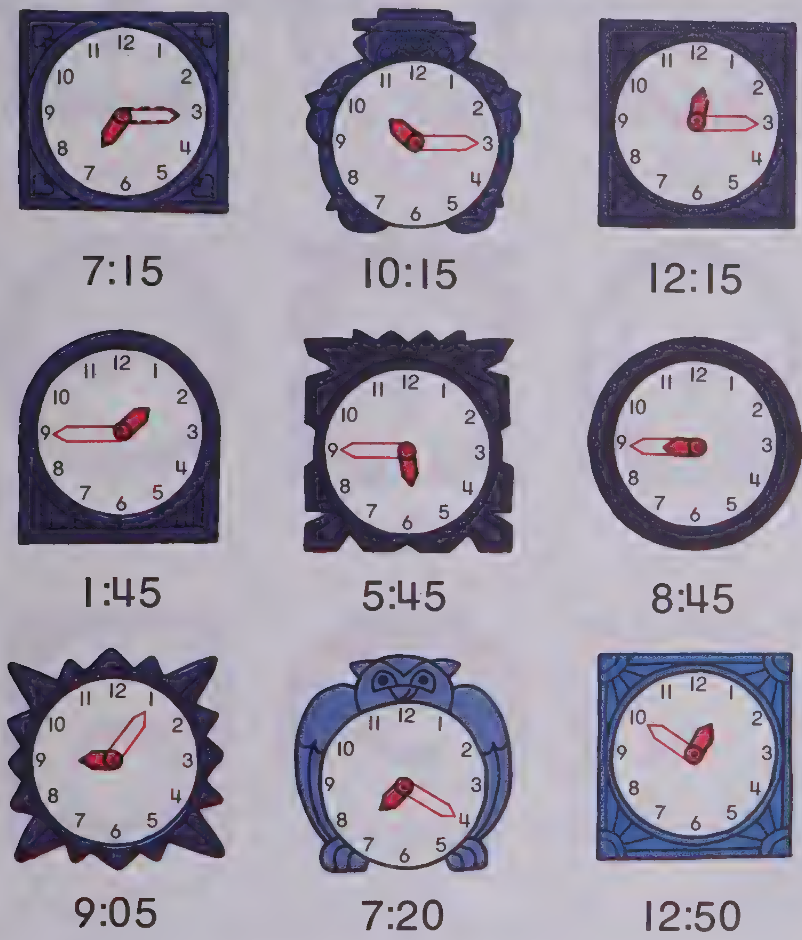
demonstration clock
individual clocks

For a quick review, use a large demonstration clock or a transparency clock on the overhead projector. Remind the class that 60 minutes total one hour. Show this on the demonstration clock. Ask the children to count hours by ones as the minute hand makes 12 sweeps around the clock and the hour hand moves slowly from one hour to the next. Point out that even if the clock is very large, it would be difficult and confusing to write the numerals for both the minutes and the hours on the face of the clock. Explain that although the numerals stand for hours, they help us remember minutes also. If we start on an hour the minute hand will be on the twelve, in five minutes it will travel to the one, so we think of five. After ten minutes, it will travel to the two, so we could think of two sets of five minutes passing. Ask the class to think of five minutes at one, ten minutes at two, fifteen minutes at three and so on. Continue this explanation by writing 0, 5, 10, 15, . . ., 55, around the demonstration clockface next to the proper numeral on the clock. Then review counting by fives, using the clock. Begin, for example, with one o'clock and move the minute hand around the clock by five's as you say, "1:00, 1:05, 1:10, 1:15, and so on, until 2:00." Continue by having the

e 38 thirty-eight



Put the minute hand on each clock.



Telling time

Teaching
Page e-38

Read the directions with the children. Point out that the hour hand is already shown in the correct position.

Work through the first exercise by having a child show the correct position of the minute hand on the demonstration clock. Direct all the children to trace the minute hand on the first clock. Remind them to make the minute hand longer and thinner than the hour hand. Tell them to complete the page by counting the minutes by fives and then drawing the minute hand to show that time.

children read from the clock with you, "2:05, 2:10, 2:15, ..." Next ask the children to take their clocks. Give them times which are multiples of five minutes past the hour to show on their clock. For example, ask them to show 3:15 or 4:20 or 8:35. Children will need many examples of this kind until they are able to master the skill.

Follow-up

The language associated with clocks, such as "fifteen after," "ten 'til," "a quarter to," and so on should be introduced as children learn to tell time since most of their time telling will be oral. Use these phrases with the children as a group until they are familiar with them.

Since each of the children has an individual clock, it would be helpful to have them work with a partner. Explain that one child should show a time on his clock and his partner should read the clock and tell the time. After showing three or four times they should switch and the first child will then read his partner's clock. To help a child who is having particular difficulty, you might pair him with a child who has a better developed ability to tell time.

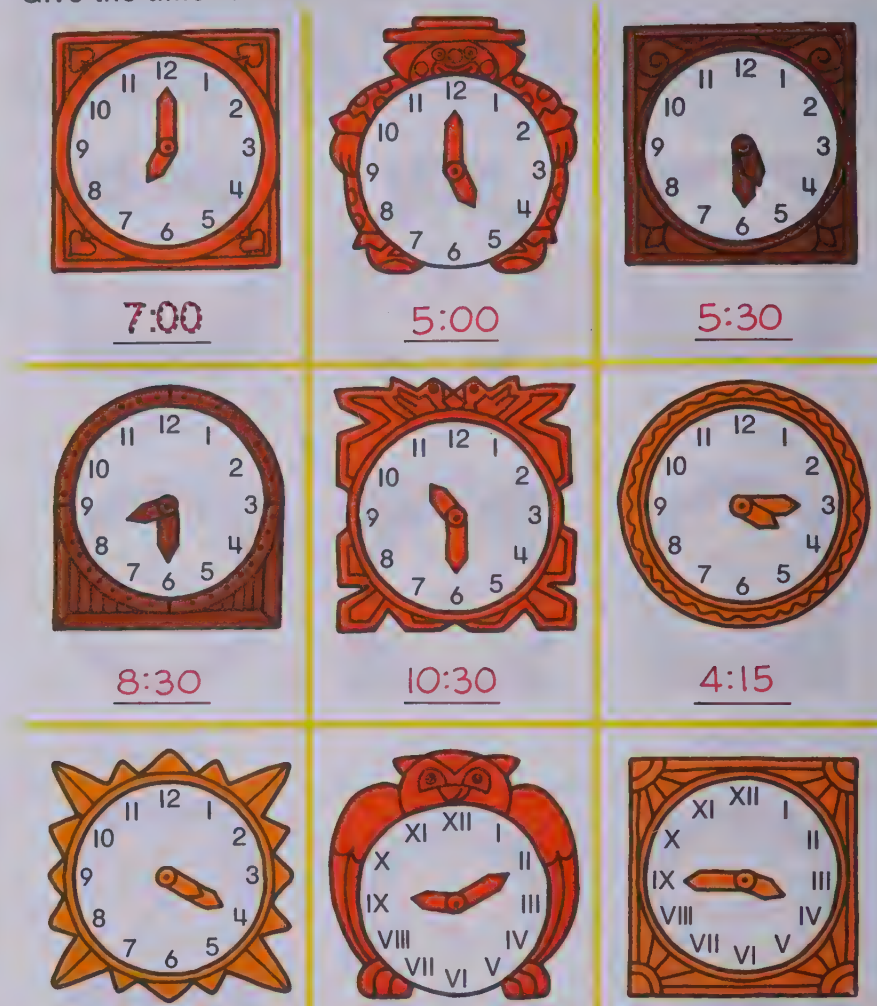
Note that activities such as this should be used throughout the year to continue to develop and refine the children's skill in telling time.

Teaching
Page e-39

Read the directions at the top of the page with the children. Point out the first clock and ask a child what time it shows. When he responds "seven o'clock," review how this is written in time notation and ask the children to trace over the dashed numerals. Ask the children to finish the remaining frames by giving the time for each clock in time notation in the space provided. As they work, move around the room to guide any children who have difficulty.

Show you know

Give the time for each clock.



7:00 5:00 5:30

8:30 10:30 4:15

4:20 9:10 3:45

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

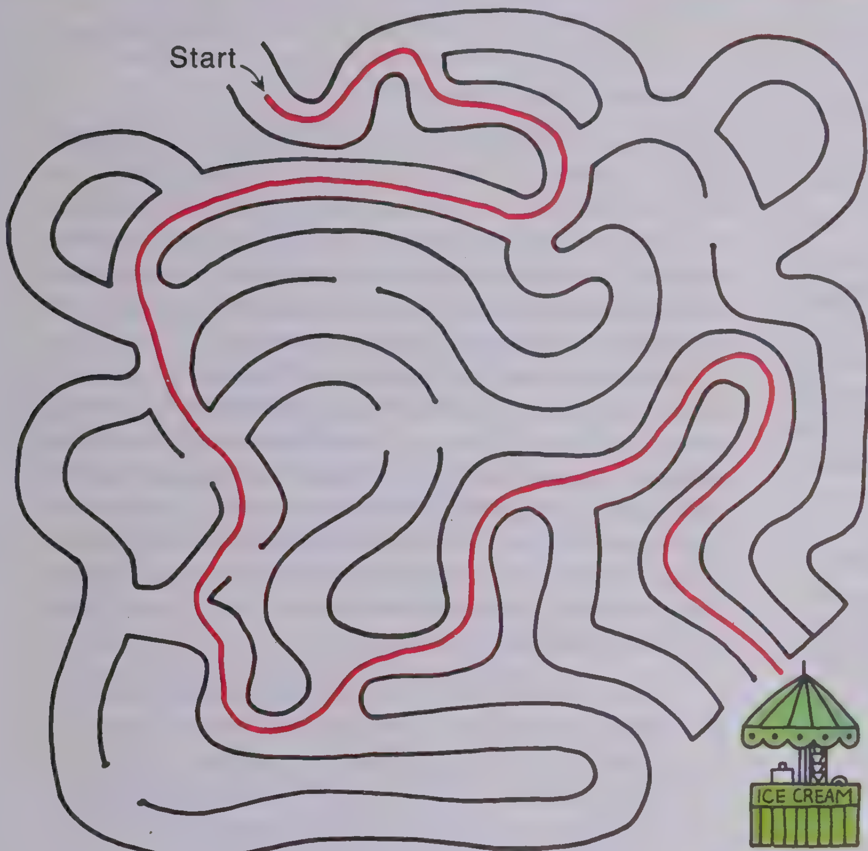
Pre-Book Activity

Materials:

large clock showing Roman numerals
individual clocks

Give the children various times orally and ask them to show these times on their individual clocks. For example, you might say: "Show me the time that is ten 'til two," or "Show on your clock forty-five minutes after twelve," "10:30," "8:15." Use various examples to review with the children how to show the correct time on their clocks and how to write the time in time notation.

Can you find the path to the ice cream stand? One path is given.



Maze

Teaching
Page e-40

Since this is a change of pace page, it should be treated with a light touch. Read the directions with the children. Explain to them that there *is* a way to get from the starting point to the ice cream booth. You might suggest that they first draw the path in pencil in case they bump into a dead end. Then when they have found the path which will get them all the way to the ice cream machine, they can color over it with a crayon.

Follow-up

In order to give the children further practice in writing time notation, particularly five, ten, or fifteen minutes after the hour, you might suggest at the beginning of a day that at certain times during the day you are simply going to call out "What time is it?" The children should keep a paper at their desks on which they record the times when you say this. At the end of the day, all the children should compare their lists with the master list which you write on the chalkboard. Remember to choose times that are multiples of five minutes beyond the hour. To make this type of activity more exciting, you might appoint a child to be the "clock-eyes" of the day and whenever he chooses, he can raise his hand and call out "What time is it?" or use some other phrase such as "Cuckoo" and then the children should record the time.

DARK GREEN MODULE, UNIT E

Addition and Subtraction to 10-Power Skill

Pages e-41 to e-52

General Objectives

To develop understanding of the concepts of addition and subtraction

To focus attention on the symbols used to express concepts of addition and subtraction

To introduce the addition and subtraction combinations for sums of 10 or less

To develop skills in addition and subtraction by using various manipulative aids

This module emphasizes three basic power skills which are used in addition and subtraction. Two of these skills involve the use of concrete materials, namely the strips which accompany this series and discrete objects or counters. The third power skill emphasizes the number line as a counting device. It is intended that each child be introduced to all of the power skills; however, children need not master all three. You might prefer to emphasize one technique more than another, according to the types of materials you have available in your classroom. Keep in mind that an ongoing objective of this module and the next is to help the children know the combination facts to 10 from memory with speedy recall. The blue module, however, will more particularly emphasize this speed skill aspect of the combinations.

Mathematics

We introduce the general concept of the sum of two cardinal numbers by first introducing the idea of the union of disjoint sets, or sets that have no common elements. The union of sets A and B, denoted by $A \cup B$, is the set containing those elements that are in A or in B or in both A and B. The following examples illustrate this definition.

Example 1.

Consider two sets A and B as follows:

$$A = \{m, n, o, p,\}, \qquad B = \{p, q, r\},$$

then

$$A \cup B = \{m, n, o, p, q, r\}.$$

Example 2.

Consider two sets R and S as follows:

$$R = \{1, 2, 3\}, \qquad S = \{15, 16\},$$

then

$$R \cup S = \{1, 2, 3, 15, 16\}.$$

Note that in Example 1, A contains 4 elements and B contains 3, whereas $A \cup B$ contains only 6 elements. Clearly, Example 1 does not illustrate the addition concept. But note that in Example 2, R contains 3 elements and S contains 2. This time the union of these sets, $R \cup S$, contains 5 elements; this union illustrates the idea we choose to call addition. These examples should emphasize the importance of the word *disjoint* in the following definition for the sum of two cardinal numbers.

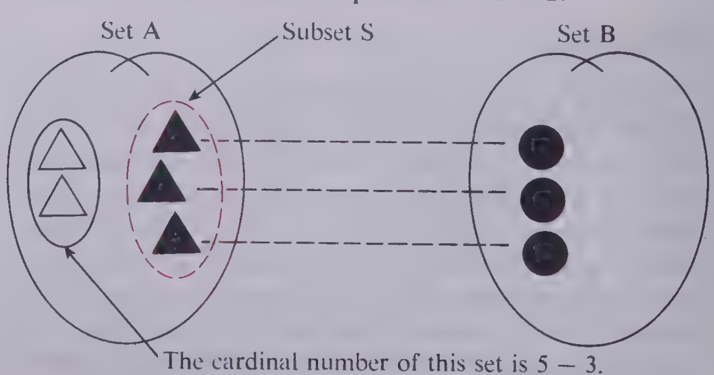
> Consider two cardinal number, a and b, and disjoint sets A and B from these cardinal numbers. The cardinal number of the union of sets A and B is the sum of the cardinal numbers a and b (written $a + b$).

It is relatively simple to present this idea to children, since the difficulty with overlapping, or intersecting, sets does not occur. For example, when children are putting two sets together, the sets are usually grouped in such a way that they have no common elements. For the children, forming the union of two sets merely requires pushing the objects together into one group.

We define subtraction in two basic ways: (1) by comparing two sets or (2) by defining the difference between the sum and one addend as the other addend. Removing objects from a given set is a clear, concrete method for introducing subtraction. Closely related to the idea of "take away" is the "comparison" idea of subtraction. A careful analysis of the definition below will show, in fact, that it encompasses both the idea of "take away" and the idea of comparing two sets.

> Let a and b be any two cardinal numbers such that $a > b$ or $a = b$. Choose sets A and B from a and b respectively. There is a subset S of A that is equivalent to B. The difference of a and b (written $a - b$) is the cardinal number of the set of objects in A other than those in S.

The diagram that follows illustrates this definition for the cardinal numbers 5 and 3. From this illustration, we obtain the subtraction equation $5 - 3 = 2$.



The definition and the illustration show clearly why subtraction is used for comparing two sets. On the other hand, it is not quite so clear how the removal of objects from a given set relates to this definition.

Using the illustration, we point out that the objects to be removed from set A to explain "take away" could be

those in the dotted ring. This set was chosen to match one-to-one with set B. However, when using the "take away" procedure, we would remark that set B consists of the numbers one, two, and three, and we would count objects "one, two, three" in set A and take them away. Thus, in the "take away" procedure, set B becomes an abstraction containing the counting numbers up to and including the number of objects we wish to remove. The idea of "take away" is merely a special part of the more general concept of subtraction associated with the comparison of two sets.

Now let us investigate a definition of subtraction in terms of addition:

> Let a, b, and c be whole numbers such that $a + b = c$. The number a is the difference $c - b$, and the number b is the difference $c - a$. In symbols,
> $$a = c - b, \qquad b = c - a.$$

According to this definition of subtraction, two subtraction equations are associated with each addition equation. There is some advantage, however, in associating only one subtraction equation with a given addition equation. For example, we like to think of $5 + 3$ as an expression for adding 3 to 5 and $8 - 3$ as an expression for subtracting 3 from 8. Therefore, it is useful to link the two equations, $5 + 3 = 8$ and $8 - 3 = 5$. For this type of example, we say that adding 3 and subtracting 3 "undo" each other. That is, we start with 5, add 3, and get 8. Then we start with 8, subtract 3, and get 5 again. In general, if a, b, and c are whole numbers such that $a + b = c$, we may link the two equations,

$$a + b = c \quad \text{and} \quad c - b = a.$$

We call this the inverse relationship between addition and subtraction.

The commutative (order) principle of addition is reviewed in this chapter. If a and b are whole numbers, then

$$a + b = b + a.$$

The purpose of the lesson is to aid in focussing attention on the four equations for a given "break-up" of a number. For instance, we know from the commutative (order) principle that if $5 + 3 = 8$ then $3 + 5 = 8$. From these two equations and from what we know about the inverse relation between addition and subtraction, we can expand the two equations to four equations: $5 + 3 = 8$, $8 - 3 = 5$, $3 + 5 = 8$, and $8 - 5 = 3$.

Teaching Dark Green Module, Unit E

Approximate Time: 6 to 9 days

Materials

counters
construction paper
demonstration number line
pictures from magazines showing sets
roll of paper about half a metre wide and 3.5 metres long
set of strips for each child
small index cards
yarn, approximately 5 metres long
45-by-60 cm sheets of newsprint

Vocabulary

addition	minus	subtract
difference	number line	subtraction
equals	plus	
equation	solve	

Many independent and small group activities with the strips, counters, and number line should accompany the children's work in this module. The pages of the module might be considered as an outline developing the three power skills emphasized. Your development of activities with the materials will depend on the materials you have available and the needs of the children. It would be helpful to extend some of the pre-book activities suggested. Children should be encouraged to work with concrete materials until they themselves want to stop using them.

Evaluation of Progress

The primary goal of this module is to have the children understand addition and subtraction. Do not emphasize speed skills at this point; instead judge how well each child understands the ideas involved and how well he can work with the power skills that are emphasized. Therefore, in any kind of testing, be sure the children have adequate materials and time to solve all the problems.

Resources for Active Learning

General Activities:

Nuffield Project: MATHEMATICS BEGINS 1, "Operations and the use of symbols," pp. 42–55, Wiley
SETS, NUMBERS AND POWERS, games, pp. 104–113, Herder and Herder
USING THE 'INVICTA' PLASTICS MATHEMATICAL BALANCE, Math Media
WORKJOBS, "Combining and Separating Groups," pp. 193–211, Addison-Wesley

Manipulative Devices:

Hainstock Blocks (Creative Publications; Lakeshore)
"Invicta" Math Balance (Math Media; Selective Educational Equipment)
Slide rule, 25-cm (Selective Educational Equipment)

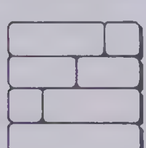
Commerical Games:

Kalah (Creative Publications; Math Media)

Investigation
Page e-41

Use the demonstration art to discuss the idea that the strips have been used to form a square. Point out that the green and white strips have been used twice in different positions. Then read the directions at the top of the page with the children. Suggest that they take their strips and cover the first square so that pairs of strips meet where the heavy black line forms a "stairstep." Stress that two strips should be used on each "step." After they have placed the strips on the square, direct them to move the strips and color the spaces to show which strips they used for each step.

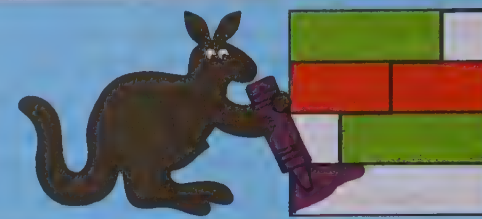
The directions for the section at the bottom of the page are much the same as for the top. Here they may choose any size square that will fit within the grid. You might suggest to some that they try to see *how many* squares they can build on the grid. An overhead projector will be a useful aid to help children who have difficulty. Keep in mind that the emphasis here should be on the size of the squares that are made and on the colors of the strips that are used, but not on the number combinations which these strips may represent.



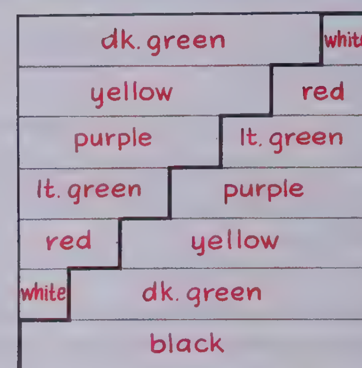
4-by-4

1 lt. gr.; 1 white
2 red
1 white; 1 lt. gr.
1 purple

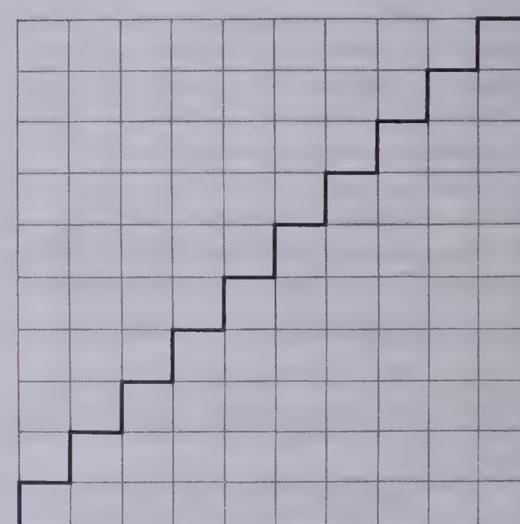
Let's do



Can you use your strips to help you color this square?



Can you build a square of your own on these stairs?
Answers will vary.



Purposes

To develop understanding of the addition and subtraction combinations to 10

To familiarize the children with the use of the strips for finding sums and differences

Many of the activities you plan for this module should be determined by the familiarities children have with the strips. Use activities such as the one on page e-41 to develop recognition of the strips by their number names when the white strip is the unit.

Preparation

Materials

one set of strips for each child

The amount of time required for freeplay with the strips will vary according to the familiarity of children with the strips. For example, you might suggest that they make some shape with the strips, such as a butterfly, house, or boat; however, if children have worked sufficiently with the strips, you might begin immediately with the investigation.

Let's talk

How many children? 7

How many animals? 5

How many plants? 5

Discussion
Page e-42

The illustration on this page is provided as a basis for an informal discussion of the combinations which children will study in this module. Read with them the questions at the top. After children have answered these questions, ask them if they can invent any questions of their own. You might help them with examples such as: "How many people are there?" "How many more bikes than scooters?" "How many shrubs?" "How many trees?" "How many adults?" Also, bring out the various subsets. For example, there are seven children: four girls and three boys. There are nine people: two adults, seven children. There are five plants: two trees and three bushes. Notice that besides reviewing or introducing combinations, this illustration also provides an opportunity for children to categorize various items. Be careful to observe any child who has difficulty in realizing that seven children and two adults may all be classified as nine people, or three dogs, and two cats may be thought of as five animals. Such a child should be given activities which stress sorting and classifying.

You might use these same kinds of questions for discussing items in the classroom. Since the main purpose of this lesson is to initiate discussion of the combinations to ten, try to limit your discussion to these combinations.

Follow-up

As a follow-up to this lesson, you might distribute a variety of pictures showing trees, children, and animals, and have the children make sets of questions similar to those used in the classroom discussion. To help the children do this, you might group them so that two or three can work together. They might paste the picture on a piece of large construction paper and print their questions at the bottom. When they have finished, suggest that they exchange pictures with another group so that they might answer the questions suggested by the other group.

Resources for Active Learning

A CLOUDBURST, Vol. 1, Nos. 1111–2, 1121, Midwest Publications

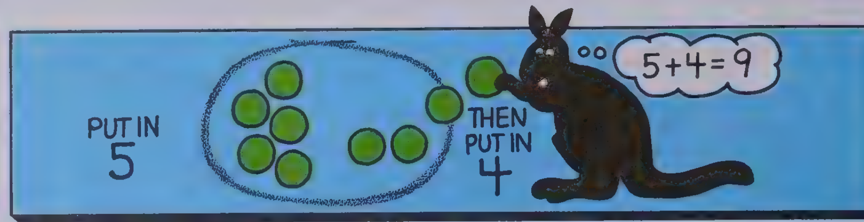
MATH ACTIVITIES, "Cuisenaire Rummy," Game 2–31, p. 37, Allyn and Bacon

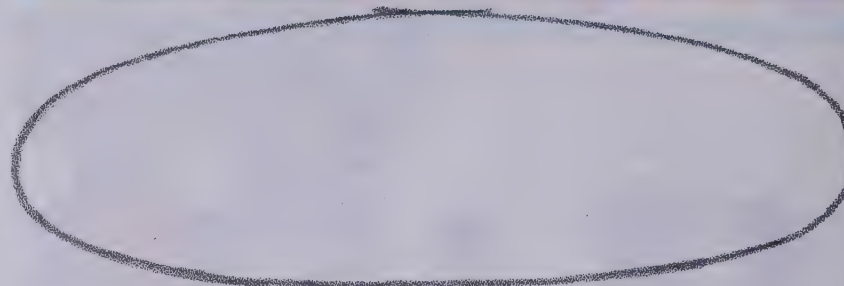
MATHS MINI-LAB, Cards 14–20, Selective Educational Equipment

Teaching
Page e-43

Have the children talk about the art at the top of the page. Explain that they are to place their counters in the ring according to the directions for each equation. You might want to work through the first equation with them. Read the frame with them saying: "Put in 4," "Then put in 3." Then ask the children to count how many counters they have in all in the ring. Direct them to write this answer in the yellow box. Then point out the equation. Relate the numerals to the number of counters they put in their ring, and point out how the addition sign is used to show the combination of their counters. Again, direct the children to write the sum in the box provided. Encourage them to do the remainder of the equations on their own.

It would be helpful to write other equations of the same type on the chalkboard for children to solve. For example, write: "Put in 8." "Put in 1." "How many in all?" "Solve the equation $8 + 1 = \square$." or "Put in 5." "Put in 2." "How many in all?" "Solve the equation $5 + 2 = \square$." You might even include an example using zero. "Put in 4." "Put in 0." "How many in all?" "Solve the equation $4 + 0 = \square$." Encourage the children to do as many examples of this kind as you feel are necessary.





Put in 4 | Then put in 3 | How many in all? 7

Solve the equation. $4 + 3 = \boxed{7}$

Put in 6 | Then put in 2 | How many in all? 8

Solve the equation. $6 + 2 = \boxed{8}$

Put in 2 | Then put in 4 | How many in all? 6

Solve the equation. $2 + 4 = \boxed{6}$

Development of addition equations

Objective

Given an addition equation for sums of 10 or less, the child will be able to find the sum by using counters or referring to illustrated sets.

This lesson contains what is review material for many children, but encourage manipulation of the counters as many will still benefit from their use.

Pre-Book Activity

Materials

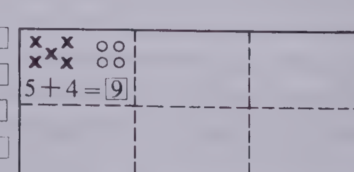
counters, 10 per child

Since page e-43 contains an activity with counters, you might choose to begin immediately with this page and use other activities after children have completed the activity on the page.

Follow-up

A duplicator sheet such as the following may give children practice working with equations and set illustrations.

$5 + 4 = \square$	$2 + 2 = \square$
$3 + 2 = \square$	$5 + 3 = \square$
$2 + 4 = \square$	$3 + 7 = \square$
$1 + 7 = \square$	$4 + 6 = \square$
$4 + 3 = \square$	



Find the sums.

$5 + 3 = \boxed{8}$

$3 + 4 = \boxed{7}$

$4 + 2 = \boxed{6}$ $5 + 4 = \boxed{9}$

$2 + 2 = \boxed{4}$ $3 + 2 = \boxed{5}$

$2 + 5 = \boxed{7}$ $7 + 3 = \boxed{10}$

$1 + 4 = \boxed{5}$ $2 + 6 = \boxed{8}$

$4 + 4 = \boxed{8}$ $2 + 8 = \boxed{10}$

4	5	4	7	7	5
+3	+0	+6	+2	+1	+5
7	5	10	9	8	10
3	3	9	1	3	8
+6	+3	+1	+6	+5	+1
9	6	10	7	8	9

Solving addition equations—sums 10 or less

TEACHING
Page e-44

Call attention to the illustrated frames at the top of the page. Ask the children to tell how many they see in each set in the first frame and relate these numbers to the numerals in the equation. Although the equations at the top use set illustrations in solving equations, many children will choose to use actual counters in solving the remainder of the equations on the page. Encourage children to use the counters for as long as they feel necessary. An ongoing objective is to reduce dependence on the counters, but this cannot be hurried. An individual child will discontinue the use of the counters when he is ready.

Supply the children with sheets of newsprint, about 45-by-60 cm, and crayons. Show them how to fold their paper to make nine approximately equal sections. Then ask the children to draw sets of objects to show each equation and then to copy and complete the equation for each section.

RESOURCES FOR ACTIVE LEARNING

MATHS MINI-LAB, Card 2, Selective Educational Equipment

Nuffield Project: COMPUTATION AND STRUCTURE (2), "The operation of addition," pp. 58–65, Wiley

MATHEMATICS

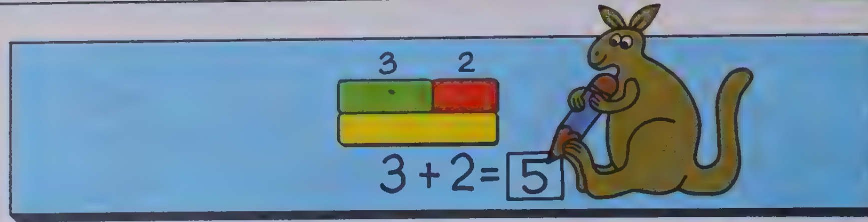
When we form the union of two sets that have no common elements, the number of the union set is the sum of the numbers of the individual sets.

○ ○ ● ● ●

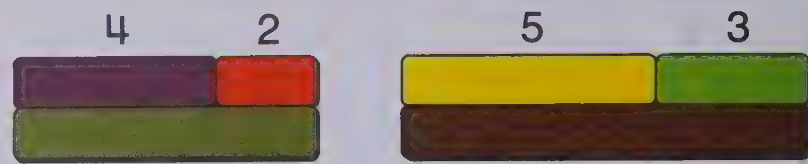
The equation describing the set above is $2 + 3 = 5$. This equation claims that the symbols $2 + 3$ represent the same number as the symbol 5. In general, an equation such as $A = B$ is a statement that A and B are different names for the same thing. Occasionally, you will see a false equation such as $2 + 3 = 6$. But when we ask children to solve the equation $2 + 3 = \square$, we expect them to write a whole number in the box which will make the statement true.

Teaching
Page e-45

If you have used the suggested pre-book activity, children should not have difficulty interpreting the illustration at the top of the page. Point out that the purple strip and the red strip may be thought of as four and two. Since these two together match the length of the dark green strip, we can relate this train to the sum $4+2=6$, so they should write 6 in the box provided. Similarly, in the next frame, the yellow strip and the light green strip together form a train that matches the length of the brown strip. Since the yellow strip makes us think of five and the light green makes us think of three, we can relate this train to the sum $5+3=8$, so they should write 8 in the box provided. Encourage the children to use their strips to complete the page independently. For each equation they should take two strips that are given as the addends and see if they can find a single strip which is the same length as the train made by those two strips. Then they should find a single strip which matches the length of that train. They should be familiar enough with the number names assigned to the strips so that they can relate this train to the equation. If necessary, work through equations with them until they know what to do. Note that the intent of this lesson is not to emphasize memory skills but to build the *power skill* for addition by using the strips.



Solve the equations.



$4+2=\boxed{6}$	$5+3=\boxed{8}$
$3+3=\boxed{6}$	$3+4=\boxed{7}$
$3+5=\boxed{8}$	$5+4=\boxed{9}$
$5+5=\boxed{10}$	$1+5=\boxed{6}$
$4+0=\boxed{4}$	$4+4=\boxed{8}$
$5+2=\boxed{7}$	$3+6=\boxed{9}$
$2+3=\boxed{5}$	$8+2=\boxed{10}$
$7+2=\boxed{9}$	$2+4=\boxed{6}$
$3+7=\boxed{10}$	$6+2=\boxed{8}$

Sums—strips

Objective

Given an addition equation with two addends whose sum is ten or less, the child will be able to find the sum by using the strips or by using a number line labelled from zero to ten.

Since pages e-45 and e-46 develop addition using two different aids, it would be possible to treat each page as a separate lesson.

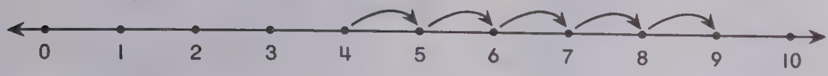
Pre-Book Activity

Materials

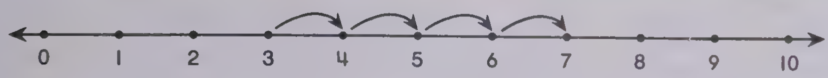
demonstration number line
strips, one set for each child

Distribute a set of strips to each child. Allow the children a brief free play introduction with the strips. Then suggest that they make a staircase with the strips to show the number values from one to ten. Review with them the numbers which can be assigned to each of the strips. "If one is the white strip, what number does the purple strip represent?" Remind the children that all these number values assigned to the strips are so assigned because the white strip here is thought of as one. Then review with the children how to make trains with the strips. Remind the children that when they make their trains, they simply place their pair of strips above a strip that they are matching as shown on page e-45.

Solve the equations.



$4 + 5 = \boxed{9}$



$3 + 4 = \boxed{7}$

$3 + 2 = \boxed{5}$	$6 + 3 = \boxed{9}$
$2 + 5 = \boxed{7}$	$4 + 4 = \boxed{8}$
$5 + 5 = \boxed{10}$	$6 + 1 = \boxed{7}$
$4 + 2 = \boxed{6}$	$3 + 3 = \boxed{6}$
$2 + 7 = \boxed{9}$	$6 + 4 = \boxed{10}$
$0 + 6 = \boxed{6}$	$4 + 3 = \boxed{7}$
$4 + 6 = \boxed{10}$	$6 + 2 = \boxed{8}$
$3 + 5 = \boxed{8}$	$8 + 1 = \boxed{9}$

Sums—number line

Teaching
Page e-46

You might want to give a separate introduction to this page using the demonstration number line. Most children will be familiar with the use of the number line but would benefit from a review. Begin by writing the addends of an equation such as $6 + 2 = \square$. Point out that they can begin with the first number and use the number line as a counting device. Some children may decide to always begin with the larger number so they will have to take fewer jumps. This is excellent thinking and represents an intuitive understanding of the order or commutative principle. Thus, if they begin at 6, jump to 7, and then to 8, they have found that the sum of 6 and 2 is 8. Since the number lines on page e-46 provide material for discussion, you might want to use them as part of your preparation for this page. Work through these examples and others until the children are comfortable with the use of the number line. Encourage the children to do the remaining equations independently. However, it is not essential that all the children use the number line to solve these equations. Allow children to choose whichever method they prefer. If some prefer to use strips, they should be allowed to use them. If some prefer to use counters, encourage them to do so. If number lines are not available, simply have them use one illustrated in the textbook. They need not show any illustration on their number lines.

Follow-up

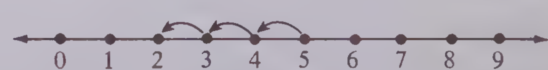
Suggest to the children that they use their strips to show all the possible combinations for some of the numbers from 1 to 10. They can do this by forming squares similar to those that they made in the investigation. However, this time they should record the equation for each pair of strips which they use.

Mathematics

The number line is used as an aid in developing combinations. Points on the line are the same distance apart and they are labelled with whole numbers. The distance between these points is called the unit distance.

The number line could point in any direction, but it is conventional to have it in a horizontal position with points representing numbers in ascending order to the right. The arrows indicate that the number line has no endpoint in either direction. Addition and subtraction equations may be shown by jumps on the number line, as shown in the figure.

$5 + 3 = 8$ 

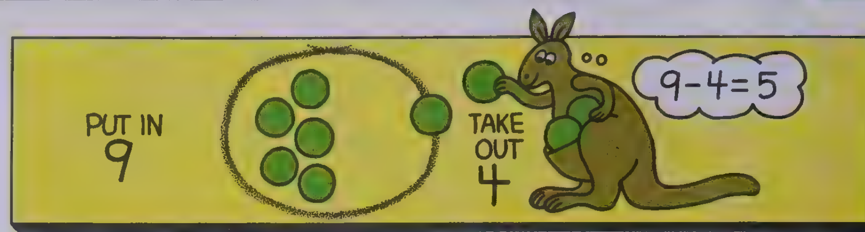
$5 - 3 = 2$ 

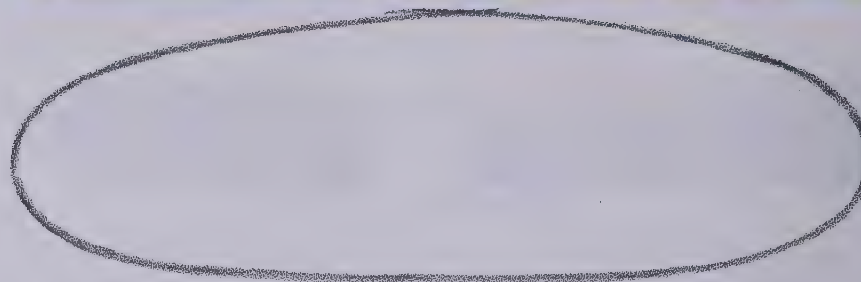
Resources for Active Learning

WORKJOBS, "Number Boards," pp. 138–139, Addison-Wesley

Teaching
Page e-47

Call attention to the illustration at the top of the page. Help the children see that the kangaroo is taking counters out of a ring. Then call attention to the ring at the top of the page. Read the first line with the children and direct them to do what it says as you read. "Put in 7." "Take out 2." "How many left?" After children have identified how many are left, point out the equation, $7-2=\square$. Ask a child to explain how to read the subtraction symbol. ("7 take away 2 equals 5" is acceptable, but gradually lead the children to read "7 minus 2" or "7 subtract 2.") Relate the numeral 7 to the number of counters they put in the ring and the numeral 2 to the number of counters they took out, then the difference 5 should be written in the box. Encourage the children to work through the remaining examples by themselves. You might write other examples on the chalkboard, such as: "Put in 3." "Take out 1." "$3-1=\square$." or "Put in 9." "Take out 5." "$9-5=\square$." Children will benefit by working through many examples of this kind.





Put in 7 | Take out 2 | How many left? 5

Solve the equation. $7-2=\boxed{5}$

Put in 6 | Take out 4 | How many left? 2

Solve the equation. $6-4=\boxed{2}$

Put in 8 | Take out 3 | How many left? 5

Solve the equation. $8-3=\boxed{5}$

Objective

Given a subtraction equation related to sums of ten, the child will be able to find the difference by using counters or relating the equation to a set illustration.

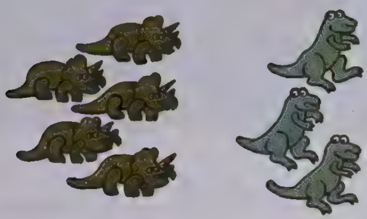
Pre-Book Activity

Materials

counters, at least 10 per child

Since page e-47 involves an activity with counters, you might want to begin immediately with the page; however, you might want to display objects and have the children work through some subtraction examples. Take a long piece of yarn and place it in a circle in front of the classroom. Then ask five children to come up and stand in the circle. Have another child write a 5 on the blackboard to record that there are five children in the circle. Ask two of the children in the circle to move out of the circle. Ask the class if anyone can record this action or write, $5-2=\square$ on the chalkboard. Then ask the class to explain how they can find out how many are left. Clearly they will only have to count those that are remaining inside the yarn. Children will enjoy an activity such as this and it relates directly to the activity with the counters which they will be doing in this lesson.

Find the differences.



$8 - 3 = \boxed{5}$

$7 - 4 = \boxed{3}$

$6 - 3 = \boxed{3}$	$9 - 6 = \boxed{3}$
$7 - 6 = \boxed{1}$	$8 - 5 = \boxed{3}$
$10 - 3 = \boxed{7}$	$7 - 5 = \boxed{2}$
$8 - 2 = \boxed{6}$	$10 - 8 = \boxed{2}$
$9 - 3 = \boxed{6}$	$4 - 0 = \boxed{4}$

7	9	5	10	8	9
−2	−1	−3	−5	−3	−4
5	8	2	5	5	5
7	10	8	6	9	10
−1	−2	−4	−2	−5	−4
6	8	4	4	4	6

Teaching
Page e-48

Call the children's attention to the illustrations in the first two frames. Stress that they should think of the total number of animals in the frame. Thus, in the first frame, there are a total of eight animals. The picture shows that 3 animals are leaving. Thus the equation is, $8 - 3 = \square$, and the difference is shown by the remaining set. Encourage the children to work through the subtraction equations independently by using either counters or drawing pictures for each equation. As the children work, move around the room. Observe and help those who are having difficulty. Also point out the vertical notation of the exercises at the bottom of the page. Help the children see that the vertical notation of $7 - 2$ may be thought of in the same way as the equation $7 - 2 = \square$. We do not speak of the vertical notation problems as equations, but they may be solved in the same manner as thinking through the equations.

Follow-up

For further emphasis on understanding subtraction, duplicate an exercise similar to the one in the next column on a worksheet, or write it on the chalkboard. Also suggest to those children having difficulty with the combinations that they make a set of flash cards. Some children will need to make cards only for the sums and differences of 6 to 10. Small index cards are suitable for making flash cards.

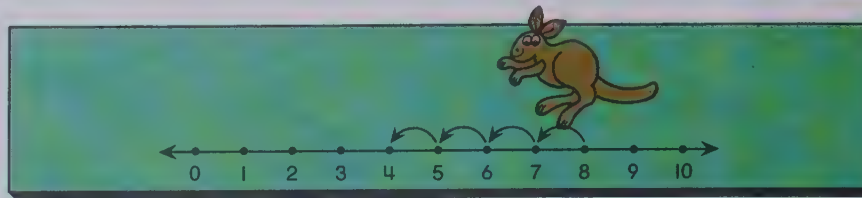
Ring the number to be taken away.
Then complete each equation.

X X X X X X X X X	$9 - 0 = \boxed{9}$	X X X X X X X X X	$9 - 5 = \square$
X X X X X X X X X	$9 - 1 = \boxed{8}$	X X X X X X X X X	$9 - 6 = \square$
X X X X X X X X X	$9 - 2 = \square$	X X X X X X X X X	$9 - 7 = \square$
X X X X X X X X X	$9 - 3 = \square$	X X X X X X X X X	$9 - 8 = \square$
X X X X X X X X X	$9 - 4 = \square$	X X X X X X X X X	$9 - 9 = \square$

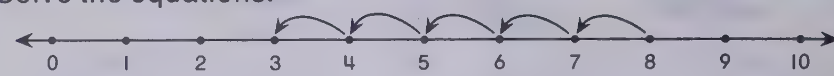
TEACHING
Page e-49

Use the illustration of the kangaroo at the top to discuss how the equation $8 - 4 = 4$ has been shown on the number line. Continue by pointing out the two equations at the top of the page. Help the children see that the first numeral in the equation is the starting point on their number line. The subtraction sign in the equation indicates that their jumps will be to the left. The second numeral in the equation indicates how many jumps they should count. In the first equation they start at 8, jump 5 spaces to the left and find out that their landing point or difference is 3. This difference should then be written in the box provided.

This page continues development of the power skill for subtraction by providing number line experiences. The number line in the middle of page e-49 is provided so that children can use it while solving the remainder of the equations on the page. They need not draw on this number line, but simply use the eraser end of their pencil or their finger to make the jumps. If some children prefer not to use the number line, allow them to use any other method which they prefer. When children are finished, you may want to have some of them demonstrate their solutions on the overhead projector or the demonstration number line on the chalkboard.



Solve the equations.



$8 - 5 = \boxed{3}$

$10 - 4 = \boxed{6}$

$6 - 2 = \boxed{4}$	$9 - 6 = \boxed{3}$
$9 - 4 = \boxed{5}$	$7 - 3 = \boxed{4}$
$8 - 6 = \boxed{2}$	$10 - 5 = \boxed{5}$
$10 - 7 = \boxed{3}$	$6 - 0 = \boxed{6}$
$7 - 4 = \boxed{3}$	$8 - 3 = \boxed{5}$
$9 - 5 = \boxed{4}$	$5 - 5 = \boxed{0}$

Subtraction—number line

OBJECTIVE

Given subtraction equations related to the sums of ten or less, the child will be able to find the differences by using the number line.

PRE-BOOK ACTIVITY

Guide the children in playing an oral game of "What's my equation?" Start out by saying, "My numbers are 8 and 5: the equation is a subtraction equation. What's my equation?" A child should then respond by writing the following on the chalkboard:

$$8 - 5 = 3$$

When children have caught on to the game, ask one of them to think of an equation, give two of his numbers and tell if the equation is addition or subtraction. Then another child should try to respond by naming the equation and writing it on the chalkboard.

Samples:

"My numbers are 4 and 2. My equation is addition.
Answer: $4 + 2 = 6$

"My numbers are 3 and 1. My equation is subtraction.
Answer: $3 - 1 = 2$

Solve.

0 1 2 3 4 5 6 7 8 9 10

$4 - 4 = \boxed{0}$	$9 - 5 = \boxed{4}$
$4 + 3 = \boxed{7}$	$7 - 3 = \boxed{4}$
$6 + 4 = \boxed{10}$	$10 - 6 = \boxed{4}$
$3 + 3 = \boxed{6}$	$8 - 2 = \boxed{6}$
$4 + 5 = \boxed{9}$	$5 - 2 = \boxed{3}$
$6 - 4 = \boxed{2}$	$7 + 3 = \boxed{10}$
$9 - 3 = \boxed{6}$	$7 - 3 = \boxed{4}$
$5 + 1 = \boxed{6}$	$6 + 3 = \boxed{9}$

$\begin{array}{r}7\\+2\\\hline 9\end{array}$ $\begin{array}{r}0\\+5\\\hline 5\end{array}$ $\begin{array}{r}2\\+8\\\hline 10\end{array}$ $\begin{array}{r}10\\-3\\\hline 7\end{array}$ $\begin{array}{r}7\\-7\\\hline 0\end{array}$ $\begin{array}{r}8\\-4\\\hline 4\end{array}$

$\begin{array}{r}4\\+3\\\hline 7\end{array}$ $\begin{array}{r}10\\-9\\\hline 1\end{array}$ $\begin{array}{r}7\\-6\\\hline 1\end{array}$ $\begin{array}{r}5\\+5\\\hline 10\end{array}$ $\begin{array}{r}3\\+5\\\hline 8\end{array}$ $\begin{array}{r}9\\-2\\\hline 7\end{array}$

Practice—addition and subtraction

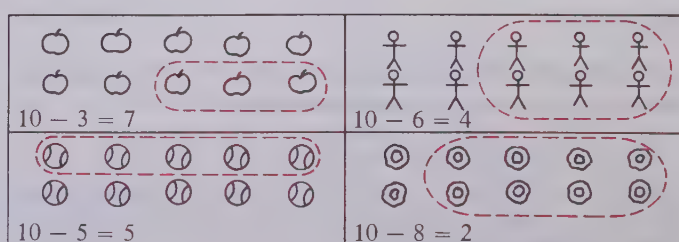
Teaching
Page e-50

Again point out the number line at the top of the page. Explain to the children that on this page both addition and subtraction equations are given. They should very carefully read each equation to determine which sign is shown. Although the number line is provided at the top of the page as a device to help the children solve these equations, they may use any other manipulative devices they prefer. You might suggest that they use the number line to solve those equations. Then, they can use another method, such as the strips or counters to check their answers. Some children may by this time recall the basic facts which they learned the previous year and be able to do these equations without reliance on manipulative devices. Again point out the examples in vertical notation at the bottom of the page. These should be solved in the same way equations are solved.

Follow-up

Use the sets made on paper plates. (See Follow-Up for text pages e-1 and e-2.) Shuffle the plates and then show them two at a time. Ask the children which set represents the greater number. Then, ask how many more there are in the larger set. Write the corresponding subtraction equation and ask the children to solve it.

You might also supply the children with 45-by-60-cm sheets of newsprint, old toy catalogues, magazines, paste, and scissors. Ask the children to fold their papers into four sections. Tell them to cut out pictures of objects, such as toys, cookies, fruit, people, and the like, and paste ten of them in each of the four sections of their papers. Ask the children to ring some subset of each set and then to write and complete a subtraction equation for each of the four sections of their papers.



Resources for Active Learning

Slide rules:

A CLOUDBURST, Vol. 1, No. 1191, Midwest Publications

MATH ACTIVITY CARDS, A5, Macmillan

WORKJOBS, "Number Cans," pp. 150–151; "Number Combination Board," pp. 168–169, Addison-Wesley

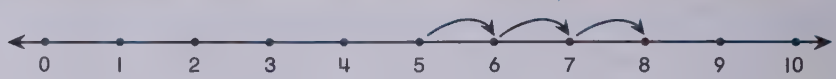
Teaching
Page e-51

Read the directions at the top of the page with the children. Explain to them that they should try to do as much of this page by themselves as possible. Remind them that they may use any materials, such as counters or strips, that they feel are necessary.

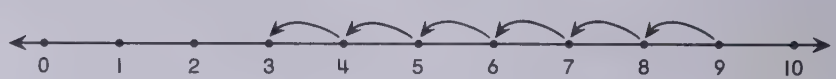
Show you know

Solve.

$5 + 2 = \boxed{7}$ $5 - 2 = \boxed{3}$



$5 + 3 = \boxed{8}$



$9 - 6 = \boxed{3}$

$6 + 3 = \boxed{9}$

$2 + 4 = \boxed{6}$

$8 - 4 = \boxed{4}$

$10 - 2 = \boxed{8}$

$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$ $\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array}$ $\begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$

$\begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array}$ $\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$ $\begin{array}{r} 10 \\ -6 \\ \hline 4 \end{array}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

As you evaluate children's work on page e-51, recall that power methods are the main focus of this module. The speed with which children find the answers should not be emphasized.

Pre-Book Activity

Materials

counters *number lines*
sets of strips

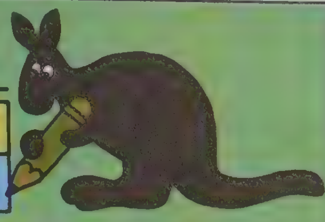
Prepare five or six stations around the classroom. At each of these, show some interpretation of an addition or subtraction equation which a group of children could study. For example, at the first station show a set illustration in which the total set contains eight items and a subset of three has been circled and crossed out.

At the second station show a set of four and a set of five which have been enclosed in a ring. At the third station, show a number line on which the small arrows show jumps from 4 to 5 to 6 to 7 to 8.

At another station, show a number line on which the arrows show jumps from 7 to 6 to 5 to 4 to 3.

Let's have fun

$$\begin{array}{r} 5 \\ -4 \\ \hline 1 \\ D \end{array} \quad \begin{array}{r} 4 \\ +1 \\ \hline 5 \\ O \end{array} \quad \begin{array}{r} 9 \\ -6 \\ \hline 3 \\ G \end{array}$$



Write the sum or difference on yellow.
Use the code to put letters on blue.

CODE

1	D
2	E
3	G
4	K
5	O
6	R
7	V
8	W
9	Y

$$\begin{array}{r} 3 \\ +4 \\ \hline 7 \\ V \end{array} \quad \begin{array}{r} 8 \\ -6 \\ \hline 2 \\ E \end{array} \quad \begin{array}{r} 9 \\ -3 \\ \hline 6 \\ R \end{array} \quad \begin{array}{r} 4 \\ +5 \\ \hline 9 \\ Y \end{array}$$

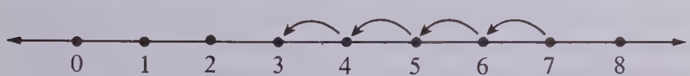
$$\begin{array}{r} 10 \\ -7 \\ \hline 3 \\ G \end{array} \quad \begin{array}{r} 5 \\ +0 \\ \hline 5 \\ O \end{array} \quad \begin{array}{r} 8 \\ -3 \\ \hline 5 \\ O \end{array} \quad \begin{array}{r} 9 \\ -8 \\ \hline 1 \\ D \end{array}$$

$$\begin{array}{r} 4 \\ +4 \\ \hline 8 \\ W \end{array} \quad \begin{array}{r} 3 \\ +2 \\ \hline 5 \\ O \end{array} \quad \begin{array}{r} 7 \\ -1 \\ \hline 6 \\ R \end{array} \quad \begin{array}{r} 9 \\ -5 \\ \hline 4 \\ K \end{array}$$

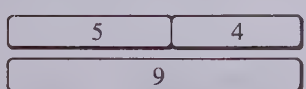
Solving a code

Teaching
Page e-52

This change of pace page should be treated with a light touch. However, explain the procedure very carefully so the children know how to decipher the code. Read the directions with them. Be sure they realize that they should first find the answer and then use it to find the letter which matches it.



At another station show a train of the yellow strip and the purple strip and place this train parallel with the blue strip.



Put out as many stations of this kind as you need so that groups of three and four children at a time might study at each station. Direct the children to number a piece of paper according to the number of stations you have. Ask a group to study the station and to discuss and figure out among themselves what addition or subtraction equation they might think of at each station you have prepared.

Follow-up

For this activity you will need at least 52 beans. Paint only one side of each of the beans by simply laying them out on newspaper or waxpaper and using a can of spray paint. Place the beans in eight containers, three in the first, four in the second, five in the third and so on. Be sure the containers have lids or can be covered by the hand so that the beans may be shaken. A child should take a container and shake the beans out onto a table. Some of the beans will land with the painted side up; some with the white side up. The child should then write an equation, either addition or subtraction, suggested by the arrangement.

BLUE MODULE, UNIT E

Missing Addends and Differences

Pages e-53 to e-62

General Objectives

To introduce the concept of missing addend

To introduce finding differences by thinking about missing addends

To provide experience with word problems in addition and subtraction

The chief purpose of this module is to begin work with missing addends in preparation for development of speed skills in finding differences. Therefore, accompanying the work in this module should be an effort toward development of speed skills or general mastery of the facts for *sums* of 10 or less. It is only when the children have sufficiently mastered sums of ten or less that they will be able to efficiently find differences by thinking about missing addends. Therefore, you should view this module as the beginning of a continuing effort throughout the Book 2 program to work toward mastery of finding differences by thinking about missing addends. That is, the overall goals of this module should be considered as ongoing objectives throughout the remainder of the school year. This module introduces the basic idea that a child need not actually memorize subtraction facts. It is our point of view that if a child knows addition facts well, then he can learn to find differences efficiently by thinking about missing addends. For example, when a child sees the problem 7 minus 4 equals box, we would like for him to think, "What number adds to four to make seven?" rather than simply memorizing the subtraction fact, $7 - 4 = 3$.

Mathematics

Related addition and subtraction equations are considered so that finding the difference of an equation such as $12 - 5 = \square$ may be thought of as finding the missing addend of the equation $5 + \square = 12$.

The missing addend concept is based on a formal definition of subtraction:

> If a, b, and c are whole numbers and $a + b = c$, then $b = c - a$.

Thus b, the difference of two whole numbers, is an addend in the addition statement.

Although children can consider subtraction solely from the interpretation of take away, this approach is very limiting. The missing addend approach lays a foundation for development of more difficult topics at a later level, particularly for mastering the combinations of 11 through 18 in Book 2.

Teaching Blue Module, Unit E.

Approximate Time: 5 to 7 days

Materials

counters, at least 10 for each child
flashcards for addition and subtraction, (optional)
paper bags for demonstration
scissors
yarn, approximately a 30-centimetre long piece for each child

Vocabulary

addend	difference	sum
combination	missing addend	

This module illustrates one of the basic trouble spots in developing abstract mathematical concepts with young children. The basic underlying concept of a missing addend is quite simple when it is demonstrated at the physical level. That is, most children can respond correctly if presented with three counters and asked the question, "How many more counters do you need to make five?" This physical experience, of course, would relate to the equation, $3 + \square = 5$ or $\square + 3 = 5$. Of course, there are countless other physical models that would demonstrate and elucidate the general concept of a missing addend. For example, one could use the centimetre strips, asking such questions as, "If you have a purple strip and a dark green strip, what strip do you need to put with the purple strip so that the two together are as long as the dark green strip?" This, of course, would illustrate the equation $\square + 4 = 6$. Again, most children can demonstrate this concept at the physical level. It is when we attempt to express these ideas with mathematical symbols that the children have difficulty. Therefore, throughout this module, you will want to accompany various physical demonstrations relating to missing addends with corresponding equations. Some children will be able to make the transition from the physical demonstrations to the equation work on the printed page much more rapidly than others; therefore, you will want to have available various kinds of materials for the children to work with through the development of the concepts in this module. Keep in mind, however, that one of the chief goals of this module is to push toward the ability to work with missing addends in the absence of physical materials.

Evaluation of Progress

Since one of the chief goals of this module is to work toward the ability to find a difference through thinking about missing addends without the use of concrete materials, your evaluation of progress should be directed toward two areas: First, you will want to evaluate the child's ability to find missing addends through use of various physical materials and relate this to the finding

of specific differences. Secondly, you will want to evaluate the child's ability to look at a given subtraction equation and think about a missing addend. Keep in mind, however, that this module is not intended as the final stage in development of these skills. Therefore, as you evaluate both kinds of skills (physical and abstract) you will want to allow considerable time for the children to think about the problems as they find their solutions. Even though this module is a step in the direction of working toward speed skills in combinations of ten or less, you will not want to overemphasize the speed skills in evaluating the children's achievement.

Resources for Active Learning

General Activities:
Basic facts games:

DEVELOPMENTAL MATH CARDS C‡4, Addison-Wesley

MATH ACTIVITIES, Games 3/31–79, pp. 97–119, Allyn and Bacon

MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Publications Ltd.

Commercial Games:

Games for practicing basic facts (refer to this list throughout the year)

Arithmecubes (Scott Foresman)

Big Zero (Creative Publications; Creative Teaching Assoc.)

Cover-Up (Selective Educational Equipment)

Equations (Creative Publications, Wff 'N Proof)

Math Match (Creative Publications)

Orbiting the Earth, addition and subtraction (Scott Foresman)

Quinto (Hammett; Selective Educational Equipment)

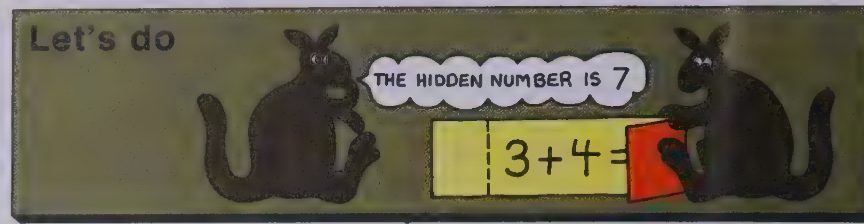
"Sum" Difference (Creative Publications; Creative Teaching Assoc.)

Sum Fun (Ideal; school supplier)

Twin-Choice (Holt, Rinehart and Winston)

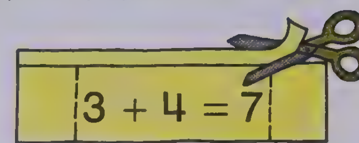
Investigation
Page e-53

Read the directions at the top of the page with the children. Explain to them that the illustration shows what they should do for each of the equations written on the yellow strips at the bottom half of their page. Point out to them that they should cut only on the heavy, dark lines. *The dashed lines are for folding and should not be cut.* After the children have cut out the slips of paper, show them how a fold may be made on each dashed line. When the fold is made on the right side, the sum is hidden. When the fold is made on the left side, the first numeral in the equation is hidden. After children know how the fold works, tell them to unfold all of their slips of paper and place them face down on the desk. Then they should work with a classmate. A child picks a color. His partner then picks up that slip and folds one side. He shows the equation to the child who called the color and that child must then identify the hidden number. The other child then unfolds the slip to check his answer. Then the two children should switch positions. Note that the children will be working with combinations either to find the sum or, when the fold is on the left, to find the missing addend. The missing addend concept might be a new concept for some children, but it should not cause difficulty because it is presented in the context of mastering already familiar addition facts.

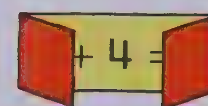


Cut and fold the slips of paper.

Step 1



Step 2



$3 + 4 = 7$	$4 + 5 = 9$
$8 + 2 = 10$	$6 + 4 = 10$
$1 + 7 = 8$	$2 + 5 = 7$
$6 + 3 = 9$	$4 + 4 = 8$
$3 + 7 = 10$	$4 + 2 = 6$

Readiness for missing addends

Purposes

To provide the children with an interesting environment in which they will be forced to think of the concept of the missing addend in terms of a hidden numeral or a hidden set of objects

To introduce the concept of missing addend

Some children might have worked with the missing addend in the first book. In this series, the missing addend in Book 1 was the second numeral in the equation. Here, however, they will be working with the missing addend as the first numeral of the equation. This is not a change in concept, but simply a change in notation. It is done to help with the development of subtraction speed skills. That is, to solve $7 - 3 = \square$, we want the child to think "backward" and read, "How much goes with 3 to make 7?" Or, $\square + 3 = 7$.

Preparation

Materials

scissors

Since the Investigation of this lesson is quite activity-oriented and children have just finished a learning unit on the addition and subtraction combinations of 10 or less, you might begin immediately with the investigation.

Let's talk

Solve the "dog-house" equations.

$\boxed{5} + 3 = 8$

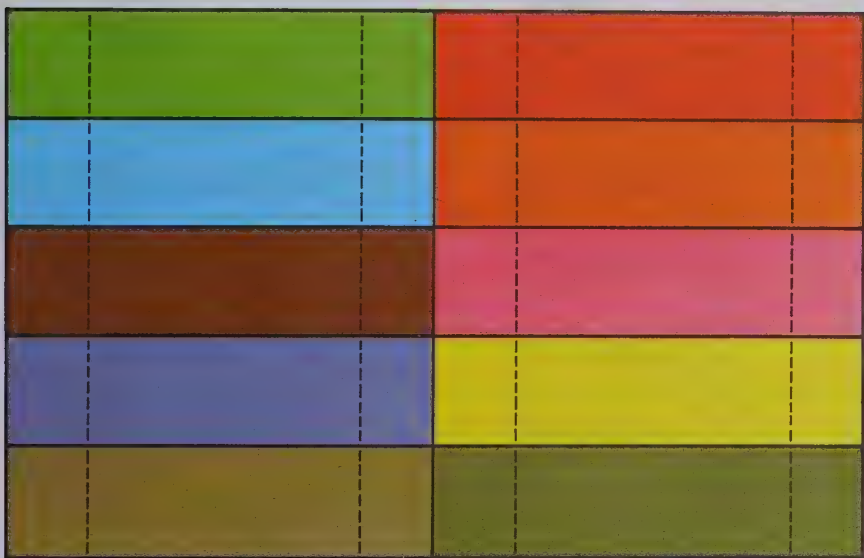
$\boxed{2} + 5 = 7$

$\boxed{4} + 2 = 6$

$\boxed{5} + 5 = 10$

$\boxed{5} + 4 = 9$

$\boxed{0} + 8 = 8$



Discussion
Page e-54

Call attention to the illustration at the top left of the page. Ask the children how many dogs they see, (3), and how many dogs according to the sign are supposed to be in the pen. Ask them where they think the dogs that they cannot see are. Finally, ask them how many dogs they think are in the dog house. After children see that the illustrated doghouse represents the number of dogs that are missing, pretend that there are other numbers of dogs in the entire pen as well as other numbers of dogs visible. Use the other equations on this page with the picture. Thus, if the sign said 7 dogs and 5 were visible, how many would be in the doghouse? If the sign said 6 and 2 were visible, how many would be in the doghouse? Help them relate the answers that they give to the missing addends that should be written in the boxes of these equations.

Follow-up

"Combo," a combination bingo-type game, can provide an excellent review of facts for small groups or for the whole class. Use scrap cardboard to make game boards measuring approximately 13-by-15 cm, or use large index cards. The simplest board usually has 25 units ruled off into 5 rows and 5 columns, with the numerals 0 through 10 filled in at random. Allow capable children to make these cards for the class.

To play the game, appoint a leader to call out addition or subtraction facts. The children should then cover the proper sum with bright paper counters. Vary the game by writing the combinations (2 + 3, 4 + 6) on the game board and having the leader call out the sums or differences. Instruct the children to cover any combination that equals the sum called out.

TEACHING
Page e-55

Call the children's attention to the illustration and sample equation in the demonstration art. Point out that the kangaroo is writing down how many dogs must be in the doghouse. There are three outside and five in all, so we have $\square + 3 = 5$. If it is necessary, work the two illustrated frames with the children. Point out to them how they might think of the number of dogs in the doghouse, plus the second number, to find the sum. The boxes are made like doghouses to help this type of thinking. Encourage the children to work through the remaining equations on this page by themselves. You might encourage them to work with the counters if they feel the need to. Again they should begin by building a set of counters equal to the sum in their equation and separate this set so that one subset is the number that is shown and then the number of the remaining subset will be the first frame or missing addend.

Solve the equations.

$\boxed{2} + 2 = 4$	$\boxed{2} + 4 = 6$
$\boxed{1} + 4 = 5$	$\boxed{4} + 4 = 8$
$\boxed{6} + 1 = 7$	$\boxed{1} + 9 = 10$
$\boxed{0} + 4 = 4$	$\boxed{4} + 3 = 7$
$\boxed{3} + 3 = 6$	$\boxed{3} + 6 = 9$
$\boxed{3} + 0 = 3$	$\boxed{6} + 2 = 8$

OBJECTIVE

Given an addition equation for a sum of 10 or less with a first addend missing, the child will be able to solve the equation.

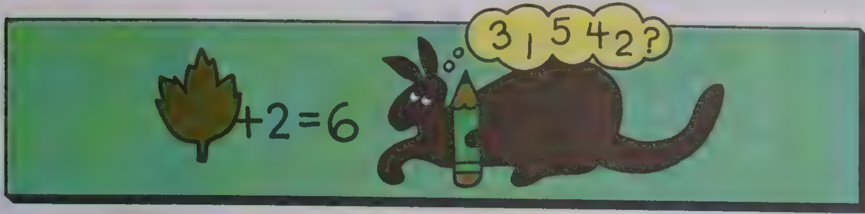
In this lesson the child works with missing addend equations. This is the first step in developing the use of missing addends in relation to finding differences. Knowledge of basic addition facts should be stressed.

PRE-BOOK ACTIVITY

Materials

10 counters (at least) for each child
yarn for each child, approximately 30 cm long

Ask the children to place on their desk or table area a set of six counters, encircled by the yarn. Then appoint children that you will call the robbers. These children should go around and remove from each set a particular number of counters. For example, if you have five children working at a table, you can have one child be the robber for that table. It is not essential that he remove the same number of counters from each set, but you might have him do so if you wish. Instruct all the children who are not the robbers to put their heads down. The robbers should then take some number of counters from each set. Then call out, "Catch the Robbers". Each child should then try to figure out how many counters were removed from his set and write an equation for this. You might want to work through a few examples together first. For example, have all of the children begin with the same number of counters, such as six, and



Find the hidden numeral.

3 + 4 = 7	2 + 2 = 4
1 + 2 = 3	1 + 1 = 2
2 + 4 = 6	3 + 3 = 6
3 + 5 = 8	1 + 9 = 10
1 + 4 = 5	2 + 7 = 9

Solve the equations.

$\boxed{3}$ + 4 = 7	$\boxed{5}$ + 1 = 6
$\boxed{6}$ + 0 = 6	$\boxed{4}$ + 5 = 9
$\boxed{2}$ + 8 = 10	$\boxed{0}$ + 5 = 5
$\boxed{3}$ + 2 = 5	$\boxed{5}$ + 2 = 7
$\boxed{6}$ + 2 = 8	$\boxed{5}$ + 5 = 10

Finding missing addends

Teaching
Page e-56

Again call attention to the illustration at the top of the page and point out that the leaf has simply covered the first number of an equation, □ + 2 = 6. The equations on this page may be thought of in the same manner. That is, a leaf is simply hiding the first number of an addition equation. At the top section of the page, the children should simply write the missing addend on the shaded leaf. At the bottom of the page, they should write the missing addend in the box provided. Again, encourage those children who seem to have the need to do so to work with counters.

quietly tell all the robbers just exactly how many counters you want them to take for the first time, such as 2. Then write on the blackboard: □ + 4 = 6. The children will then have to figure out what number of counters the robber took and put that number in the frame. As the children understand what they are to do, you might gradually challenge them to write as well as solve the equations.

Follow-up

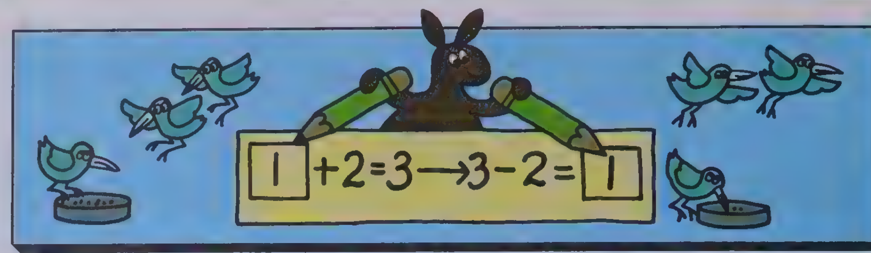
A game called "In the bag" may help the children find missing addends. Invert eight paper cups on a demonstration table, and make sure that all the children can identify the total number of cups in the set. Then place five paper cups in a large paper bag while the children put their heads down. Say: "There are three paper cups on the table." "How many are there in the bag?" Continue by putting different numbers of cups in the bag and varying the number of the initial set.

To give children further practice in finding missing addends, duplicate a worksheet similar to the one following. Direct the children to find the missing addends.

□	7	4	□	□	3	6	2
+ 3	+ □	+ □	+ 4	+ 6	+ □	+ □	+ □
8	10	7	9	8	9	10	10
8	2	□	□	4	5	□	□
+ □	+ □	+ 8	+ 4	+ □	+ □	+ 5	+ 2
10	7	9	8	6	7	10	9

Teaching
Page e-57

Relate the sample problems at the top of the page to the development of the lesson. Encourage the children to make up stories to correspond to the pictured sets. Then ask one child to solve the addition equation and another, the subtraction equation. Stress the relationship between "doing" and "undoing" by using arrows as shown in the text. Finally, direct the children to do the last four exercises independently.



Solve the equations.

$\boxed{3} + 2 = 5 \longrightarrow 5 - 2 = \boxed{3}$

$\boxed{2} + 4 = 6 \longrightarrow 6 - 4 = \boxed{2}$

$\boxed{1} + 3 = 4 \longrightarrow 4 - 3 = \boxed{1}$

$\boxed{4} + 2 = 6 \longrightarrow 6 - 2 = \boxed{4}$

$\boxed{5} + 4 = 9 \longrightarrow 9 - 4 = \boxed{5}$

$\boxed{4} + 3 = 7 \longrightarrow 7 - 3 = \boxed{4}$

Objective

Given an addition equation in which the first numeral is missing and a related subtraction equation, the child will be able to find the difference for the subtraction equation by finding the missing addend of the addition equation.

This lesson should help children relate the missing addend equations to their related subtraction equations.

Pre-Book Activity

Materials

counters
paper bags

Put some number of counters in a paper bag without letting the children know how many counters you are putting in. For example, put in four counters. Then tell the children that you want to add to the number of counters in the bag. Call a child to come up and add, for example, three other counters. Then put the contents of the bag on the table so that everyone can see that there are now seven counters. Ask the children if they can figure out how many counters were in the bag. Have the children talk about ways of figuring out how many counters you put in the bag to begin with. Eventually, elicit from the children that in order to find how many counters there were in the beginning, they might remove the counters that they added and then count the ones that are remaining. For example, if you had put four counters into the bag and a child had put three counters into the bag, another child might come up and remove three

Solve the equations.

$\boxed{2} + 1 = 3$	$\boxed{1} + 2 = 3$
$3 - 1 = \boxed{2}$	$3 - 2 = \boxed{1}$
$\boxed{7} + 2 = 9$	$\boxed{0} + 3 = 3$
$9 - 2 = \boxed{7}$	$3 - 3 = \boxed{0}$
$\boxed{5} + 2 = 7$	$\boxed{2} + 4 = 6$
$7 - 2 = \boxed{5}$	$6 - 4 = \boxed{2}$
$\boxed{5} + 1 = 6$	$\boxed{7} + 3 = 10$
$6 - 1 = \boxed{5}$	$10 - 3 = \boxed{7}$
$\boxed{4} + 3 = 7$	$\boxed{4} + 4 = 8$
$7 - 3 = \boxed{4}$	$8 - 4 = \boxed{4}$

The inverse relation

Teaching
Page e-58

With the children, review the pairs of equations in each frame and show how they are related. Go through the first one or two frames with the class if necessary.

When you are certain that the children know what to do, ask them to complete the page by themselves. Move around the room and help any child having difficulty.

counters from the bag. Then the number of counters in the bag would be the same as the number that you had put in the bag. Work through many examples of this kind with the children. If the children are capable, they might work as partners. One partner may put counters into the bag, and then direct his partner to put another number of counters in the bag. Finally this partner should count the total and figure out how many were in the bag to begin with.

Follow-up

You might encourage children to use sets to show the inverse relation. For example, as they study the inverse relations shown on the following worksheet, different colored counters or the beans painted on one side (as suggested on page e-52 of the previous module) would be suitable materials.

Find the pattern. Complete the table.

Start	6	2	8	4	7	1	5	3
Add	+3	+5	+2	+6	+4	+9	+8	+7
Sum	9	7						
Subtract	-3	-5	-2	-6	-4	-1	-8	-7
Finish	6	2						

Resources for Active Learning

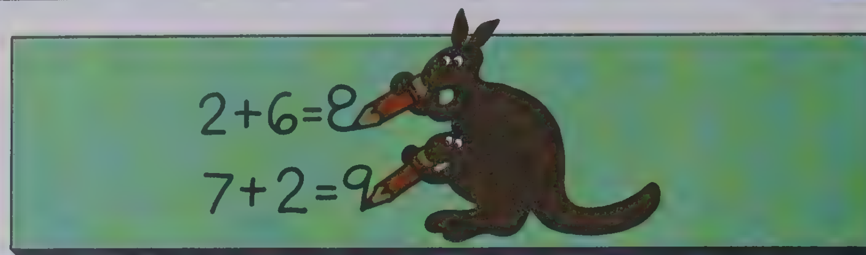
MATH ACTIVITY CARDS, "Addition Chart," A2, Macmillan

Teaching
Page e-59

Although most of the equations on this page are to be worked independently by the children, be sure that they understand that they are free to use any materials they choose. Point out that the left frame contains only addition equations and the frame on the right contains only subtraction equations. Also remind them that the differences in the subtraction equations may be found by thinking of missing addends. For example, in the first subtraction equation $6 - 4 = \square$, they might read this backwards and think: "What number should I add to 4 to get 6?" Even after children have completed these equations, it would be helpful to work through this type of thinking with them. Note that the vertical notation problems at the bottom may be thought of in the same way as the equations at the top. Even for the subtraction problems written in vertical notation, a child might think: "What number must I add to 3 to get 6?"

$$\begin{array}{r} 6 \\ -3 \\ \hline ? \end{array}$$

This missing addend is their answer.



Solve.

$4 + 3 = \boxed{7}$	$6 - 4 = \boxed{2}$
$5 + 1 = \boxed{6}$	$5 - 1 = \boxed{4}$
$3 + 6 = \boxed{9}$	$9 - 3 = \boxed{6}$
$0 + 5 = \boxed{5}$	$10 - 4 = \boxed{6}$
$7 + 3 = \boxed{10}$	$7 - 6 = \boxed{1}$
$4 + 4 = \boxed{8}$	$8 - 8 = \boxed{0}$

$$\begin{array}{r} 6 \\ +0 \\ \hline 6 \end{array} \quad \begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array} \quad \begin{array}{r} 5 \\ +3 \\ \hline 8 \end{array} \quad \begin{array}{r} 8 \\ +1 \\ \hline 9 \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \quad \begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$$

$$\begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array} \quad \begin{array}{r} 8 \\ -1 \\ \hline 7 \end{array} \quad \begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array} \quad \begin{array}{r} 10 \\ -2 \\ \hline 8 \end{array} \quad \begin{array}{r} 5 \\ -0 \\ \hline 5 \end{array} \quad \begin{array}{r} 9 \\ -7 \\ \hline 2 \end{array}$$

Objective

Given an addition or subtraction equation related to the sums of 10 or less, the child will be able to find the sums or differences.

Pre-Book Activity

Materials

one set of strips per child

Ask the children to take the black strip and the yellow strip and put them in parallel positions. Then ask the children if they can figure out what strip they have to put with the yellow strip to make a train matching the black strip. When a child responds, "red," ask if anyone can think of an equation which would show what they have just done. Since the yellow strip represents 5, they should think of 5. Since they had a blank space, they should write a box. $5 + \square$ was to equal the length of the black strip, so their equation should be $5 + \square = 7$, or $\square + 5 = 7$.

$\square + 5 = 7$ $\qquad$ $7 - 5 = \square$

$2 + 5 = 7$ $\qquad$ $7 - 5 = 2$

As they discover what strip was missing, they should write the 2 in the equation. Work through several examples of this kind until children can build equations with missing addends by using the strips.

Short Stories

1 Pam saw 5 birds.
3 flew away.
How many stayed? 2

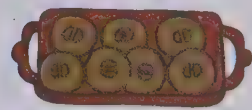
2 Tom had 5 apples.
He ate 4.
How many were left? 1

3 Sue paid 5 cents for a pencil.
She paid 4 cents for paper.
How much did she spend? 9

4 Ted ate 2 candy bars.
Bob ate 3 candy bars.
How many did they eat? 5

5 Jane walks 4 blocks to school.
She walks 4 blocks home.
How far does she walk? 8

6 Fred had 3 cents.
He found 5 cents.
How much does
he have now? 8



7 Betty had 7 cookies.
She and Mary each ate 1.
How many were left? 5

8 Jack has 6 fish.
Dick has 3.
How many in all? 9

9 10 chairs are in the room.
6 children are in chairs.
How many empty
chairs? 4

10 6 girls and 4 boys were
at a party. How many
children were there? 10

Story problems

Teaching
Page e-60

Although these short stories deal with combinations of 10 or less with which the children are familiar, you will probably want to work carefully through this page with them to help them become accustomed to doing story problems. You might prepare separately for this page by giving the children oral story problems before they actually begin the work on this page. For example, develop story problems about children in your class in everyday situations, such as:

"Jerry and Jeff were riding their bicycles when they heard the chimes of the ice cream man. They stopped to count their money. Jerry had 6¢ and Jeff had 4¢. How much did they have altogether?"

Give the children an opportunity to make up story problems on their own. If possible, encourage the children to read these short stories and work them independently. Be sure the children understand that they may use counters or other concrete materials while solving the problems. It would also be helpful for them to draw pictures for some of the problems. Note that exercises 7 and 9 many be turned into comparison subtraction problems simply by asking: "How many more cookies are there than children?" or in problem 9: "How many more chairs are there than children?" However, only discuss these questions with a mature group of children.

Follow-up

To motivate children to practice addition and subtraction facts, draw "ladders" on the chalkboard and write combinations on each rung. Top the ladders with stars, increasing the number of stars to correspond with the difficulty of the combinations. Choose a capable child to monitor each ladder and give out gummed stars as they are earned. Assign children to begin at the bottom of the ladders and give the sums and differences just loud enough for the monitor to hear. Each child who reaches the top, gets the number of stars drawn there. Encourage the children to collect more stars by trying the ladders with more difficult combinations.

☆	☆ ☆	☆ ☆ ☆	☆☆☆☆
6 + 0	3 + 4	6 + 2	9 − 5
2 + 3	7 − 2	7 − 6	10 − 6
4 + 1	2 + 5	8 − 4	4 + 5
2 + 2	8 − 6	3 + 5	2 + 7
0 + 5	1 + 9	4 + 5	7 + 3
3 + 2	10 − 3	9 − 7	9 − 1

You might also give the children sheets of newsprint and crayons. Instruct them to fold the paper into three sections and write the numeral 10 in a corner of the top section. Tell the children to write as many combinations for 10 as they can think of in that section. Ask them to write combinations that make 9 in the next section and those for 8 in the third section. Let the children use manipulative materials, but do not allow them to copy from tables for this particular activity.

Teaching
Page e-61

Point out to the children that the top frames of equations are missing addend equations and that they should think, for example, "What number plus 3 equals 6?" Point out that all the equations in the frames at the middle of the page are subtraction equations. They may use manipulative materials to solve these equations if they wish, but remind them to try to do these equations as much as possible without materials and that they may again solve these by finding the missing addend. You might observe with the children the inverse relationship between the equations in these frames and those in the top frames. The exercises at the bottom are also related: the vertical notation addition exercises are related to the subtraction exercises below them. You may choose to point out this inverse relationship to the children before or after they have completed the page. In any event encourage the children to complete the page independently.

Show you know

Solve.

$\boxed{3} + 3 = 6$	$\boxed{6} + 4 = 10$
$\boxed{1} + 8 = 9$	$\boxed{2} + 3 = 5$
$\boxed{6} + 2 = 8$	$\boxed{5} + 3 = 8$
$\boxed{2} + 5 = 7$	$\boxed{3} + 6 = 9$
$6 - 3 = \boxed{3}$	$10 - 4 = \boxed{6}$
$9 - 8 = \boxed{1}$	$5 - 3 = \boxed{2}$
$8 - 2 = \boxed{6}$	$8 - 3 = \boxed{5}$
$7 - 5 = \boxed{2}$	$9 - 6 = \boxed{3}$

$\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$	$\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$	$\begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array}$	$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$	$\begin{array}{r} 8 \\ +2 \\ \hline 10 \end{array}$
$\begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array}$	$\begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array}$	$\begin{array}{r} 10 \\ -4 \\ \hline 6 \end{array}$	$\begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array}$	$\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array}$	$\begin{array}{r} 10 \\ -2 \\ \hline 8 \end{array}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Since this is a review lesson, you might choose to review the combinations of 10 or less with a game such as "Cross Over the Bridge."

Direct two teams of children to line up on opposite sides of the room. Suggest that the children imagine a bridge across the open space at the front of the room. The first two members of each team compete against each other in answering flash cards showing addition and subtraction combinations to 10. When a flash card is shown, the first one to give a correct answer may stay on his side of the river. The other person must "cross over the bridge" and go to the end of the other team's line. At the end of the game, the team with the most members is the winner.

Let's have fun

Complete the counting. Color every third box.

Complete the skip counting by threes.

0	3	6	9	12	15	18	21	24

Complete the counting. Color every fourth box.

Complete the skip counting by fours.

0	4	8	12	16	20	24	28

Skip counting

Teaching
Page e-62

Treat this change of pace page with a light touch. Read the directions with the children. At the top of the page they should color every third box, then they should complete the skip counting by threes. Show how the green boxes underneath the path can be related to the numbers which they colored on the path. Similarly in the bottom part of the page, they should color every fourth box. Then they should complete the skip counting by fours, again relating the blue boxes at the bottom of the page to the blue boxes they colored on the path. Use this page to orally review skip counting.

Follow-up

Many children will enjoy a simple decoding exercise such as the following.

Write the correct numerals in the answer boxes.
Then write the corresponding letters in the message boxes.

								Code			
	3	8	9	5	3	7	8	0	C	5	G
	+4	−7	−5	+3	+0	+2	−6	1	A	6	S
								2	N	7	H
Answer	□	□	□	□	□	□	□	3	F	8	E
								4	V	9	U
Message	□	□	□	□	□	□	□				

Resources for Active Learning

Number patterns:

DEVELOPMENTAL MATH CARDS, C[1]14, E[1]9, Addison-Wesley

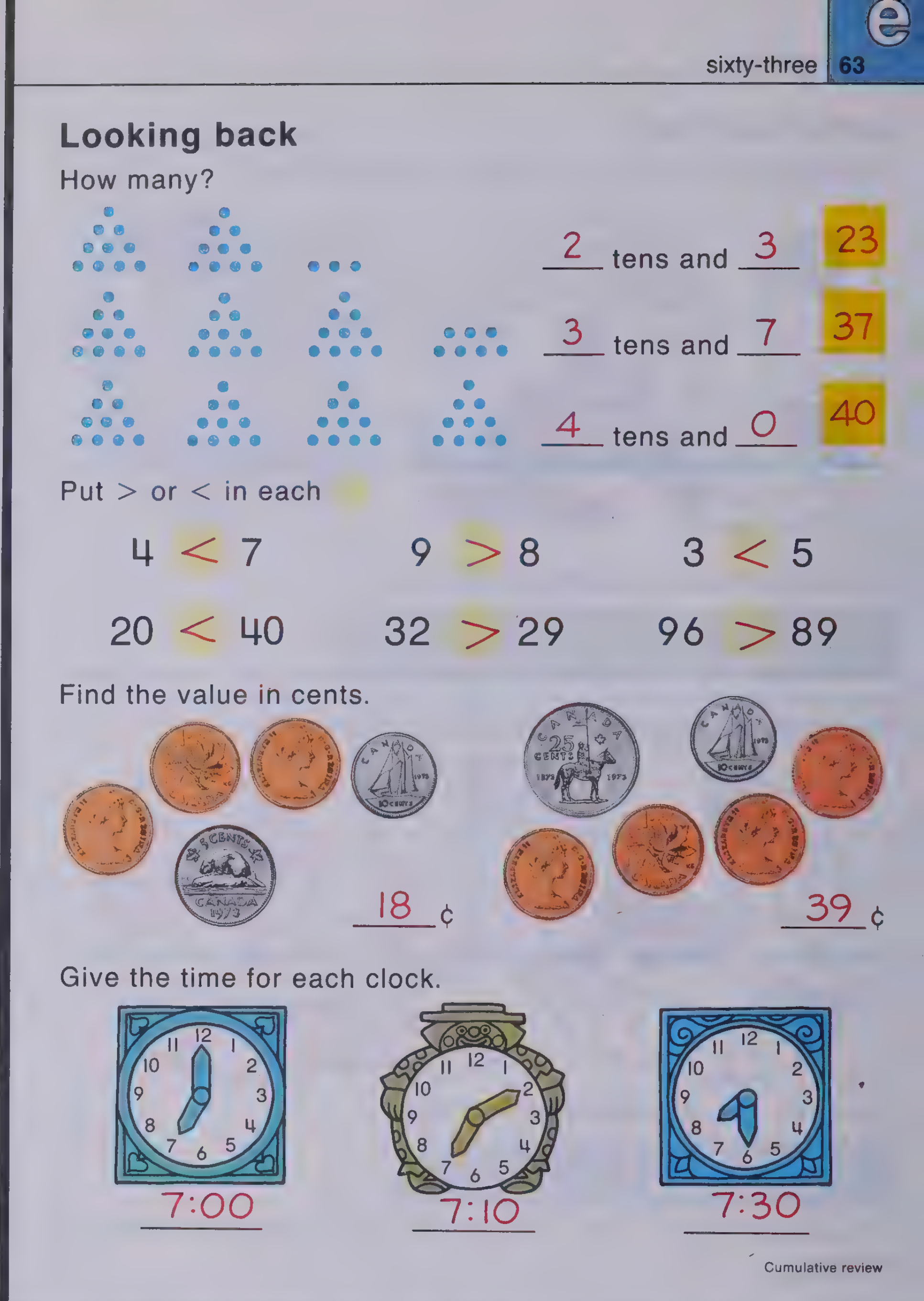
MATH ACTIVITY CARDS, A43, Macmillan

MATHEX: Matching and Graphing No. 1, Activity 6, pp. 13–14; Numeration No. 2, pp. 37–41, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 42–57, Wiley

Teaching
Page e-63

Since the directions for this page differ for each section, it will be important to give careful directions to the children. With some children you might want to work through each frame, giving the directions and waiting for them to complete the work. In the first frame, explain that they should count the number of dots shown and express it in terms of how many tens and how many ones and then write the two-digit numeral in the yellow box provided. For the next section the children simply write in the greater than or less than sign to show the comparison between each pair of numbers. Children should not have difficulty with the straightforward directions in the bottom two frames.



e

Looking back

How many?

2 tens and 3 — 23

3 tens and 7 — 37

4 tens and 0 — 40

Put > or < in each

4 < 7   9 > 8   3 < 5

20 < 40   32 > 29   96 > 89

Find the value in cents.

18 ¢   39 ¢

Give the time for each clock.

7:00   7:10   7:30

Cumulative review

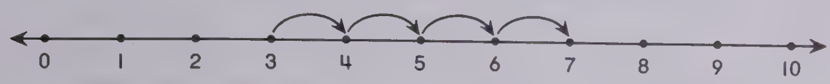
Objective

The child will demonstrate his ability to work with the concepts presented in Unit E.

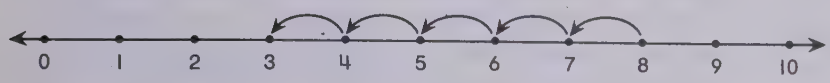
Pre-Book Activity

Use an oral game to review any concepts with which the children have had particular difficulty throughout this book. For example, the "What's My Rule?" game might be used. Change your rules, so that you review not only the basic facts, but also topics such as grouping by tens. For example, you might ask the children to give you a two-digit number less than one hundred (such as 32). Your response should be to describe that number in terms of tens and how many more. Thus, you would say 3 tens and 2. When a child thinks he knows your rule, he should fold his arms. Then you should give him a two-digit number such as 45 and he should respond with the phrase 4 tens and 5. Another rule might be simply to give two numbers and ask the children to respond "greater than" or "less than."

Solve the equations.



$3 + 4 = \boxed{7}$



$8 - 5 = \boxed{3}$

Add.

5	4	5	3	7	1	4
+2	+4	+3	+6	+3	+8	+5
7	8	8	9	10	9	9

Subtract.

9	10	8	7	10	6	9
−3	−4	−6	−7	−3	−3	−5
6	6	2	0	7	3	4

Solve the equations.

$\boxed{2} + 3 = 5$ $5 - 3 = \boxed{2}$

$\boxed{3} + 4 = 7$ $7 - 4 = \boxed{3}$

$\boxed{8} + 2 = 10$ $10 - 2 = \boxed{8}$

Cumulative review

Teaching
Page e-64

Explain to the children that on this page they will have an opportunity to show their skill with the addition and subtraction combinations they have studied in this Unit. Point out the number lines provided in the first frame. In the second frame, be sure the children understand that they should find the sums and differences. Finally, direct them to solve the equations in the last frame. If you prefer, this entire review may be used as a class activity to summarize the work of this Unit.

Follow-up

A game such as Combo might be used to conclude the work in this module. See page 83 for directions.

Basic Principles

Pages f-1 to f-10

General Objectives

To introduce the order (commutative) principle of addition

To introduce the use of parentheses

To introduce the grouping (associative) principle of addition

To introduce the child to the fact that the grouping and order principles may be generalized into a "rearranging" principle

Following the introductory investigation with strips, the children are introduced to the idea of *order* in addition. That is, by working equations in pairs such as $2 + 3 = 5$ and $3 + 2 = 5$, the children are led to see that $2 + 3 = 3 + 2$ and, in fact, that this idea holds for any two whole numbers.

Following this, the children are introduced to the use of parentheses in equations with three addends. They are led to discover that the parentheses *group* the two addends they are to add first. They also discover that changing the grouping does not change the sum.

Mathematics

Two important basic principles for addition are introduced in this module. The first principle is the *order* principle (*commutative* principle) which states that for any whole numbers a and b, $a + b = b + a$. That is, changing the order of two addends does not change the sum in addition. Since addition is often demonstrated by the union of disjoint sets, this principle is readily grasped by most children.

The second basic principle is called the *grouping* principle (*associative* principle). This principle is stated as follows:

If a, b, and c are whole numbers, then

$$a + (b + c) = (a + b) + c.$$

The expression $a + (b + c)$ indicates that the sum of b and c is to be added to a. The expression $(a + b) + c$ indicates that c is to be added to the sum of a and b. The grouping principle of addition states that each of these expressions represents the same number. In other words, given three numbers in a definite order, we obtain the same result whether we add the sum of the last two numbers to the first or add the third number to the sum of the first two.

Before discussing the grouping principle with the children, you must explain what the parentheses mean. You may do this most easily by telling children that the parentheses show which operation is to be performed first.

Since addition is both commutative and associative, we may change both the order and the grouping of addends without changing the sum. For this reason, it is possible to omit parentheses entirely in addition problems involving more than two addends. Thus, instead of writing $(4 + 3) + 2$ or $4 + (3 + 2)$, we may simply write $4 + 3 + 2$. Furthermore, the addends may be *rearranged* in various ways such as $4 + 2 + 3$, $2 + 3 + 4$, $3 + 4 + 2$ without affecting the sum.

Teaching Yellow Module, Unit F

Approximate Time: 5 to 7 days

Materials

counters
cubes or blocks
large cards showing numerals from 1 to 9
objects for set demonstrations (optional)
sets of clothespins of different colors
set of strips for each child

Vocabulary

associative	grouping principle	parentheses
commutative	order	
grouping	order principle	

You will notice the terms grouping principle and order principle in the vocabulary. If you have a particularly adept group of children, you may choose to introduce the names commutative principle and associative principle. Some children enjoy learning and using these technical terms.

Encourage the children to verify their work with the equations by using strips, counters, or other concrete devices. You might extend the development of these pages by using the number line and/or set demonstrations. However, most children will benefit from an approach which allows them to manipulate concrete objects themselves as some of the pages require.

Evaluation of Progress

Your evaluation of the children's achievement in this unit should be based primarily on how well the children understand and use the two basic principles. This, of course, includes how well they understand that three numbers may be added in any order, and that addends may be rearranged without changing the sum.

Resources for Active Learning

Manipulative Devices:

Beads and string (CCM School Materials; Childcraft; Ideal)
Cogno-Board (Teaching Resources)
Colored beads and pattern cards (Lakeshore; Teaching Resources)
Cubical counting blocks, (Milton Bradley; school supplier)
Pegboards (school supplier)
Sigma Chips (Scott Scientific)

Commercial Games:

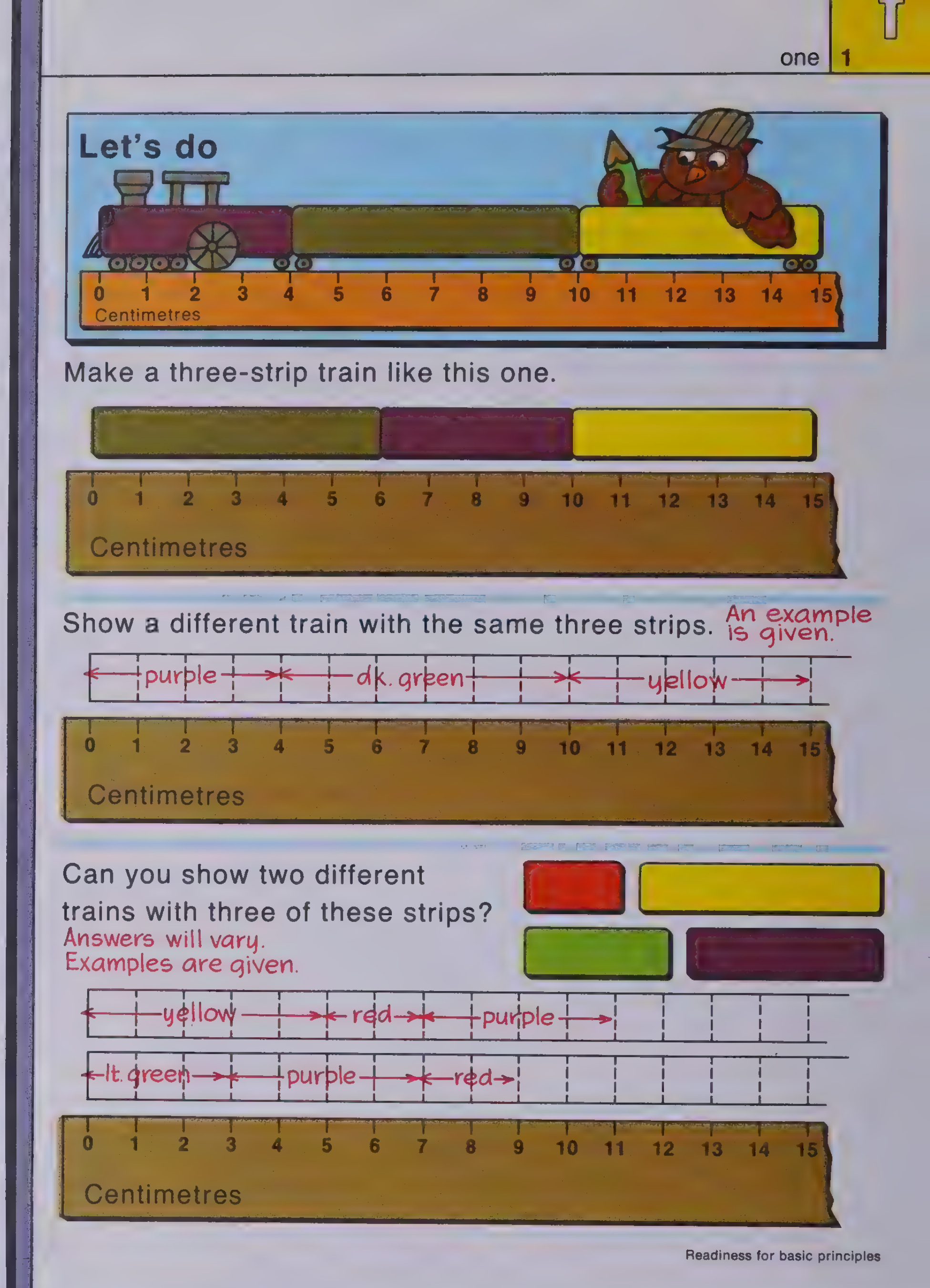
Games of Strategy:

Matrix (Creative Publications)
Tac Tickle (Gamco; Wff'N Proof)
TUF (Creative Publications; Cuisenaire Co.; TUF)
Wff (Childcraft; Cuisenaire Co.; Wff'N Proof)
Scan (Math Media)
Think-A-Dot (Childcraft; Cuisenaire Co.; Edmund Scientific)

Investigation
Page f-1

Call attention to the demonstration art at the top of the page. Point out that the owl has used the top edge of the centimetre ruler as a number line. The children should take their strips and build the three-strip train on top of the green, purple, and yellow train illustrated in the first frame. Then explain that they should use those same three strips to make a different train in the blank space above the centimetre ruler in the next frame. After they have rearranged their strips, instruct them to color in the spaces to show the train that they made. Then explain to them that in the bottom section of the page, they should choose three strips from the four strips that are shown. Again they should make a three-strip train. There are spaces for them to show two different trains. Again, instruct them to build their trains, then color the spaces to show the trains they built. As the children work, emphasize that the length of the trains is the same even when the strips are rearranged. If the children want to refer to the strips by their number names, encourage them to do so.

Note that it is necessary to direct the children to begin their train above the zero mark of the illustrated centimetre ruler.


one 1

Let's do

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Centimetres

Make a three-strip train like this one.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Centimetres

Show a different train with the same three strips. An example is given.

purple dk. green yellow

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Centimetres

Can you show two different trains with three of these strips?
Answers will vary.
Examples are given.

yellow red purple

lt. green purple red

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Centimetres

Readiness for basic principles

Purpose

To provide readiness for the idea that addends can be rearranged in any way and still give the same sum

The strips provide an excellent aid in developing the concept of rearranging addends. The length of a train of two or three strips remains constant even though the arrangement of the strips changes.

Preparation

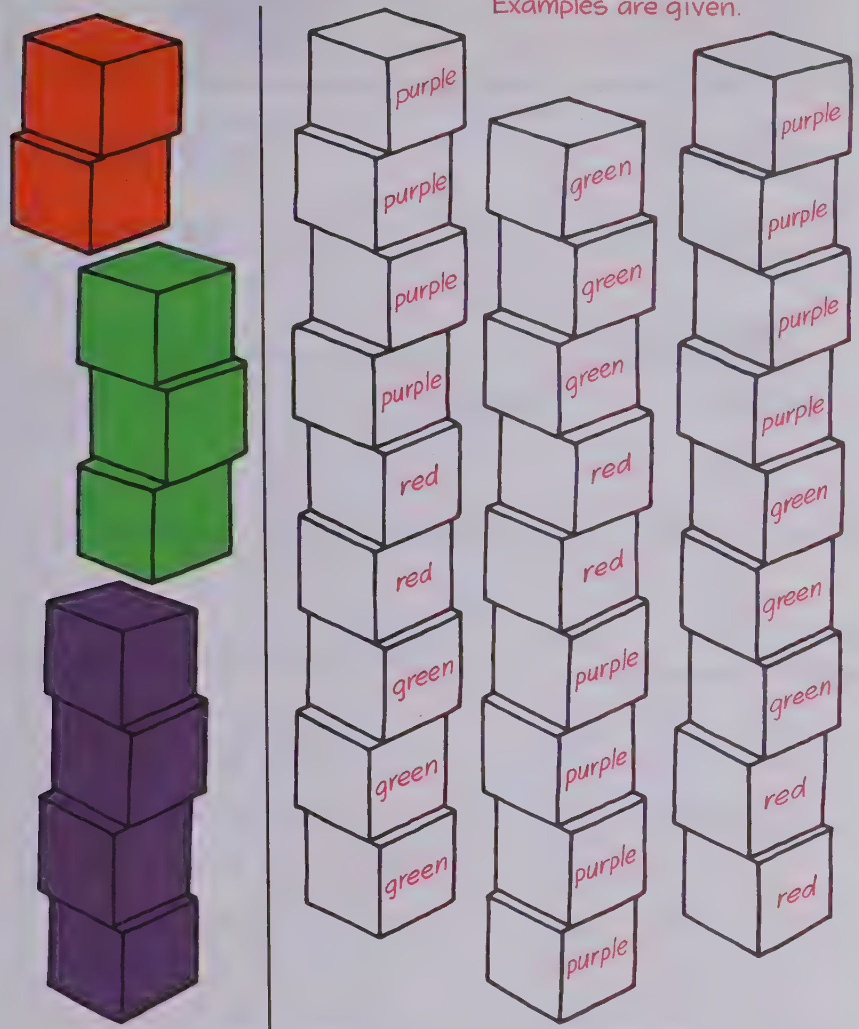
Materials

a set of strips for each child

Encourage the children to participate in free play with the strips. For example, suggest that they make a house, a railroad car, or an airplane. Then ask the children to build various sets of trains using two strips. For example, ask them to make a train with their dark green strip and their red strip and see if they can find one strip which is the same length as these two. Give them other examples. Then ask them to make a train of three strips. Explain to them that in today's book page they will be working with three-strip trains.

Let's talk

Color the towers to show different ways to stack the blocks. Answers will vary. Examples are given.



Readiness for basic principles

Discussion
Page f-2

Guide children in doing the activity on page f-2 before discussing the concept of rearranging addends. Instruct the children that the same color blocks are glued together. That is, the green blocks cannot be separated from each other, nor can the red or purple. Explain to them that they are to color each tower differently to show how they might stack the colored blocks together. This activity contains the germ of the idea for rearranging addends, that is, no matter how the blocks are stacked, the stack is still nine blocks high. After the children have finished the coloring and the investigation on the previous page, centre the discussion on the fact that rearranging the strips does not change the length and rearranging the blocks does not change the height. Here, no matter how the blocks are stacked, they still form a stack of nine blocks.

Follow-up

Encourage children to explore the number of possible arrangements which might be made with three or four blocks, or three or four cards. For example, print a different letter of the alphabet on each of three cards. Ask the children to guess how many arrangements they might make with these cards and then to verify their guess:

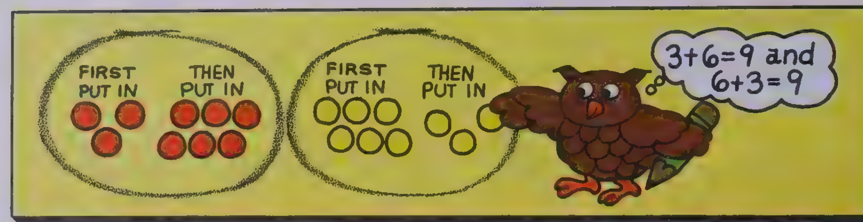
A	B	C	B	A	C
A	C	B	C	A	B
B	C	A	C	B	A

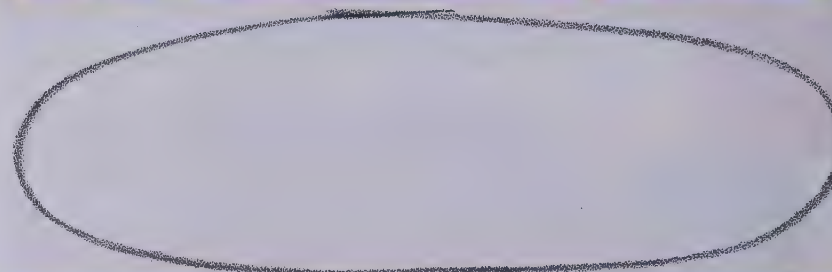
They might chart their findings:

Number of Cards	Arrangements
1	1
2	2
3	6
4	24

Teaching
Page f-3

Point out the ring at the top of the page and the equations written beneath it. Direct the children to put in 5 counters. Then they should put in 4. The sum of $5+4$ answers the question "How many?" and this they should write in the yellow space provided. Be sure children relate the $5+4$ equation to the $4+5$ equation. In the second instance, they first put in the 4 counters and then put in the 5. The pairs of equations in the frames at the bottom should also be worked with counters in the ring. You might read with the children, giving them the directions, "First put in 3." "Then put in 5." Wait for them to write the sum. Then read, "First put in 5." "Then put in 3." Again, wait for them to write the sum. Point out that the sums are the same even when the addends are rearranged. Use as many examples of this kind as you think are necessary for the children to understand that the order of addends in an equation may be changed without changing the sum.





First put in		Then put in		How many?
5	+	4	=	9

First put in		Then put in		How many?
4	+	5	=	9

Solve the equations.

$3 + 5 = \boxed{8}$	$7 + 2 = \boxed{9}$
$5 + 3 = \boxed{8}$	$2 + 7 = \boxed{9}$
$4 + 2 = \boxed{6}$	$6 + 4 = \boxed{10}$
$2 + 4 = \boxed{6}$	$4 + 6 = \boxed{10}$

Commutative (order) principle

Objective

Given an addition equation with two addends, the child will know that the order of the addends may be reversed without changing the sum.

This lesson introduces the order principle, the next lesson will introduce the grouping principle. Even though each principle is occasionally used in isolation from the other, the main purpose of this module is treated on pages f-7 where it is observed that because of *both* principles—the order principle and the grouping principle—we can rearrange addends in any order.

Pre-Book Activity

Materials

counters, 10 per child
set of strips for each child

Since the children work with the counters on page f-3, use the pre-book activity to develop the idea of the order principle by using the strips. For example, ask the children to make a train with the purple strip and the yellow strip. Then ask them how long the train would be if they were to rearrange the order of these two strips. Some children may compare the length of strips of the first train to the single blue strip, then change the order of the strips. Others may take another yellow strip and match these, making two parallel trains, one with the first strip yellow and the second strip purple, the other with the first strip purple and the second strip yellow.

Complete the matching.

3 + 5	6 + 1
2 + 4	7 + 2
1 + 6	5 + 3
2 + 7	1 + 3
3 + 1	4 + 2

Solve the equations.

$7 + 2 = \boxed{9}$ $\qquad 5 + 3 = \boxed{8}$

$2 + 7 = \boxed{9}$ $\qquad 3 + 5 = \boxed{8}$

Complete each addition table.

+	4	3
4	8	4 + 3 7
3	3 + 4 7	6

+	5	3
5	10	5 + 3 8
3	3 + 5 8	6

Commutative (order) principle

Teaching
Page f-4

Since there are three separate sections on this page, children will need particular guidance to know what to do. At the top of the page, they should simply match the sums on the left with the sums on the right. It would also be helpful to have them explain how these sums are different and how they are the same. They are different in order of addends, but they are the same because they represent the same number. In the second section, they need simply solve the equations. Again have them point out what they notice that is the same about each pair of equations. The same addends are used; they are simply added in reverse order. Finally, at the bottom of the page, help the children complete the addition table. As they fill in the space for 4 + 3 and for 3 + 4, ask them what they observe. Be sure they realize that both sums may be represented by the same single numeral.

Follow-up

Develop a worksheet similar to the following one to stress the order principle for addition.

Complete each equation.

3 + 1 = 1 + □	6 + □ = □ + 6
2 + 0 = □ + 2	□ + 4 = 4 + □
7 + □ = 3 + 7	0 + □ = □ + 0
□ + 4 = 4 + 5	2 + □ = □ + 2
3 + □ = 5 + 3	□ + 1 = 1 + □
2 + 3 = □ + 2	□ + 3 = 3 + □
1 + 5 = 5 + □	5 + □ = □ + 5
□ + 2 = 2 + 6	□ + 7 = 7 + □

Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB, Cards 38–43, Responsive Environments Corp.

SETS, NUMBERS AND POWERS, Games, pp. 117–120, Herder and Herder

MATHEX: Numeration No. 2, "Commutative . . . Addition," pp. 33–36, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: MATHEMATICS BEGINS ①, "Addition," pp. 54–55, Wiley

Teaching
Page f-5

Call the children's attention to the three sets displayed in the first exercise. Ask the children to identify the number of each set. Next, point out the numerals 3, 2, and 4 in the equation below the oranges. Then direct the children to observe the grouping shown in the picture by the shading, and also the grouping shown by the parentheses and symbols in the equation. In particular point out the shaded 5 over the $(3 + 2)$ in the first equation and the shaded 6 over the $(2 + 4)$ in the second. Show that in the first equation they are to add 3 and 2 first, and then add 4. Ask the children to do these steps to show that $3 + 2 = 5$ and that by adding 4 to 5 they get the final sum, 9. Then ask them to fill in the box.

As you direct the children's attention to the second picture, observe that the same three sets of objects are illustrated here. Point out that this time the sets on the right are grouped and that the parentheses show this idea in the equation. Tell the children to add 2 and 4 and then to add this sum to 3 to get the total. Be sure that they observe that even though the groupings differ, the number of objects displayed in both pictures is the same.

Now ask the children to finish the page. Point out that in each pair of equations the only difference is in the way the grouping is done.

Solve the equations.

$(3 + 2) + 4 = \boxed{9}$

$3 + (2 + 4) = \boxed{9}$

$(1 + 5) + 3 = \boxed{9}$

$1 + (5 + 3) = \boxed{9}$

$(1 + 3) + 4 = \boxed{8}$

$1 + (3 + 4) = \boxed{8}$

$(2 + 3) + 5 = \boxed{10}$

$2 + (3 + 5) = \boxed{10}$

$(3 + 4) + 2 = \boxed{9}$

$3 + (4 + 2) = \boxed{9}$

Associative (grouping) principle

Objective

Given three addends, the child will find the sum by grouping the first addend with the second or the second with the third as in the equations:

$$(2 + 3) + 4 = \square$$
$$2 + (3 + 4) = \square$$

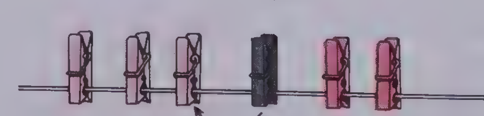
Pre-Book Activity

Materials

cord or rope for "clothesline"
sets of clothespins of different colors (if possible, 3 different colors)

Snap groups of the plastic colored clothespins onto the long edge of a box or onto a stout cord held by two children. Be sure that the color of the middle group contrasts with that of the other two groups. For example, select 3 red, 1 yellow, and 2 green clothespins. Ask the children how many clothespins are clipped to the edge of the box or cord. When someone answers correctly, slide the middle group next to the first group. Then ask the children to count that group, $3 + 1$, *first*, and then add the total to 2 to get 6, You might write the following equations on the chalkboard.

$$(3 + 1) + 2 = 6$$
$$4 + 2 = 6$$



Then regroup the clothespins, this time placing the middle pin with the last group. Ask the children to count

Solve the equations.

$(4 + 1) + 3 = \boxed{8}$

$4 + (1 + 3) = \boxed{8}$

$(2 + 3) + 5 = \boxed{10}$

$2 + (3 + 5) = \boxed{10}$

$(2 + 1) + 3 = \boxed{6}$	$(4 + 3) + 2 = \boxed{9}$
$2 + (1 + 3) = \boxed{6}$	$4 + (3 + 2) = \boxed{9}$
$(1 + 6) + 3 = \boxed{10}$	$(5 + 2) + 1 = \boxed{8}$
$1 + (6 + 3) = \boxed{10}$	$5 + (2 + 1) = \boxed{8}$

Associative (grouping) principle

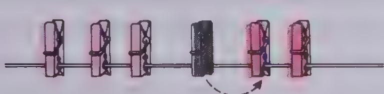
Teaching
Page f-6

Again point out the three sets at the top of the page. The children should identify the numbers of these sets as 4, 1, and 3. Call their attention to the sets that have been grouped and to the way that the parentheses in the equation demonstrate this idea. Instruct the children to complete the first equation by first adding 4 and 1, and then adding 3 to this sum.

Move on to the second row of apples and point out that these sets are the same as those above, except that the two sets on the right are grouped this time. Again point out that the parentheses in the equation demonstrate this grouping. Ask the children to solve this equation by first adding 1 and 3, and then adding 4 to the sum. Point out that 8 is the solution for each equation. Work through the next illustration similarly. Finally, work through the set of examples at the bottom of the page, with the children, eliciting methods for obtaining answers from them. Observe that the sum obtained by using either grouping is the same in each pair of equations.

the first group and then add the resulting 3 to 3 to get 6 again. Show this grouping also in an equation on the board.

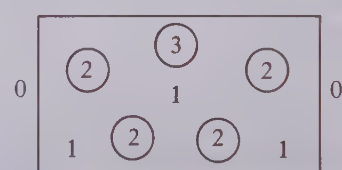
$$3 + (1 + 2) = 6$$
$$3 + 3 = 6$$



Encourage the children to repeat this same procedure with several other groupings of clothespins. Emphasize that the parentheses in the equations show what to do first.

Follow-up

Make a target-practice board by fastening five aluminum-foil pie pans to a large (75-by-90-cm) sheet of tagboard with brass paper-fasteners. For scoring, paint numerals, as shown. Lay the target board on the floor and mark a throwing line two metres from it. Let each child throw three beanbags and keep his own score. As they keep score, encourage the children to use parentheses to show which pair of numbers to add first.



Resources for Active Learning

MATHS MINI-LAB, Cards 23–36, Selective Educational Equipment

SETS, NUMBERS AND POWERS, p. 120, Herder and Herder

Teaching
Page f-7

The activity on this page is very similar to an investigation. Point out the question at the top and read it with the children. Explain that they should record, in the spaces provided, the different ways they may arrange these numbers. Then they should record the sum for each arrangement. When the children have found four different ways to add the numbers, use this as a basis for discussion. Record all of the six possibilities on the chalkboard, asking different children to contribute their choices.

In a similar manner, work through the different ways of adding 1, 4, and 5. Be sure to point out during this lesson that we can write a three addend equation without using parentheses if we want. This is because of the order and grouping principles that they have just discovered.

Can you show four different ways to add the numbers 2, 3, 4? 2+4+3 and 3+4+2 can be used also.

4 + 2 + 3 = 9

2 + 3 + 4 = 9

3 + 2 + 4 = 9

4 + 3 + 2 = 9

Show four ways to add 1, 4, 5. 5+4+1 and 5+1+4 can be used also.

1 + 4 + 5 = 10

4 + 1 + 5 = 10

1 + 5 + 4 = 10

4 + 5 + 1 = 10

Objective

Given an equation with three addends, the child will be able to find the sum by adding them in any order.

Pre-Book Activity

Materials

large cards showing numerals from 1 to 9

Give two children each a large numeral card and ask them to come to the front of the room and stand side by side. Also give a small symbol card with a plus sign on it to another child and have him stand between the first two children. Ask a child to record on the chalkboard the order in which the two children stand. For example, if the numerals are 4 and 5, the child would record $4 + 5$. Then ask the two children holding the cards if they can rearrange themselves so that the numerals appear in a different order. They should then exchange places and the child at the blackboard should record $5 + 4$.

Then give three children each a numeral card and two other children plus sign cards. Ask the children with the numeral cards to arrange themselves in any order. Again have a child record their arrangement on the chalkboard. When they have done this for the first grouping and one equation has been written, ask the three children holding the cards if they can rearrange themselves. Have a child at the chalkboard record their rearrangement. Ask them to do this until they have rearranged themselves in all possible ways. For three cards there are six possibilities. Use as many examples as seem necessary to stress that addends may be rearranged in any order.

Solve.

3 + 1 + 2 = 6

4 + 1 + 4 = 9

5 + 2 + 1 = 8

1 + 4 + 3 = 8

4 + 2 + 1 = 7

5 + 2 + 3 = 10

2 + 3 + 3 = 8

4 + 3 + 2 = 9

2 + 1 + 3 = 6

3 + 4 + 3 = 10

3 + 2 + 5 = 10

3 + 2 + 1 = 6

1	2	4	2	3	2
2	3	1	4	3	3
+ 1	+ 1	+ 2	+ 4	+ 3	+ 2
4	6	7	10	9	7

4	2	3	5	4	1
1	1	4	2	3	5
+ 1	+ 2	+ 2	+ 3	+ 1	+ 4
6	5	9	10	8	10

Rearranging addends

Teaching
Page f-8

Call the children's attention to the equations at the top of the page. Point out that the numerals are not grouped with parentheses and observe with the children that the numerals may be added in any order. Point out that although the numerals are written in vertical notation at the bottom of the page, they may add them as they do the equations. Encourage the children to work the page independently. As they work, it would be helpful to move around the room and ask individual children to think aloud a problem for you, particularly vertical notation problems at the bottom of the page. If necessary, point out to a child that he may begin to add a column either from the top or from the bottom.

Follow-up

Prepare three dice on which you have marked the numerals 1, 2, 3, 4, 1, 2. Direct the children to play in groups of three or four. Suggest that each child take a turn rolling the three dice. For each roll all of the players should try to see how many equations they can write using the three numerals as addends. It would be well to use an egg timer or the like to establish a time limit. For more capable groups, suggest that they use parentheses in each equation. For a roll in which each numeral is different, these children should be able to find no more than 12 equations. When two numerals that are the same appear they can find only 6 equations, including the various groupings with parentheses. You might have them decide what to do when three numerals that are the same appear.

Resources for Active Learning

Patterns:

MAPPING GAMES, 6, 7, Webster, McGraw-Hill

MATHEX: Matching and Graphing No. 1, Activities 1–5, 7, 8, pp. 12–14, Encyclopaedia Britannica Publications

NOTES ON MATHEMATICS IN PRIMARY SCHOOLS, pp. 24–33, Cambridge University Press

Nuffield Project: BEGINNINGS 1, "Needlecrafts," pp. 14–17, Wiley

SIGMA CHIPS (manual), pp. 2–3, Scott Scientific

THINK AND COLOR, pp. 28–37, Educational Science Consultants

Teaching
Page f-9

Since this is a review page, you might wish to have the children work on it independently. The children need only solve the equations and put the sum in the appropriate space. Again point out that the vertical notation at the bottom may be thought of as simply solving an equation.

Show you know

Solve.

$5 + 2 = \boxed{7}$

$2 + 5 = \boxed{7}$

$3 + 6 = \boxed{9}$

$6 + 3 = \boxed{9}$

$(1 + 4) + 1 = \boxed{6}$

$1 + (4 + 1) = \boxed{6}$

$(2 + 1) + 4 = \boxed{7}$

$2 + (1 + 4) = \boxed{7}$

$(3 + 1) + 2 = \boxed{6}$

$3 + (1 + 2) = \boxed{6}$

$(2 + 4) + 4 = \boxed{10}$

$2 + (4 + 4) = \boxed{10}$

$4 + 1 + 2 = \boxed{7}$

$1 + 2 + 3 = \boxed{6}$

$3 + 2 + 3 = \boxed{8}$

$2 + 4 + 2 = \boxed{8}$

$3 + 3 + 4 = \boxed{10}$

$5 + 2 + 2 = \boxed{9}$

2	3	5	6	1	4
1	2	1	2	7	4
+2	+4	+2	+2	+1	+1
5	9	8	10	9	9

Objective

The child will demonstrate his ability to work with the concepts which are presented in this module.

Pre-Book Activity

Materials

one set of strips per child

After a brief time for free play, direct the children to take the red strip and two purple strips and to form a train which will be as long as the orange strip. Then explain to the children that you want them to work with this train of three strips and to remove two strips from one end and to replace these two strips with one other strip so that they will have a train with two strips which will be the same length as their original three strip train.

red | purple | purple
$2 + 4 + 4 = \square$

red | brown
$2 + 8 = \square$

or

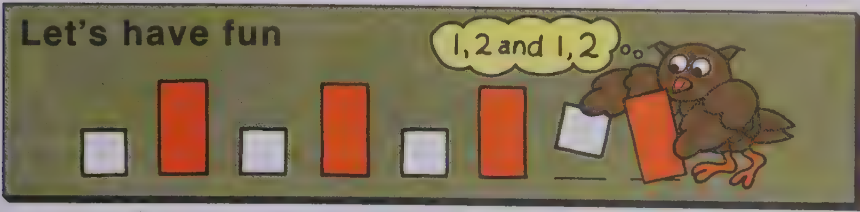
red | purple | purple
$2 + 4 + 4 = \square$

dark green | purple
$6 + 4 = \square$

Write these equations on the chalkboard, and insert the parentheses to show this grouping. Work through other examples, according to the children's need and ability.

If you prefer, you might also use an oral activity in which you give the children sums in groups of three. For example, ask them to follow you very carefully and add: $(2 + 1) + 4$, or $(4 + 5) + 1$, and so on.

Let's have fun



Study the pattern. Can you show the next two strips?

Patterns

Teaching
Page f-10

This change of pace page might need some explanation. Read the directions at the top of the page with the children. Explain that they should first study the pattern. (Go over the first pattern with them so that they know what a pattern is.) The first pattern shows a repetition of the white, and two red strips. Explain that they should study the strips shown and when they have figured out what the pattern is they should use their strips to trace around and color the next two strips needed to complete the pattern.

Follow-up

Duplicate worksheets or show tables such as the following on the chalkboard.

Find three numbers which added together make the sum shown on the outside. (Other answers are possible.)

2	2	3	7
2	2	3	7
3	3	1	7
7	7	7	

2	1	7	10
5	3	2	10
3	6	1	10
10	10	10	

Resources for Active Learning

Magic Squares, (etc.):

A CLOUDBURST, Vol. 1, Nos. 8311–8331, Midwest Publications

ENRICHMENT OF ARITHMETIC, pp. 2/32–33, Webster, McGraw-Hill

Franklin Series: PATTERNS AND PUZZLES, pp. 27–29, Lyons and Carnahan

Nuffield Project: COMPUTATION AND STRUCTURE ②, pp. 66–69, Wiley

TEACHING AIDS FOR ELEMENTARY MATHEMATICS, p. 107, Holt, Rinehart and Winston

ORANGE MODULE, UNIT F

Sums to 18 – Power Skill

Pages f-11 to f-24

General Objectives

To develop addition combinations for sums 11 to 18 (power methods)

To strengthen skills with combinations of 10 or less

To provide word problem experiences

The emphasis in this module is to develop in the child the ability to work out the sums of 18 or less. Later an emphasis will be placed on the mastery of these facts, but at this time the emphasis is upon power skills which enable the child to find the sums independently with the aid of various manipulative and reasoning devices. Counters, strips, and the number line are developed as valid power skill aids. The technique of "making ten" is also treated and considered as a power skill in finding these larger sums.

Mathematics

This module develops the sums 11 through 18 from the viewpoint of power skill as opposed to speed skill. First, the idea of joining or forming the union of two sets is given as a method for finding sums. Secondly, concepts associated with length are used in the form of the centimetre strips and jumps on the number line to find sums. The strong emphasis upon the use of these mathematical ideas, union of sets and length, should not be interpreted as an indication that the speed skills, or memorization of the basic facts, will be ignored. This is the key time to develop the underlying mathematical concepts. The speed skills will be emphasized in later modules.

One of the most significant mathematical features of this module is the use of the grouping principles to find sums from 11 through 18. This process requires a knowledge of place value. As an illustration, consider the combination $8 + 5$. First, 5 is broken into two parts, so that the sum of 8 and one of these parts is 10. Next, the grouping principle is used to combine this part with 8 to make 10; then, place value is used to combine 10 and the other part of 5. This process is shown below in equation form:

$$\begin{aligned} 8 + 5 &= 8 + (2 + 3) \\ &= (8 + 2) + 3 \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

Similarly, 8 could be broken apart like this:

$$\begin{aligned} 8 + 5 &= (3 + 5) + 5 \\ &= 3 + (5 + 5) \\ &= 3 + 10 \\ &= 13 \end{aligned}$$

To show the mental process, it is usually easier if the smaller of the two unequal addends is broken apart.

Teaching Orange Module, Unit F

Approximate Time: 7 to 10 days

Materials

centimetre rule (optional)
counters, at least 18 per child
demonstration number line
flannelboard and felt objects to be used in set demonstrations
large grocery bag
one set of strips per child
overhead projector (optional)
paper cups, 18

Since the addition concepts developed in this chapter are not new to the children, there are no new words for them to master. Children should be given freedom to express their preference for particular methods for finding sums. It is intended that each child be introduced to each of the methods developed and gain some understanding of each. Then each child should develop at least one method which he can use effectively to solve equations. Many children will be able to effectively use all of the methods presented.

Evaluation of Progress

Since the principal objective of this module is to help children develop power skill methods for finding sums, you should observe children's ability to effectively use the various methods suggested. A child who can effectively use the counters, strips, or number line to find sums of 11 through 18 has some basic understanding of the addition operation and should be able to solve addition problems independently.

Resources for Active Learning

Slide rules:

A CLOUDBURST, Vol. 1, No. 1191, Midwest Publications

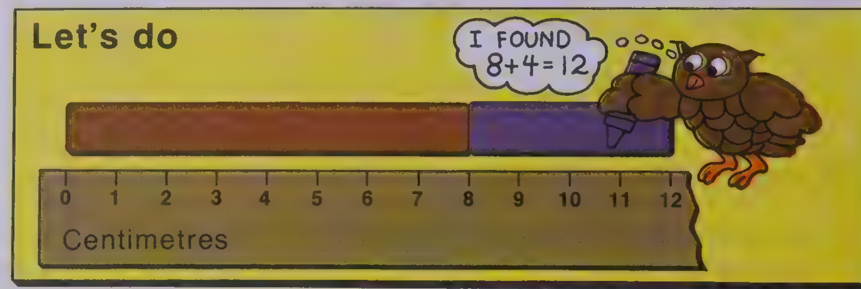
MATH ACTIVITY CARDS, A5, Macmillan

USING THE 'INVICTA' PLASTICS MATHEMATICAL BALANCE, Math Media

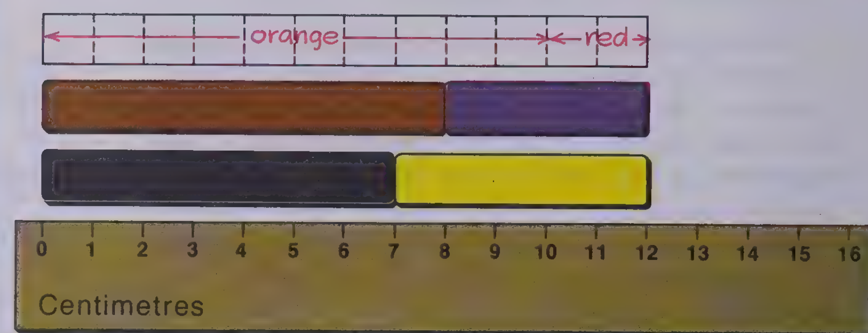
Investigation
Page f-11

Read the directions with the children. Be sure they understand that for each number they should use only two strips. When they work on the bottom of the page, they should choose either 13, 14, 15, or 16. Again, the element of choice here is important to the child. Then they should find three pairs of strips which will "make" their number when placed above the centimetre ruler shown. The coloring is intended only as a means of recording which strips were used. If you prefer, you might have the children simply show you their three pairs of strips. After children have worked on this investigation for a while, encourage them to tell each other the strips which they used for different numbers. For example, ask: "How many chose the number 15?" Then ask these children to explain to the others which strips they used for their three trains. It is not necessary at this time to write equations for these trains, but if the children are capable, you may choose to have them do so.

Let's do



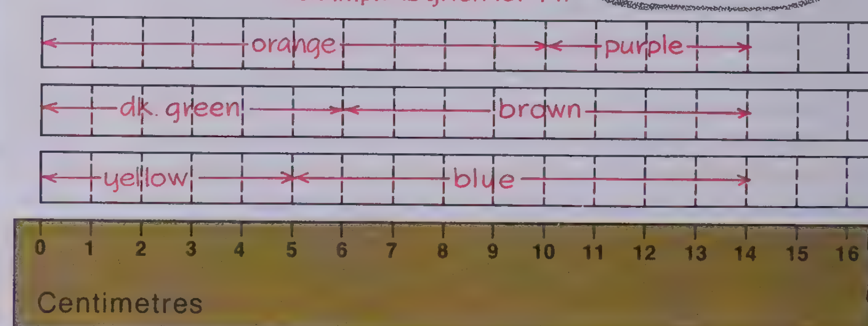
Show another pair of strips that "make 12". One example is given.



Choose one of these numbers.
Can you show three pairs of strips for your number?

13 14
15 16

Answers will vary.
An example is given for 14.



Purposes

To introduce the children to the study of the combinations of 11–18

To provide readiness for the power skills used to develop the combinations for 11–18

Preparation

Materials

one set of strips per child

Have the children take a pair of strips and make a train for a sum less than 10. For example, ask them to find a pair of strips which matches the length of the blue strip. When all of the children have done this, ask volunteers to write equations on the chalkboard to show their groupings of the strips.

dark green	light green

$6 + 3 = 9$

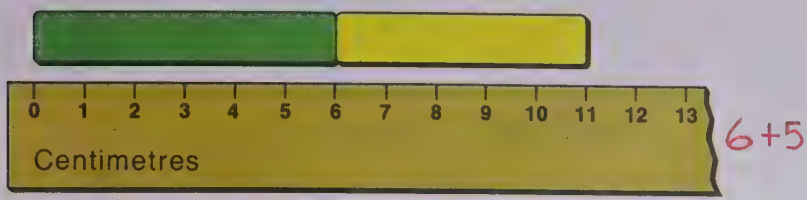
yellow	purple

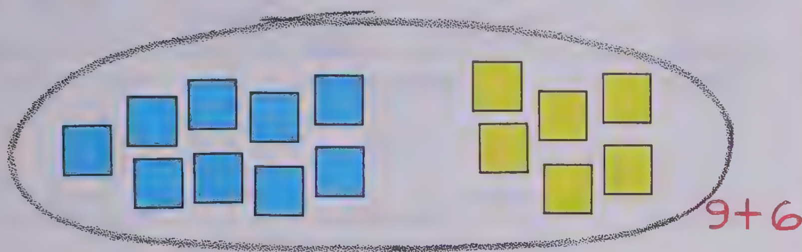
$5 + 4 = 9$

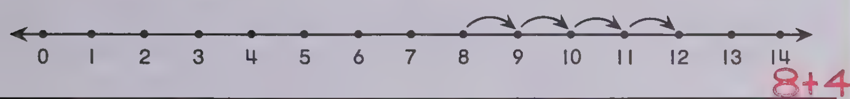
Following a variety of such experiences, begin the investigation.

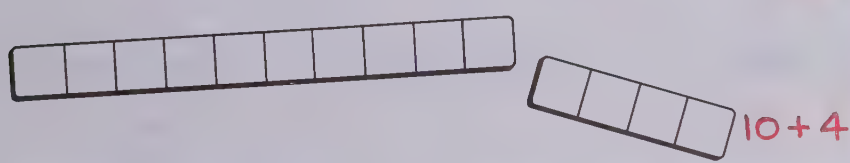
Let's talk

What sum does each picture show?







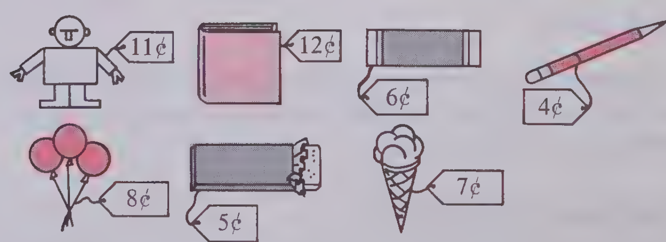


Discussion
Page f-12

The illustrations on this page provide a basis for discussion of the various ways to find sums of 11 through 18. For example, the first picture may be related to the investigation in which the children built a train with their strips to find the sum of a number between 11 and 18. The second illustration shows how addition may be used in counting money or may be thought of simply as combining two sets. The combination or union of two sets of different colored counters is shown in the third frame. The frame with the number line suggests a counting device to find the sums. The final illustration is a picture of the backside of the strips marked in centimetre units. Since the ten strip is illustrated, this may be used to show that the ten centimetre strip may be thought of as one ten and the four strip as four.

Follow-up

Distribute to the children the sets of coins which were used in Unit E. Suggest that they use these as counters to help them determine how much two or three of the items that you have pictured on the board would cost. Also you might tell them to suppose that they have 20¢ and figure out what items they might be able to buy. Instruct them to find the total cost of the items and figure out what change they should receive. If you wish, you might also use a duplicating master such as the one shown.

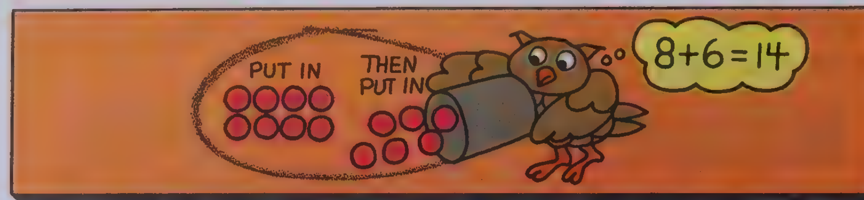


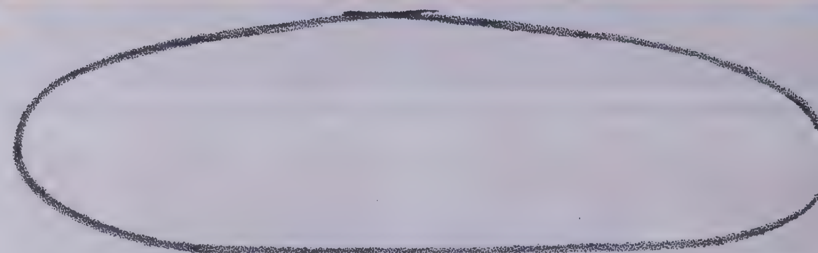
First Item	Second Item	Total Cost	Change from 20¢
11¢	4¢	15¢	5¢

Teaching Page f-13

Work through the first few equations as necessary. If you have worked through the introductory sections with the children, you might want them to simply follow the directions on the page. It would also be helpful to write other equations such as the following on the chalkboard.

Put in		Then put in		How many?
4	+	7	=	□
8	+	6	=	□
5	+	7	=	□
8	+	4	=	□





Put in		Then put in		How many?
6	+	7	=	13
9	+	6	=	15
7	+	7	=	14
5	+	9	=	14
8	+	5	=	13

Objective

Given an addition equation for sums 11–18, the child will be able to find the sum by using counters or by referring to illustrated sets.

Pre-Book Activity

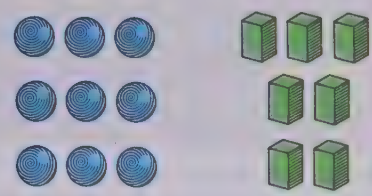
Materials

at least 18 counters per child

Since the first page of this lesson has an investigation type activity, you might choose to distribute the pages before you follow these introductory suggestions. Suggest to the children that they use the ring at the top of the page as a ring for their counters. Direct them first to put in three, then to put in four. Write on the chalkboard $4 + 3 = \square$ and ask a child to write the sum in the box and to explain its relation to the counters. Work through other examples that deal with combinations of ten or less. Then continue to the equations that deal with the combinations of 11 through 18 given on the page.

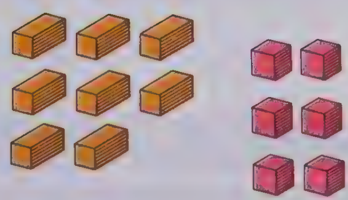
Find the sums.

$7 + 4 = \boxed{11}$



$9 + 7 = \boxed{16}$

$8 + 8 = \boxed{16}$



$8 + 6 = \boxed{14}$

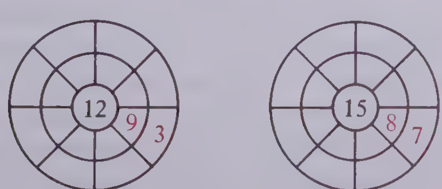
$6 + 5 = \boxed{11}$	$7 + 8 = \boxed{15}$
$4 + 8 = \boxed{12}$	$8 + 9 = \boxed{17}$
$9 + 3 = \boxed{12}$	$8 + 7 = \boxed{15}$
$6 + 7 = \boxed{13}$	$9 + 9 = \boxed{18}$

Teaching
Page f-14

Read the directions at the top of the page. Call the children's attention to the first frame. Point out that the illustration contains seven cubes and four pyramids (don't belabor the new words) and relate these sets to the numerals shown in the equation. Direct children to write the sum in the box. Suggest that they continue to do the remainder of the page independently. Point out that they might use their counters for the equations at the bottom if they wish or they may draw set pictures.

Follow-up

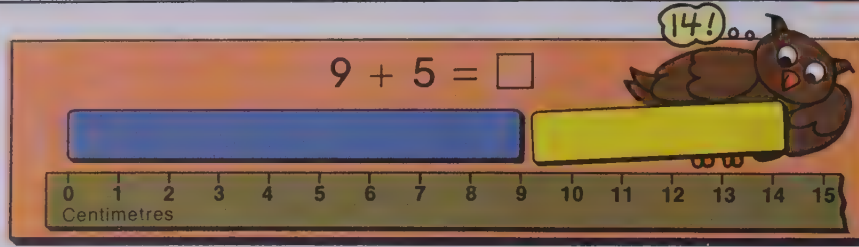
The following type of worksheet may be given to the children with the suggestion that they find the sums by using the counters. Explain that a sum is at the centre of the wheel and that all the combinations should lead to it like spokes. Use numbers 11 through 18 as sums in the centre.



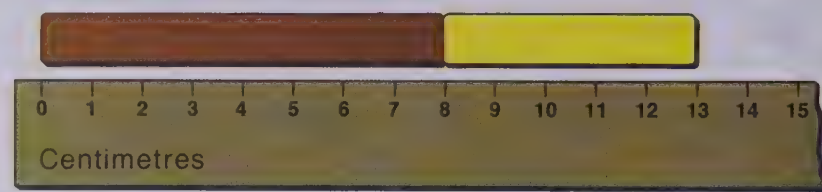
Teaching
Page f-15

Ask children to identify the strips shown above the centimetre ruler in the demonstration art. In particular ask them to observe where the 9 strip ends. Ask them if they can tell where the 5, or yellow, strip will end. Be sure children see that at the end the 5 strip will be above 14 and that $9 + 5 = 14$. Then work through the first example. Help them realize that the train of the 8 strip and the 5 strip together will make a train which ends at the sum of $8 + 5$. Work through the next example, and again help children to relate the train of the 9 strip and the 6 strip to the equation $9 + 6 = \square$, and the number on the centimetre ruler where this train ends, to the sum 15. Suggest other examples to the children until you are sure they see how the centimetre ruler may be used with the trains to help them solve the equations.

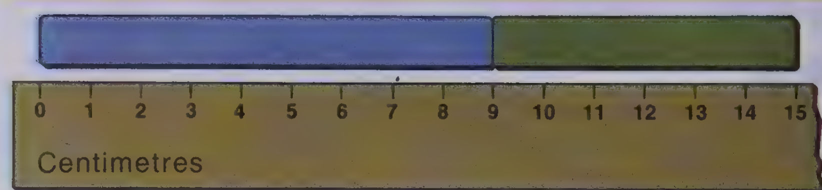
As you develop the bottom half of the page, observe with the children that they may use either number in the equation as a starting point and then use the other number as the number of jumps. Finally, the sum may be read at the tip of the last arrow or jump. Ask volunteers to show on a demonstration number line the jumps for particular equations you give them.



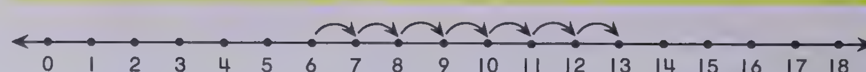
Solve the equations.



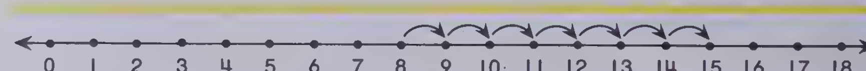
$8 + 5 = \boxed{13}$



$9 + 6 = \boxed{15}$



$6 + 7 = \boxed{13}$



$8 + 7 = \boxed{15}$

Objective

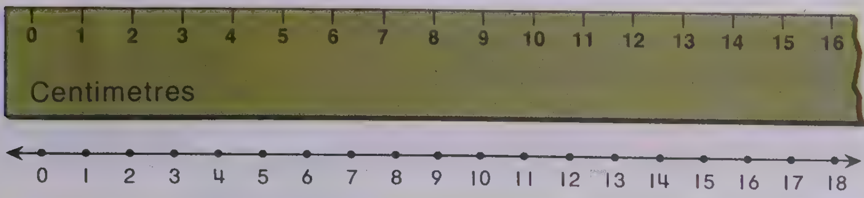
Given an addition equation for a sum of 11–18, the child will be able to find the sum by using the strips or the number line.

Pre-Book Activity

Materials

centimetre ruler (optional)
centimetre ruler for overhead projector (optional)
demonstration number line
strips

Since the material on page f-15 may be developed into an investigation type activity, you will probably wish to begin immediately with the lesson page.



Find the sums.

4 + 9 = 13	8 + 6 = 14
6 + 5 = 11	9 + 2 = 11
7 + 7 = 14	9 + 7 = 16
4 + 8 = 12	5 + 8 = 13
7 + 6 = 13	9 + 3 = 12
6 + 9 = 15	7 + 8 = 15

$$\begin{array}{r}5\\+9\\\hline 14\end{array}\quad\begin{array}{r}5\\+7\\\hline 12\end{array}\quad\begin{array}{r}9\\+4\\\hline 13\end{array}\quad\begin{array}{r}9\\+9\\\hline 18\end{array}\quad\begin{array}{r}4\\+8\\\hline 12\end{array}\quad\begin{array}{r}8\\+7\\\hline 15\end{array}$$

$$\begin{array}{r}7\\+4\\\hline 11\end{array}\quad\begin{array}{r}9\\+8\\\hline 17\end{array}\quad\begin{array}{r}7\\+5\\\hline 12\end{array}\quad\begin{array}{r}9\\+5\\\hline 14\end{array}\quad\begin{array}{r}6\\+0\\\hline 6\end{array}\quad\begin{array}{r}8\\+8\\\hline 16\end{array}$$

Sums to 18—number line

Teaching
Page f-16

Call the children's attention to the illustrated centimetre ruler and the number line at the top of the page. Explain that as an aid in working out these equations, they can choose to use either the strips and the centimetre ruler or the number line. Encourage the children to work some of the equations in one way and some in another. It would be helpful to use the overhead projector and a transparent centimetre ruler with the strips to develop understanding of this method. Notice that the examples at the bottom are in vertical notation. Again stress with the children that these may be worked in the same manner as equations.

Follow-up

The use of the strips in finding sums may be further developed by the idea of a "5 adder" or "8 adder" and so on. For example, suggest that the children take the yellow strip and the illustration of the centimetre ruler and find the sum 6 + 5 in the following manner. Since 6 is the first number in the equation, put the yellow strip or the "5 adder" at the 6 mark on the ruler. Then read the number where the strip ends. This will be the sum 11. In order to find the sum for 7 + 8, the brown, or 8 strip would be used as an "8 adder." Thus, the end of this strip would be placed at the 7 mark on the ruler and the sum for 7 + 8, 15, would be found at the end of the strip. This same idea may be used later for subtraction.

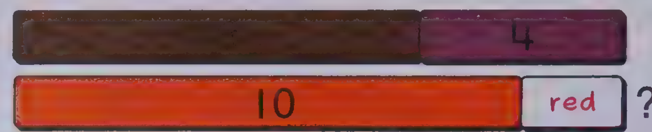
Resources for Active Learning

MATH WORKSHOP: Games and Enrichment Activities, "Cuisenaire Rods . . .," pp. 44–49, Encyclopaedia Britannica Educational Corp.

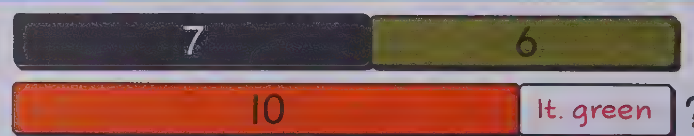
Teaching
Page f-17

You can think of this page as an investigative activity by simply asking the children: "Can you make a train using the orange strip and one other to match the top train shown in each frame?" They should record what they find by coloring the missing strip. When they have done this, they should solve the equation. Use the demonstration art to explain this activity. Then depending on the capability of the children, you might want to work through some of the frames with them. For example, in the first frame discuss how the brown strip and the purple strip relate to the equation 8 + 4. Then point out how the ten strip plus some other strip can be made into a train to match the 8 and 4 strip. When children discover what strip can be used with the ten strip, ask: "What number is ten and two?" and relate this to the sum; 8 + 4 is the same as 10 + 2.

Color the missing strip. Then solve the equation.



8 + 4 = 12



7 + 6 = 13

9 + 5 = 14

8 + 7 = 15

Objective

Given an addition equation for a sum of 18 or less, the child will be able to solve the equation by thinking of the sum of 10 and another number.

Pre-Book Activity

Materials

one set of strips for each child

Since page f-17 makes extensive use of the strips and place value concepts, give the children an opportunity to show various teen numbers using an orange strip and one other. For example, ask them: "How would you show the number 12?" They should use one orange strip and one red strip. Or ask: "How would you show the number 15?" They should respond that they would use the orange strip and the yellow strip.

Give the missing numbers.

7 and 5	10 and 2
8 and 6	10 and 4
5 and 8	10 and 3
7 and 7	10 and 4
9 and 6	10 and 5
6 and 7	10 and 3
8 and 8	10 and 6
4 and 7	10 and 1

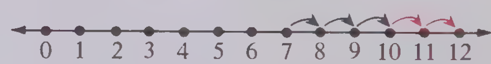
Solve the equations.

$7 + 5 = \boxed{12}$	$9 + 6 = \boxed{15}$
$8 + 6 = \boxed{14}$	$6 + 7 = \boxed{13}$
$5 + 8 = \boxed{13}$	$8 + 8 = \boxed{16}$
$7 + 7 = \boxed{14}$	$4 + 7 = \boxed{11}$

Teaching
Page f-18

Work through the first two or three examples with the children and relate what they should think mentally to their activity with the strips on the previous page. For example, if they are to think 7 and 5, they should think, "What number do I need to add to 7 to get 10?" "Since that number is 3, I take 3 from 5 and have 2, leaving me 10 and 2." Some children will need to use the strips or counters to figure this out. However, the main emphasis is on the mental power of making 10 to find the sum. Some children will be able to do this quickly; others will need much work before they can make ten mentally. Notice that the equations at the bottom relate to the sums written at the top.

You might also suggest that they use the number line to figure out how to rephrase sums. For example:



7 and 5 is the same as 10 and 2

Exercises such as this may often be used to give children who understand how to do the work mentally an opportunity to explain their thinking to the rest of the class.

Follow-up

"Guess the Rule" games such as the following will challenge many of your children.

Table One	
3	13
5	15
6	
7	
	14
	18
9	
2	

Table Two	
7, 8	10, 5
9, 2	10, 1
6, 5	,
3, 9	,
,	10, 3
,	10, 4
9, 7	,
6, 9	,

"Guess the Rule"

Urge them to study the patterns, guess the rule, and then complete the tables according to that rule. The rule for Table One is "Add 10," and the rule for Table Two is "Regroup to tens and ones." Note that there are several possible number pairs for the first column of Table Two.

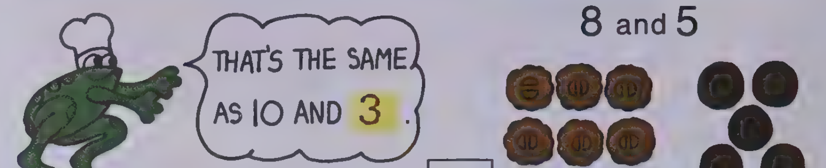
Resources for Active Learning

MATHEX: Operations No. 3, "Operation Big Ten," pp. 16–23, Encyclopaedia Britannica Publications Ltd.

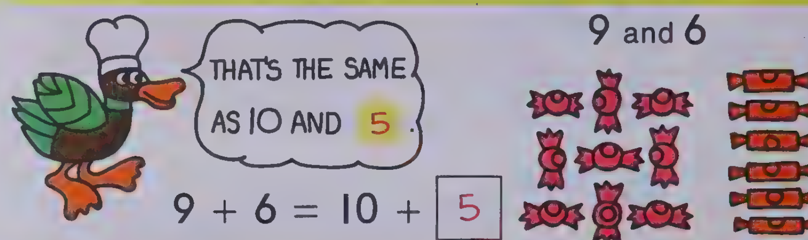
Teaching
Page f-19

Use the illustration to continue development of the concepts stressed in the previous lesson. Ask a child to read the directions at the top of the page. Then point out the sets of 8 and 5 in the first frame. Ask someone to explain how the frog is thinking of these two sets. You might suggest to the children that they circle the group of ten and then find out how many are not in the circle, showing that $8 + 5$ is the same as $10 + 3$. Continue with the next frame, again asking a child to explain how the duck is thinking of the two sets of nine and six. Again you may want to have the children circle the group of 10 and count those that are not in the circle. Work through the last frame in a similar manner. Finally, help the children relate the equations at the bottom to the three frames that they just worked.

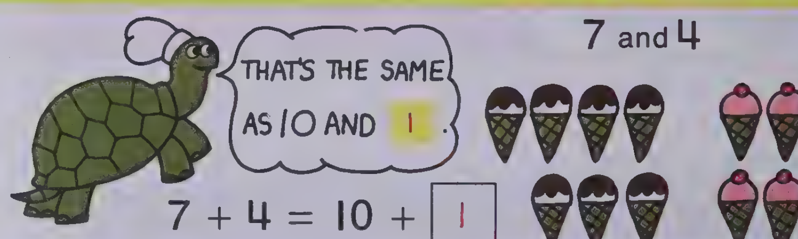
Give the missing numbers.



$8 + 5 = 10 + \boxed{3}$



$9 + 6 = 10 + \boxed{5}$



$7 + 4 = 10 + \boxed{1}$

Solve the equations.

$8 + 5 = \boxed{13}$ $9 + 6 = \boxed{15}$ $7 + 4 = \boxed{11}$

Sums to 18—regrouping to make 10

Objective

Given an equation for a sum of 18 or less, the child will be able to solve the equation by thinking of the sum of 10 and another number.

Pre-Book Activity

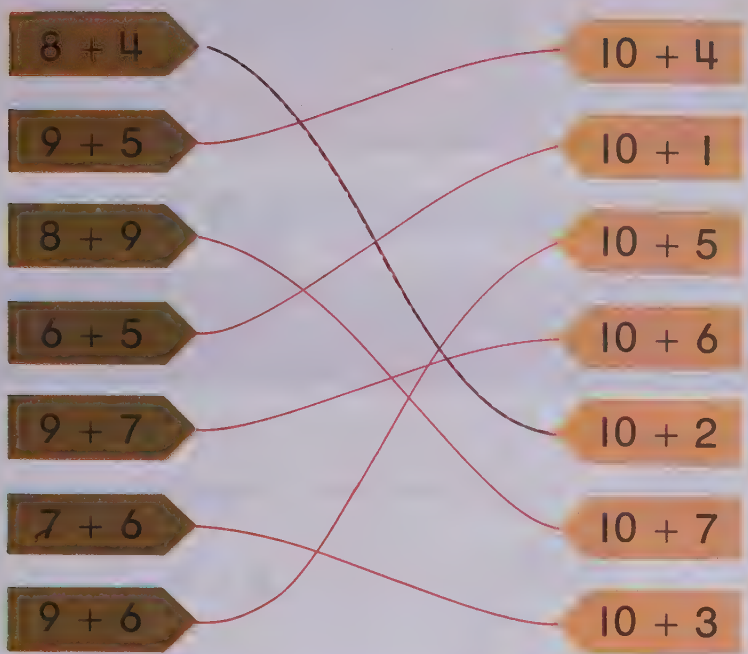
Materials

flannelboard
objects for set demonstration

Exhibit a set of 7 and a set of 8 beside each other on the flannelboard or chalkboard in easily recognized patterns such as those shown in the next column.

Place the symbols $7 + 8$ beneath the sets. Ask the children to remove enough objects from the set of 8 to make a set of 10 when they are placed with the set of 7. Explain that they now have a set of 10 and 5 more, which makes 15 in all. Therefore, the sum of 7 and 8 is 15. Following several demonstrations of this type, give the children some combinations whose sums are greater than 10 to see if they can do the regrouping mentally and arrive at the sum.

Complete the matching.



Mark each incorrect answer with an X.

Solve the equations. *Billy W.*

$9 + 6 = 10 + \boxed{5}$	$8 + 8 = 10 + \boxed{6}$
$7 + 4 = 10 + \boxed{1}$	$7 + 3 = 10 + \boxed{\cancel{1}}$
$6 + 6 = 10 + \boxed{\cancel{3}}$	$9 + 4 = 10 + \boxed{3}$
$8 + 5 = 10 + \boxed{\cancel{2}}$	$5 + 9 = 10 + \boxed{\cancel{5}}$

Sums to 18—regrouping to make 10

Teaching
Page f-20

Read the directions at the top of the page with the children. Ask them how 8 + 4 could be regrouped to 10 and some number of ones. Tell the children to choose their answer from the second column. As soon as a child says that 8 + 4 could be regrouped to 10 + 2, direct the children to trace the matching line. Ask the children to complete the exercise by such matching.

Now move down the page to Billy's paper. Observe with the children that Billy has also been doing problems by regrouping, but that he has made some mistakes. Direct the children to mark each incorrect answer with an X and to write the correct answer beside the answer given.

Follow-up

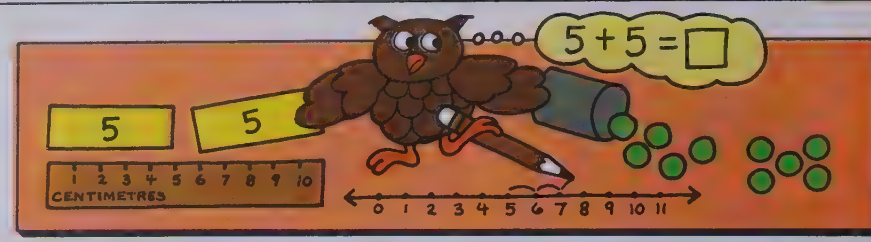
If some children need another approach to regrouping, using manipulative materials, provide 18 paper cups and a large grocery bag for the demonstration table. Write an equation such as $8 + 6 = \square$ on the chalkboard. Show 8 paper cups on the table and place 6 in the bag. Ask a child to take enough cups out of the bag to make a total of 10 cups on the table. Then ask the class to tell you how many cups remain in the bag. Emphasize that the regrouping of the cups shows that 8 + 6 = 10 + 4. Work out other combinations with the children as needed.

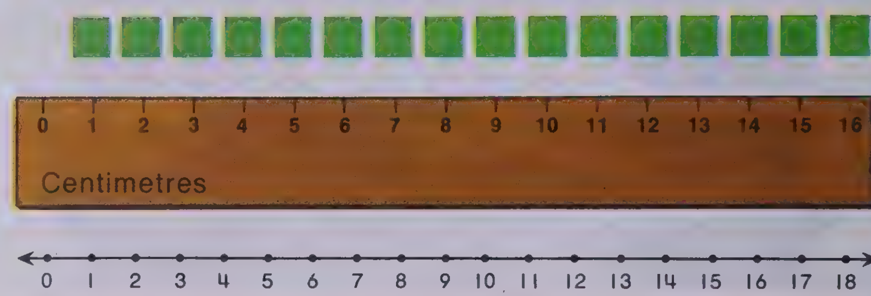
Resources for Active Learning

MATHEX: Operations No. 3, "Operation Big Ten," pp. 16–23, Encyclopaedia Britannica Publications Ltd.

Teaching
Page f-21

Call attention to the illustration at the top of the page. Point out that these represent various methods that the children have just reviewed and should use to find the sums on this page. Encourage the children to work independently. Also encourage them to use the method they prefer to solve the equations. If some children know these facts by memory, of course, they may simply put down the answers and not use any of the power methods. For such a child though, it would be helpful to ask him to explain how one of these methods might be used to figure out a sum even though he does know the sum by memory.





Find the sums.

$9 + 4 = \boxed{13}$ $8 + 8 = \boxed{16}$

$2 + 8 = \boxed{10}$ $5 + 6 = \boxed{11}$

$7 + 7 = \boxed{14}$ $9 + 5 = \boxed{14}$

$5 + 7 = \boxed{12}$ $6 + 7 = \boxed{13}$

$7 + 4 = \boxed{11}$ $8 + 6 = \boxed{14}$

8	7	4	6	6	9
+ 5	+ 8	+ 8	+ 4	+ 6	+ 8
13	15	12	10	12	17

Sums to 18—power

Objective

Given an equation for a sum of 11–18, the child will be able to find the sum by using counters, strips, the number line or "making ten."

Pre-Book Activity

Materials

counters
demonstration number line
objects for set demonstrations
strips

It would be well at this time to review with the children the various methods which they have been studying to find the combinations for 11 through 18. For example, write the combinations 8 + 7 under the number line and ask one of the children to come to the demonstration number line and show seven jumps beginning at eight. Then ask the children to participate in demonstrations with sets of objects. For example, ask a group of eight children and a group of five children to regroup themselves into groups of ten and three more so that they can determine that the set of 8 and 5 is 13. Then, give the children examples in which they can use their strips. For example, tell them that you want them to find the sum of 7 and 6. Ask a child to explain what strips he should use and how he might use a centimetre ruler to show this. If children are capable and understand these methods, conduct a short drill to give them an opportunity to regroup numbers mentally. You might do this by giving them a sum such as 5 + 6 and asking them not just to give the sum, but to give the regrouping with 10.

f 22 twenty-two

Picture Problems

1. 9 and 7 played kick . How many played? 16
2. Each costs 7 . How much for 2 ? 14¢
3. John saw 9 and 3 . How many in all? 12
4. 6 at one . 6 at another . How many children in all? 12
5. Randy put 8 on one page of his . He put 7 on another page. How many in all? 15
6. An costs 9 . A costs 3 . How much for both? 12¢
7. Tam saw 6 in a . She saw 8 in a . How many in all? 14
8. Chris saw 5 and 6 . How many in all? 11
9. Ann had 9 . She bought 8 more. How many does she have now? 17
10. Beth bought 2 . Each cost 9 . How much in all? 18¢

Picture problems

Teaching
Page f-22

Depending on the reading ability of the children, you may or may not want to read some of these problems with them; in any case, encourage the children to do most of the thinking through of the problem by themselves. You might do this by suggesting, after you read a problem, that the child show a picture to explain how he interprets the problem. For example, in problem 5, the child might draw eight stamps on a page and another set of seven stamps on another page and finally count or solve the sum $8 + 7$ by any one of his power methods.

You might prefer to divide the children into groups and encourage them to read the problems and work the solutions out together. You might take others and help them with the reading, but again encourage them to do the thinking through of the problem on their own. With another group you might want to concentrate on the use of manipulative materials as you read the problems. The important point is to adapt this page to the children's capabilities.

Thus, if you say $5 + 6$, a child would respond that this is the same as 10 plus 1, or 11. If you say, 9 plus 5, a child might respond that this is the same as 10 plus 4, or 14.

Follow-up

Provide each child with 18 counters or discs, a large sheet of newsprint, and crayons. Ask the children to fold their papers into four sections and label the sections 15, 16, 17, and 18. Suggest that they use the counters to help them and write as many combinations for 15 as they can in the section for 15. Direct them to complete the page by finding and writing combinations for 16, 17, and 18 in the same way.

You might also distribute single-column addition tables like the following which may be solved using the power skill methods developed in this module.

Add 8	
3	
4	
5	
6	
7	
8	
9	

Add 8	
7	
5	
3	
9	
8	
4	
6	

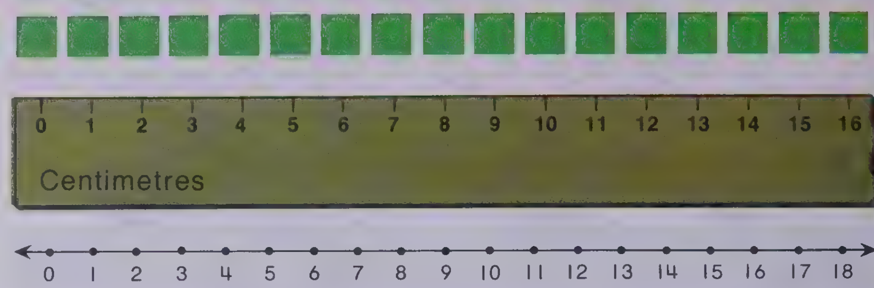
Add 9	
2	
3	
4	
5	
6	
7	
8	
9	

Add 9	
5	
2	
7	
6	
4	
8	
9	
3	

Again point out the illustrations of the methods studied in this module. Although it is expected that many 7- and 8-year olds will eventually master the combinations through 18, the emphasis in this module is on developing power skills. Thus, children should understand that to solve these equations they may use any method they prefer. Again point out that the examples written in vertical notation at the bottom may be worked out in the same manner as the equations.

Show you know

Find the sums.



$6 + 5 = \boxed{11}$ $8 + 8 = \boxed{16}$

$7 + 7 = \boxed{14}$ $5 + 9 = \boxed{14}$

$4 + 9 = \boxed{13}$ $7 + 8 = \boxed{15}$

$8 + 4 = \boxed{12}$ $4 + 7 = \boxed{11}$

$9 + 7 = \boxed{16}$ $8 + 9 = \boxed{17}$

$$\begin{array}{rrrrrr} 9 & 7 & 5 & 9 & 7 & 6 \\ +3 & +9 & +7 & +5 & +6 & +9 \\ \hline 12 & 16 & 12 & 14 & 13 & 15 \end{array}$$

$$\begin{array}{rrrrrr} 6 & 6 & 8 & 9 & 6 & 8 \\ +8 & +7 & +7 & +9 & +6 & +5 \\ \hline 14 & 13 & 15 & 18 & 12 & 13 \end{array}$$

Module review

OBJECTIVE

The child will demonstrate his ability to work with the concepts presented in this module.

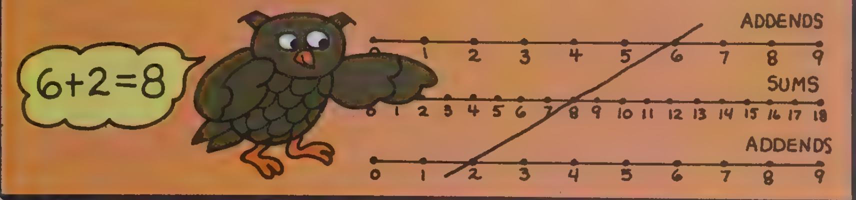
PRE-BOOK ACTIVITY

If you used the pre-book activity suggested for the previous lesson, you will have already reviewed the basic power skills studied in this module; therefore, for this pre-book activity, it would be helpful to review all of the doubles that the children have been working with. For example, say: "I'm thinking of a double. The sum is 18. What is my double?" After a child has responded 9 and 9, continue similarly until you have completed all the basic fact doubles. Next provide exercises in pairs such as 9 + 9, followed immediately by 9 + 8; or 6 + 6, followed immediately by 6 + 7. In this way the children will be introduced to the method of working from known combinations to unknown combinations.

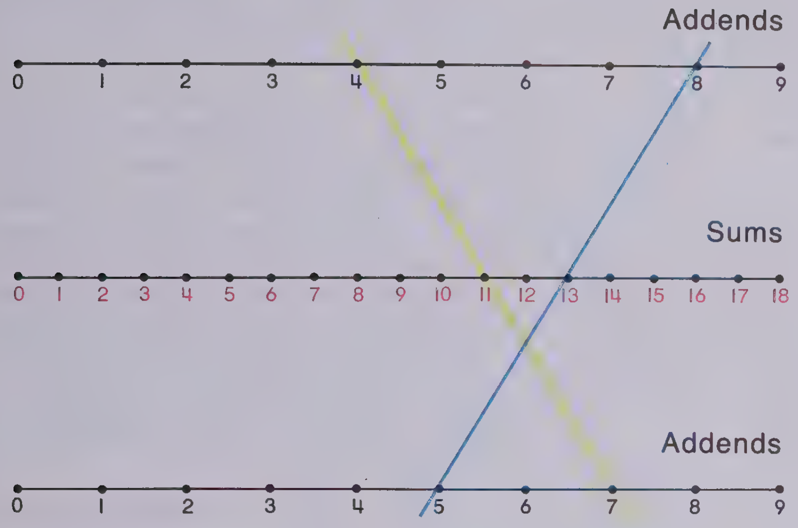
FOLLOW-UP

Provide additional practice in recognizing doubles and in using them to find other combinations by duplicating a worksheet similar to the one in the following column or writing it on the board.

Let's have fun



The figure below is called a **nomograph**.
The blue line shows that 5 + 8 = 13



What does the yellow line show? 7+4=11 or 4+7=11

Can you find some other sums using your nomograph?

TEACHING
Page f-24

Although this is a change of pace page, many children will need to examine it carefully before they can figure out how the nomograph is used. First, help them with the word nomograph. With some children, you might suggest that they work together to figure out how this nomograph is used. With others you will want to work through the questions yourself. Children will be aided by the use of any available straightedge as they do this. They may even use their orange strip for this purpose; however, a longer ruler would be more helpful. Notice that each line independently relates the addends of the two extremes to the sum in the middle; that is, the line that relates $4 + 7 = 11$ is not related in any way to the line that intersects $5 + 8 = 13$. If children enjoy working with the nomograph, you might help some of them construct such a nomograph to be placed on a bulletion board. Then yarn might be used in place of the lines to show an equation.

Find the sums.

4 +4	4 +3	4 +5	5 +5	5 +4	5 + 6
6 +6	6 +5	6 +7	7 +7	7 +6	7 + 8
8 +8	8 +7	8 +9	9 +9	9 +8	9 +10

Table One	
7, 5	2
3, 8	1
6, 8	4
9, 8	
4, 6	
8, 5	
7, 9	
8, 7	

Table Two	
6, 6	2
9, 3	2
,	5
,	6
,	4
,	
,	
,	

If more capable children enjoyed the "Guess the Rule" games suggested for page f-18, create additional tables such as the following. This activity should be considered enrichment for those who have mastered the power skills.

Remind the children to study the patterns, guess the rule, and then complete the tables. Explain that they can make up the last three problems in the second table using the rule for that table. Note: The rule for the game is "Subtract 10 (after regrouping to tens and ones)."

RED MODULE, UNIT F

Differences to 18 – Power Skills

Pages f-25 to f-36

General objectives

To develop subtraction combinations associated with sums 11 through 18 (power skill methods)

To provide word problems experiences

To introduce the use of subtraction when making comparisons

The power skill methods for subtraction developed in this module correspond to the methods used in the previous module to develop power skills for addition. Strips, counters, and the number line receive the main emphasis. Subtraction related to comparison of sets rather than as take away is introduced. One-to-one matching is an important technique used to develop an understanding of how subtraction can be used to show comparison of sets. The module closes with the usual review page and a change of pace page which may be used to develop skill in counting by fives.

Mathematics

This module involves little in the way of new mathematical concepts. Subtracting concepts involving numbers less than ten have been developed in Unit E. This module simply extends the ideas that have already been developed and applies them to finding differences relating to sums 11 through 18 by means of power-skill methods.

Teaching Red Module, Unit F

Approximate Time: 6 to 9 days

Materials

centimetre ruler (optional)
at least 18 counters per child, if possible 36 for each child
one set of strips for each child

Since the same materials that were used in the previous module are used here, children should again be given freedom to explore each and to develop a preference for a particular method. Notice that in this module the strips are not used in relation to a missing addend. They are presented for use as a "subtractor." The concept of missing addend and the corresponding representation with strips will be developed in a later module which stresses speed rather than power. Encourage the children to use the manipulative materials as they find them helpful. Some children will prefer to work without manipulatives. An ongoing objective is that they eventually work without the aid of concrete materials.

Evaluation of Progress

Since this module stresses understanding how to find differences for combinations related to sums 11 through 18, most of your evaluation should be based upon daily observation of the children. Recall that it is more beneficial for a child to develop ability to effectively use one method, than simply be exposed to all of them. However, the child who understands each method and can use any of them shows a clear grasp of the subtraction operation and of the power skills used to solve subtraction equations.

Resources for Active Learning

General Activities:
For information related to slide rules, refer to the Introduction to the orange module of Book F.

Ideas for graphing:
DEVELOPMENTAL MATH CARDS, C-6, 17, E-4, Addison-Wesley
MATH ACTIVITY CARDS, A6, 7, Macmillan
MATHEX: Matching and Graphing No. 1, pp. 16–25, Encyclopaedia Britannica Publications Ltd.
MATHS MINI-LAB, Card 3, Selective Educational Equipment
Nuffield Project: PICTORIAL REPRESENTATION $\boxed{1}$; MATHEMATICS BEGINS ①, ". . . Sorting," pp. 10–31, Wiley
USING THE 'INVICTA' PLASTICS MATHEMATICAL BALANCE, Math Media

Manipulative Devices:
"Invicta" Math Balance (Math Media; Selective Educational Equipment)

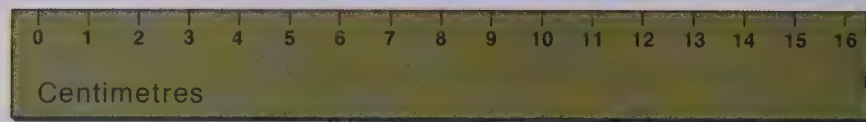
Investigation
Page f-25

Direct the children to use their yellow strip. Identify the yellow strip which the owl is using as a "5-subtractor." Help them see that the right end of the "subtractor" can be put at the 12 and the difference may be read from the left end. Work through as many examples using the "5-subtractor" as necessary to help children understand the idea. Then point out the four strips from which they may choose a subtractor to use for the equations at the bottom of the page. Notice that the subtractor which they choose will give them the second number in their subtraction equations and then they may use this to find the difference which they will write in the box. As they work, move around the room to see that children are putting the numbers in the proper spaces.

Let's do

$12 - 5 = 7$

$11 - 5 = 6$ A 5-subtractor



Use a 5-subtractor to solve this equation.

$13 - 5 = \boxed{8}$

Choose one of these subtractors.

Can you use your subtractor to help you complete these equations? Answers will vary.

Your number	Your number
$10 - \_\_\_ = \square$	$13 - \_\_\_ = \square$
$11 - \_\_\_ = \square$	$14 - \_\_\_ = \square$
$12 - \_\_\_ = \square$	$15 - \_\_\_ = \square$

Differences to 18 – power

Purposes

To provide an introduction to the development of power skills for subtraction

To provide readiness for the study of comparison subtraction

To provide readiness for the use of the number line and the strips in subtraction

Preparation

Materials

one set of strips for each child

Since the work in this lesson uses only materials and ideas that have previously been used, little or no preparation is required.

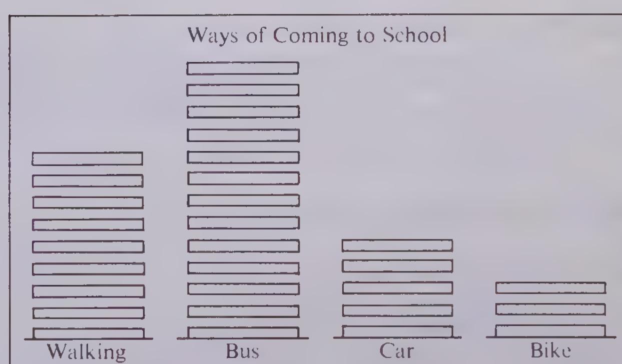
Let's talk

Discussion
Page f-26

The illustration on this page should provide the basis for a discussion of subtraction as take away. For each group of items, discuss how many will be left after each of the five children have one. For example, if each of the five children have a cookie then how many cookies will be left? If each of the five children have a hot dog, how many hot dogs will be left? Encourage those children who have difficulty responding to put 13 counters which represent the 13 cookies in the picture on their desk and remove 5. This illustration may also be used to prepare the children for comparison without doing actual one-to-one matching. This idea will be more thoroughly developed later in the learning unit. It is suggested here simply to relate comparison to take away. A discussion of this type will not only introduce children to the topic of study in this module, it will also give you an idea of the children's understanding of subtraction and of the need that they will have for developing the various power skills studied in the module.

Follow-up

As a follow-up for this lesson you might help the children make a chart which you can use throughout the module for developing comparison ideas. From among various topics suitable for a graph (favorite ice creams, kinds of pets, sizes of shoes), help the children choose a topic and make a graph. You might, for example choose to show ways they come to school. Ask children to explain their ways of coming to school and write these at the bottom of the graph. Then have each child place a marker over the column appropriate to his transportation. As work through the module progresses, use the various columns to discuss comparisons.



Allow such an activity to springboard into studies and comparisons of other types, such as charting teams and scores of games played on the school playground, or graphing the number of trucks and the number of buses which pass the school window and so on.

Teaching
Page f-27

Point out the ring at the top of the page and explain to the children that they should put in the number of counters indicated by the first number in the equation. For example, for the first equation, they should put 13 counters in the ring. Remind them that the symbol used in the equation means to subtract, so they should take out 6 counters. Most children will be able to work through these equations without much assistance, but move around the room and give guidance where necessary. If children have difficulty fitting their counters in the ring, suggest that they simply use their desk top.

Put in		Take out		How many?
13	–	6	=	7
11	–	4	=	7
14	–	7	=	7
12	–	8	=	4
15	–	9	=	6

Differences to 18 – sets

Objective

Given a subtraction equation related to sums 11–18, the child will be able to find the difference by using counters.

Pre-Book Activity

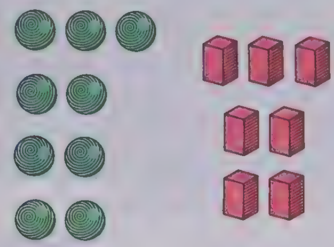
Materials

at least 18 counters per child

Since page f-27 uses the counters in an activity exercise set, you might wish to begin the lesson by making a circle in the centre of the classroom using a large piece of string or yarn. Then ask children in groups of 12, 13, or 14 to enter the circle. When, for example, 12 children are in the circle, ask a child come up and take some number of children, such as 7, back to their places. Then ask volunteers to explain what equation might be used to show the action that occurred. Work through several examples, and write the equations on the chalkboard. After several examples of this kind, introduce the children to the text page.

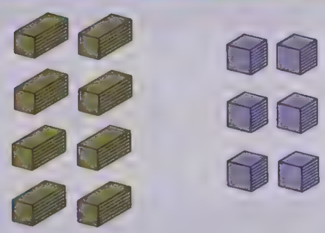
Solve the equations.

$11 - 4 = \boxed{7}$



$16 - 7 = \boxed{9}$

$16 - 8 = \boxed{8}$



$14 - 6 = \boxed{8}$

$13 - 4 = \boxed{9}$	$14 - 7 = \boxed{7}$
$11 - 8 = \boxed{3}$	$17 - 8 = \boxed{9}$
$15 - 6 = \boxed{9}$	$12 - 3 = \boxed{9}$
$12 - 5 = \boxed{7}$	$16 - 9 = \boxed{7}$

Teaching
Page f-28

This page shows how subtraction equations may be solved using set illustrations. Suggest that the children first count how many items are pictured in the illustration. Then you might suggest that they cross out the number that they are supposed to subtract. For example, in the first frame they will count 11 objects. Suggest that they cross out four of these objects since the equations reads 11 minus 4 equals □. The equations at the bottom of this page may be worked either with the aid of counters or by drawing pictures of sets to match each equation. Stress that the important thing is for them to be able to figure out the differences with the aid of various materials and methods; they should not be expected to know these differences from memory at this time.

Follow-up

Some children might be interested in the subtraction method of breaking apart a number and subtracting enough to make ten and then subtracting the number which remains to be subtracted. This technique may be developed with the use of the counters. For example, ask the group of children working with you to place 16 counters in the circle or on their table area. Explain that you want them to subtract 7 from 16, that is, you want them to solve the equation $16 - 7 = \square$. First ask them to take away as many as are needed to have 10 left. Discuss that taking 6 away will leave 10. Then ask how many more must they remove to have taken away a total of 7. Show that

$$16 - 6 = 10$$
$$10 - 1 = 9$$
$$16 - 7 = 9$$

After you work though similar examples, stress that if they like this way of thinking they should strive to develop it as a mental power skill.

For children not capable of this type of activity you might prepare a worksheet such as the following:

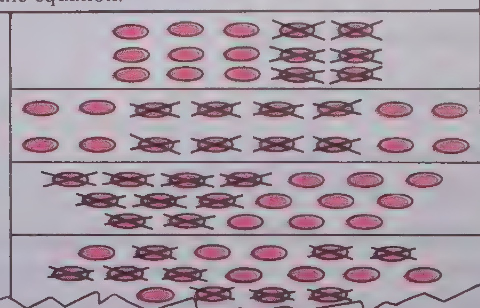
Match each equation with a picture.
Then solve the equation.

$18 - 9 = \square$

$15 - 6 = \square$

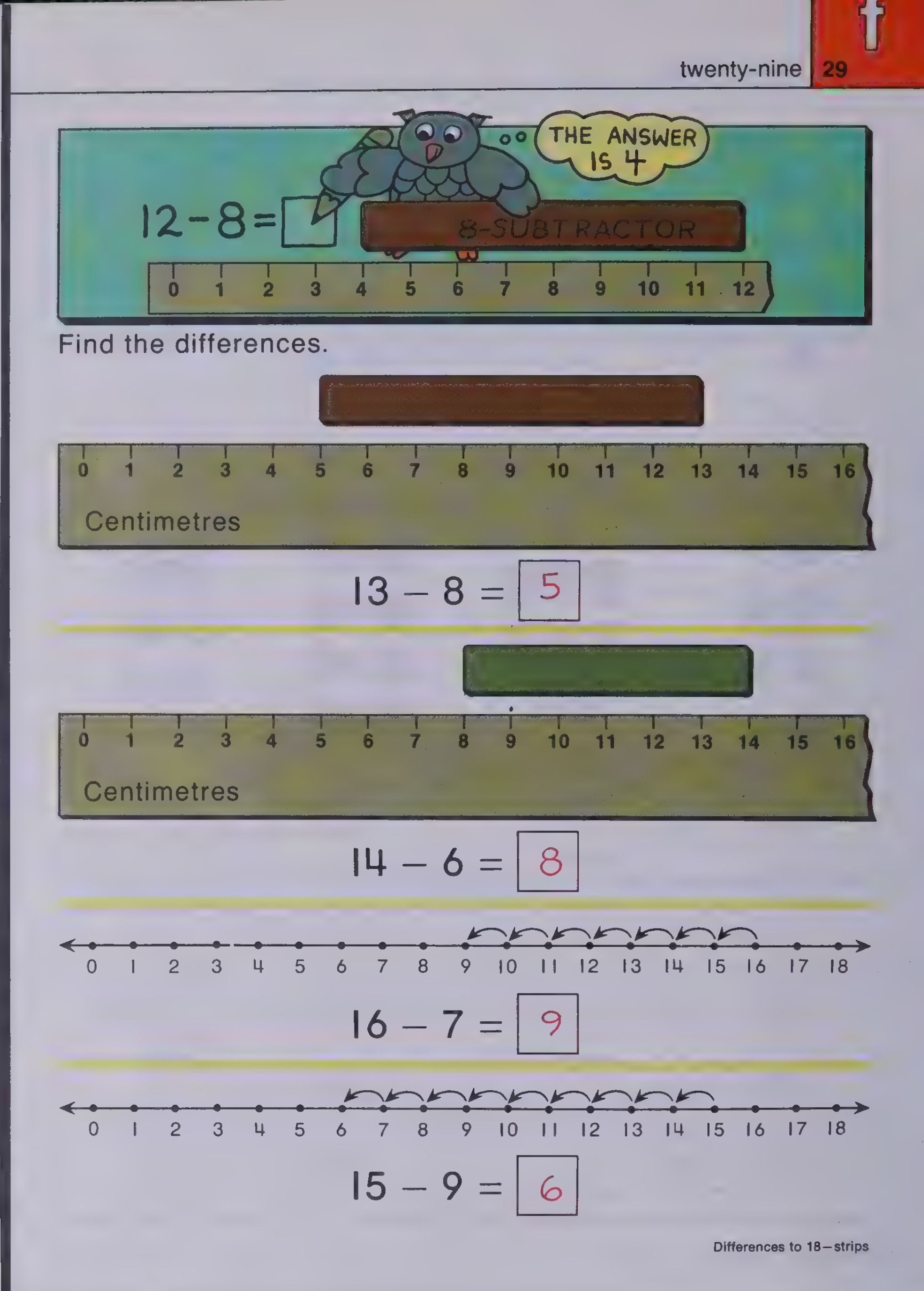
$17 - 9 = \square$

$16 - 8 = \square$



Teaching
Page f-29

Call attention to the frame at the top and ask the children to use their brown strip. Point out that since the equation reads 13 − 8, they should place their 8 or brown strip at the mark 13 and then read the difference from the left end of the strip. Work similarly through the second frame. You might also use the illustrated centimetre rulers to work with other strips as subtractors. For example, ask children to solve equations such as 14 − 9 = □, 12 − 5 = □, or 13 − 6 = □ and so on. Finally, relate the use of the strips to the number line. To subtract on the number line, the children should locate the first numeral in the equation on the number line and then count to the left the number of spaces indicated by the second numeral. They will land at the number which is the difference. Use the demonstration number line to work through several examples. Ask volunteers to show the jumps on the number line for equations which you write on the chalkboard.


twenty-nine 29

12−8=□

Find the differences.

13 − 8 = 5

14 − 6 = 8

16 − 7 = 9

15 − 9 = 6

Differences to 18—strips

Objective

Given a subtraction equation related to sums of 18 or less, the child will be able to find the difference using strips or a number line.

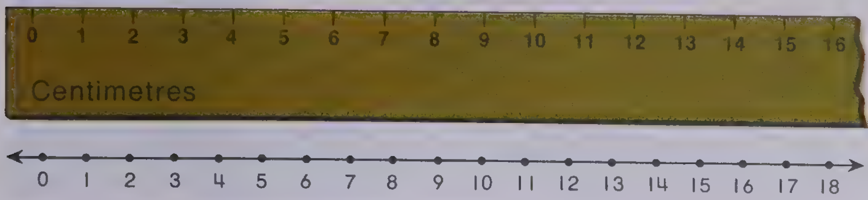
Pre-Book Activity

Materials

demonstration number line
one set of strips for each child

Remind the children of the investigation at the beginning of this unit. Recall with them how they used the yellow strip as a 5-subtractor by putting it on the edge of the centimetre ruler. If you have centimetre rulers, you might wish to distribute one to each child and have them work out a few equations. If not, use page f-29.

Find the differences.



12 − 9 = $\boxed{3}$	13 − 5 = $\boxed{8}$
11 − 3 = $\boxed{8}$	17 − 9 = $\boxed{8}$
15 − 6 = $\boxed{9}$	10 − 3 = $\boxed{7}$
14 − 7 = $\boxed{7}$	14 − 6 = $\boxed{8}$
16 − 8 = $\boxed{8}$	12 − 5 = $\boxed{7}$
13 − 6 = $\boxed{7}$	11 − 4 = $\boxed{7}$

14	13	15	11	18	12
−8	−8	−7	−6	−9	−6
6	5	8	5	9	6

13	16	15	12	14	11
−4	−7	−8	−8	−5	−8
9	9	7	4	9	3

Differences to 18—number line

Teaching
Page f-30

Explain to the children that the centimetre ruler and the number line at the top of the page are provided for their use in finding differences. Suggest that they use either the strips on their ruler or count on the number line to solve the equations. Note that the exercises at the bottom are written in vertical notation. Again point out that these may be solved in the same way as an equation.

Follow-up

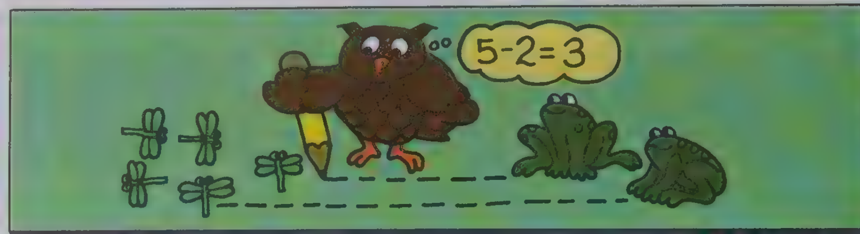
Direct the children in a number line activity. Roll out a floor number line. Divide the children into two teams of four or five each. Write a table similar to the one below on the chalkboard. Be sure there is one row for each child playing.

Start at 0		
Forward	Back	Land
5	2	
7	1	
	5	2

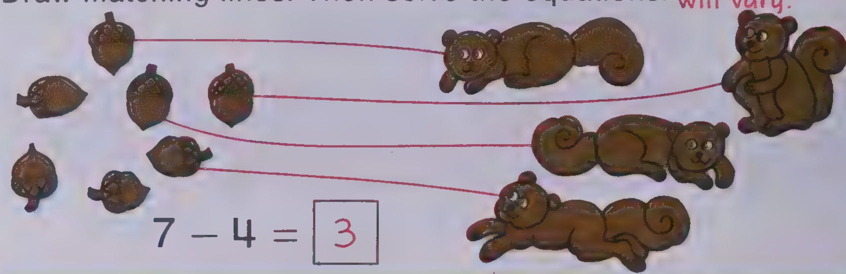
As each member of a team takes a turn, he should first try to guess the missing number on the table. If he guesses correctly, he scores two points; if he does not want to make a guess or guesses incorrectly, he can score one point by jumping out the problem on the number line. A child from the competing team should then take a turn. The team with the most points wins.

Teaching
Page f-31

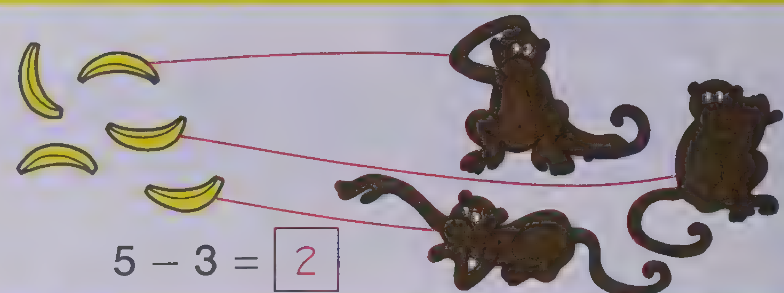
Use the demonstration art as an introduction. Then work through the first frame with the children. Suppose with the children that each squirrel was going to eat one nut. Ask them first to figure out if there are enough nuts for each squirrel to have one. Ask: "Will there by any nuts left over?" "How many nuts will be left over?" Then use the equation written below the illustration to point out the relationship between the numerals in the equation and the comparison situation. The equation tells us that there will be three nuts left over since one nut has been matched with each of the four squirrels. Although the equations on this page are simple, the idea of comparison subtraction, which is the main emphasis in this lesson, is more difficult. When you ask the question: "How many nuts will be left?" you are talking about take away subtraction, but when you ask the question: "How many more nuts are there than squirrels?" you are talking about comparison subtraction. Ask the children to draw the matching lines between the squirrels and nuts to help them see that there are 3 more nuts than squirrels. Work through the next illustrations similarly. Note that the subtraction examples on this page tell two things: how many are left and how many more of one thing than another. This is not always the case in comparison subtraction, as will be seen in other examples the children study.



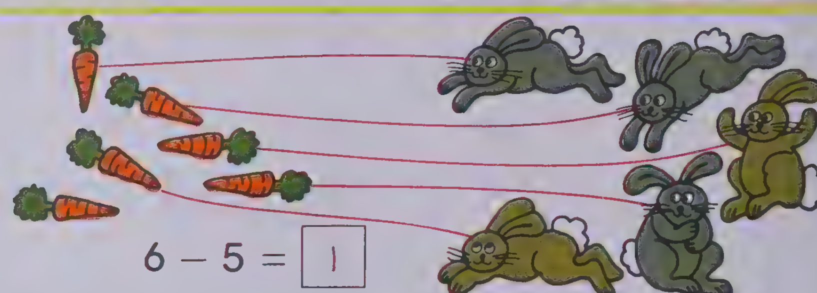
Draw matching lines. Then solve the equations. Matchings will vary.



$7 - 4 = \boxed{3}$



$5 - 3 = \boxed{2}$



$6 - 5 = \boxed{1}$

Subtraction – comparison

Objective

Given two sets both of which contain less than 18 elements, the child will be able to compare the sets and solve a related subtraction equation.

Pre-Book Activity

Materials

counters, at least 36 per child or for every two children
sheets of newsprint (optional)

Distribute the counters and pieces of newsprint for recording. Ask the children to build two sets with their counters so that one set is larger than the other. Explain that you want them to compare their sets, that is, to figure out which set is larger and how many counters larger it is. Ask children to talk about ways of finding out which set is larger. Some methods might be: matching each counter from one set with those of the other, or counting the number of counters in one set, counting the number of counters in the second set and comparing the two numbers. You will want to develop this discussion briefly and then go on to the text page.

Short Stories

1 Sue had 12 peanuts.
She ate 5.
How many left? 7

2 Mike caught 11 fish.
He gave away 6.
How many did he keep? 5

3 Fran had 13 balloons.
7 of them popped.
How many now? 6

4 Mary has 4 glasses
and 7 straws.
How many more
straws than glasses? 3

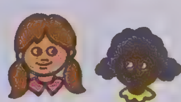
5 There are 15 children
in the pool. 8 are
girls. How many are
boys? 7

6 Jan saw 14 ladybugs.
9 flew away.
How many now? 5

7 Tom caught 6 fish.
Bill caught 10.
How many more did
Bill catch? 4

8 Beth had 15 cents.
She spent 8 cents.
How much does she
have now? 7¢

9 Spiders have 8 legs.
Ants have 6.
How many more legs
do spiders have? 2



10 Kathy is 8 years old.
Alice is 12. How much
older is Alice? 4

Teaching
Page f-32

It would be helpful to work through many of these problems with the children, not only to help them read the problem, but also to develop the comparison idea. Most of the problems on the page deal with the take away subtraction, but problems 4, 7, 9, and 10 should be used to develop the idea of comparison subtraction. In order to do this, for example, with problem 4 you might have the children draw four glasses. Then tell them that they have seven straws and that they should use as many straws as they need so that each glass has one straw. Ask them how many straws they have left over. Relate this question to the question: "How many more straws are there than glasses?" Again in problem 7, if children have difficulty with the comparison idea, have them draw Tom's 6 fish and Bill's 10 fish and then match Tom's 6 fish with 6 of Bill's fish. They will then find that Bill caught 4 more fish than Tom.

Follow-up

The use of graphing with the children will give you an opportunity to develop many more comparison ideas. Topics for graphs may spring from any discussion or interests which the children have. For example, ask them to find out how many different kinds of pets they have, or how many brothers and sisters each child has, or distances travelled from home to school, either in terms of city blocks or km. If you live in a transient area, compare how many cities or different places they have lived in during their lifetime. They might do some measurement and compare heights, weights, or sizes of shoes. Have the children suggest their own ideas. The more they become involved in such a graphing project, the better you will be able to develop the comparison subtraction ideas from the finished graph.

Resources for Active Learning

MATHEX: Operations No. 3, "Subtraction by Matching," p. 4, Encyclopaedia Britannica Publications Ltd.

Teaching
Page f-33

Even though generally drill doesn't enter the study of power skills, this is the first page on which both addition and subtraction equations are given. This is done primarily to evaluate a child's ability to recognize which operation to perform and also to give him practice in changing from one operation to the other as he works through the page. Note that it is still not intended that a child know all of these facts from memory. Any manipulative devices may be used as aids to help him find the sums and differences. However, if a child is able to work through the problems mentally, encourage him to do so. With slower groups of children it would be helpful to have them explain how they worked out a few examples after they have completed the page. Children should be encouraged to work this page independently.

Find the sums and differences.

7 + 7 = 14	12 − 4 = 8
6 + 5 = 11	15 − 6 = 9
5 + 7 = 12	11 − 4 = 7
9 + 7 = 16	12 − 7 = 5
6 + 8 = 14	10 − 3 = 7
8 + 7 = 15	17 − 9 = 8

8	6	3	16	14	10
+ 3	+ 4	+ 9	− 7	− 5	− 5
11	10	12	9	9	5
8	14	9	8	15	13
+ 8	− 8	+ 6	+ 9	− 7	− 9
16	6	15	17	8	4

Subtraction practice

Objectives

Given both addition and subtraction equations, the child will be able to find the sums or differences.

Given short stories which require either simple addition or simple subtraction, the child will be able to determine which operation to use and solve the problem.

Pre-Book Activity

Since this lesson emphasizes the recognition of the addition and subtraction operations, you might write on the chalkboard parts of equations such as these shown in the next column. Then tell the children that you are going to either add or subtract the two numbers and that you will write down only your answer. Then they should write the sign which shows what you did between the two numbers.

9 − 7 = 2
9 + 7 = 16
8 − 4 = 4
12 − 7 = 5
8   7 = 15
7   4 = 3
7   4 = 11

Follow-up

Examples such as the following may be used to extend activities with short stories. Small groups of children may work on these together. After someone reads the problem, they should first tell which operation to use and

Short Stories

1 One spider has 8 legs. How many legs on 2 spiders? 16

2 Ted scored 16 points. Fred scored 9. How many more did Ted score? 7

3 6 girls and 7 boys came to the party. How many children? 13

4 Marty baked 15 cookies. She ate 6. How many left? 9

5 Nancy had 8 cents. She earned 7 cents. How much does she have now? 15¢

6 There are 8 chairs at the table. 5 children sit down. How many empty chairs? 3

7 Crayfish have 10 legs. Grasshoppers have 6. How many more legs do crayfish have? 4

8 Peg bought 2 bows for 7 cents each. How much did she spend? 14¢

9 Sally is 7 years old. Her sister is 13. How much older is her sister? 6 years

10 There are 6 apples and 7 oranges in the bowl. How many in all? 13

Story problems

Teaching
Page f-34

The short stories on this page are problems which involve both addition and subtraction. According to the children's reading ability, you may encourage them to work the pages by themselves or read the problems with them, but encourage them to think through the solution of the problem by themselves. Suggest that they use counters to help them solve the problems or draw pictures while they are thinking about each problem. It is not necessary that they write an equation for each problem, but some children would benefit from doing so.

then find the answer. If you work these with the children, you may wish to include an intermediate step, such as having a child write the equation on the board with a placeholder in it, before having them attempt to solve the problem mentally.

You might write the following short stories on task cards for children to solve.

1) 8 birthdays in January. 6 were girls'. How many boys' birthdays in January?

2) 17 marbles. 9 are yellow; the rest green. How many are green?

3) 15 children built a snow fort. 10 were boys. How many were girls?

4) 8 chocolate-chip cookies. 10 raisin cookies. How many cookies in all?

For more work with addition and subtraction, give the children the following exercises.

Write + or − in each ○.	
9 ○ 7 = 10 + 6	5 ○ 9 = 16 ○ 2
8 ○ 2 = 12 − 2	7 ○ 6 = 15 ○ 14
3 ○ 9 = 5 ○ 7	11 ○ 2 = 4 ○ 9
14 ○ 7 = 2 ○ 5	16 ○ 8 = 5 ○ 3

Resources for Active Learning

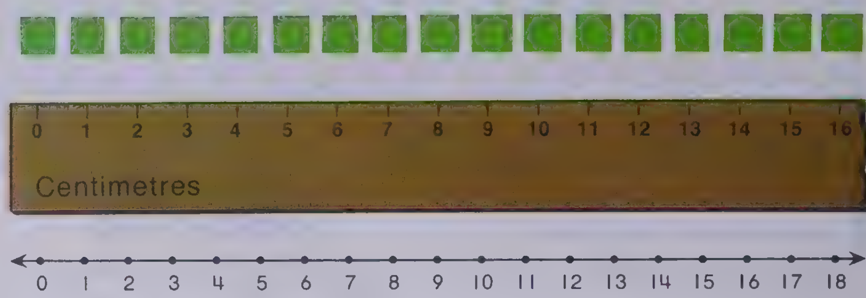
DEVELOPMENTAL MATH CARDS, "Charts," C-18, Addison-Wesley

TEACHING AIDS FOR ELEMENTARY MATHEMATICS, "Number Sentence," p. 21, Holt, Rinehart and Winston

Teaching
Page f-35

Point out the counters, ruler, and number line at the top of the page which indicate methods which the children may use to solve these equations. However, also point out that if they can find the answer simply by thinking through the problem, they might do so and then possibly check their answer by using one of the methods shown. Encourage the children to work through this page independently. Be sure they understand that they are free to choose whichever method they wish to use to solve these exercises. Help any children who have difficulty reading the word problems at the bottom of the page.

Show you know



Solve.

$12 - 5 = \boxed{7}$ $\quad$ $16 - 8 = \boxed{8}$

$15 - 7 = \boxed{8}$ $\quad$ $12 - 4 = \boxed{8}$

$14 - 5 = \boxed{9}$ $\quad$ $14 - 7 = \boxed{7}$

$13 - 6 = \boxed{7}$ $\quad$ $13 - 9 = \boxed{4}$

$11 - 2 = \boxed{9}$ $\quad$ $17 - 8 = \boxed{9}$

14	12	11	8	6	7
− 6	− 3	− 4	+ 4	+ 5	+ 7
8	9	7	12	11	14

Alan had 15 cents. He spent 8 cents. How much does he have left? 7¢

Sara lives 12 blocks from school. Ann lives only 5. How much farther does Sara live? 7

Module review

Objective

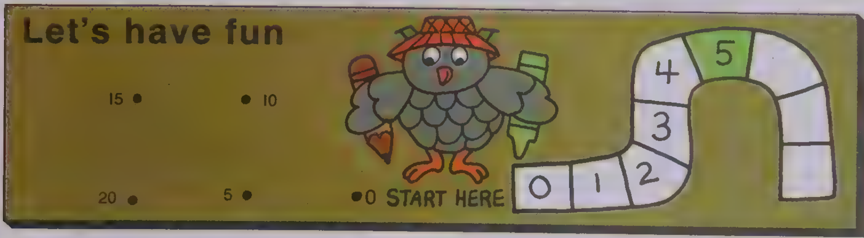
The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Materials

counters
demonstration number line
strips

Have volunteers work through the various methods they have studied. Write an equation such as $15 - 9 = \square$. Ask a child to show with counters or with pieces of felt on the flannelboard a way in which he can solve this problem. Then ask another child to show how to solve the problem with the strips. Ask a third child to solve the problem on the demonstration number line. The child who uses the counters may simply build a set of 15 and remove 9 or he may build a set of 15 and a set of 9 and match them one-to-one, showing that there are 6 not matched. The child who uses the strips should be given a centimetre ruler to work with. To work the equation using this method, he would simply place the right edge of his 9-strip at the 15 mark and read off where the left-edge of his 9-strip falls. Remind the children how they used the different strips as subtractors. The child who uses the demonstration number line might simply begin at 15 and count back 9 spaces. Notice that it is not necessary for a child to draw small arrows on the number line. He may simply do the jumping with his finger.

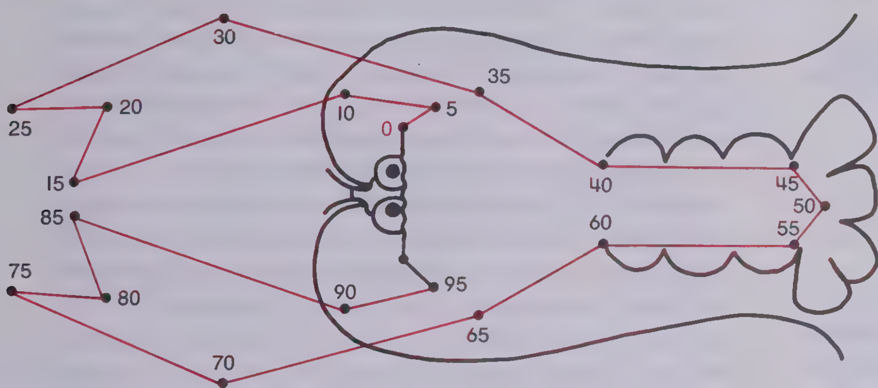


Complete the counting. Color every fifth box.

Complete the skip counting by fives.

0	5	10	15	20	25	30	35	40	45	50	55

Skip count by fives to connect the dots.



Teaching
Page f-36

Read the directions with the children. Explain that after they fill in the numerals, they should color every fifth box, then they should complete the skip counting by fives. The dot picture at the bottom of the page should also be connected by counting by fives. If necessary, orally review counting by fives before children do this page.

Follow-up

Develop an oral activity in which you give problems and ask the children to tell you what operation was used to find the solutions. For example:

1) If 7 boys and 8 girls brought insect books to school, how do you know 15 insect books were brought?

2) In Room 2A, 9 girls are Brownies in Troop 10. The rest of the Troop are in Room 2B. If 17 Brownies are in troop 10, how can you prove that there are 8 Brownies in Room 2B?

If the children's reading ability allows you might duplicate problems such as those in the next column.

Solve the first 3 problems, then make up two of your own.

1) 14 skaters in the club. 8 skaters raced. How many did not race?
2) 15¢ allowance. 9¢ for pencil and eraser. How much change?
3) 9 boys and 7 girls do jobs. How many room helpers this week?
4) . . .
5 . . .
6) . . .

Resources for Active Learning

Refer to page e-62 for resources pertaining to number patterns.

Measurement

Pages f-37 to f-46

General Objectives

To introduce the basic measurement concepts of length, perimeter, and area

To familiarize the child with the unit, 'centimetre'

To provide experiences in exploring volume through work with liquid measure

The first few lessons of the module concentrate on the concept of length. Non-standard units such as the paper clip should be explored by the children and serve as an introduction to the use of the standard unit, the centimetre. Children are given an opportunity to make centimetre rulers using the punchout units available separately to use with this book. The concept of length is extended to include finding the length of a path consisting of several line segments and is finally related to the idea of finding the distance around a figure, that is, of finding the perimeter.

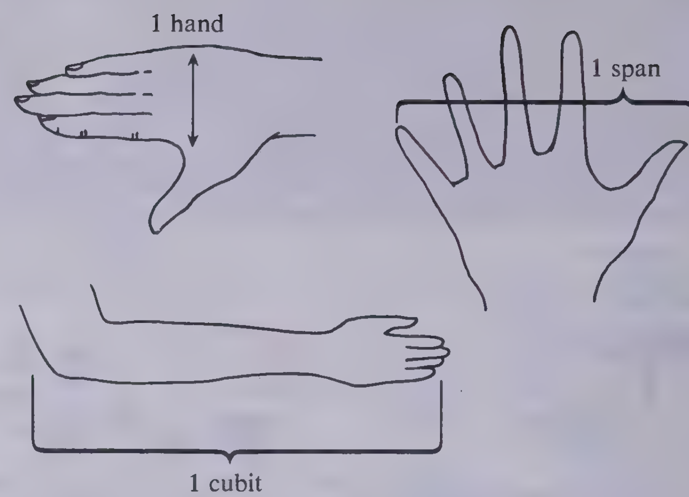
The concept of area is introduced. The module concludes with a review page and a change of pace page which encourages exploration of liquid measure.

Mathematics

Measurement is a process of associating numbers with certain objects. Before an object in the real world can be measured, a unit must be chosen. The number 1 is assigned to this unit. The measuring process then consists of comparing the object to be measured with this unit, usually by counting the number of units it takes to "cover" or "fill up" the object. Thus, the choice of the unit determines which numbers are assigned to other objects. This process is illustrated below.

The Measurement Process		
S stands for object to be measured. →	1. Select a unit. 2. Divide object into units. 3. Count the number of units. →	A number for the measure of S

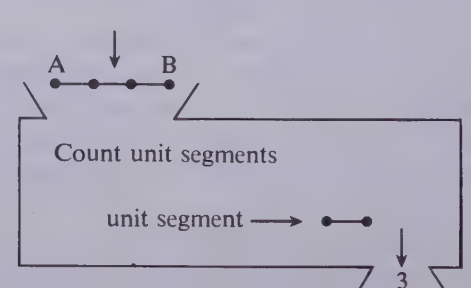
We should observe that *the choice of units is arbitrary,* as shown in the following figure.



Early units for measuring length were frequently related to parts of the body. For example, the width of the palm, roughly 10 cm, makes a *hand.* Horses are measured by hands. If you spread your thumb and little finger as far apart as possible, you have a unit of measure called a *span*, about 23 cm. A biblical unit of measure, the *cubit,* is the distance from the elbow to the tip of the fingers, about 46 cm. Goliath is said to have had a height of "6 cubits and a span." Sailors still use a measure called a *fathom,* about 1.8 metres. This was originally the distance between outstretched fingertips when the arms are stretched out to each side at shoulder height.

Words like *height* and *distance* are often used to describe the idea of *length*. For example, we speak of the length of a piece of string, the distance between Don Mills and Montreal, and the height of a flagpole; but the same idea is present in all of tnese instances. We mean that we choose a unit of measure and then give a number that tells approximately how many of these units it would take, lying end to end along a straight path, to reach from one point to another.

The length of segment AB is a number that compares the size of the segment to be measured with the size of a unit segment. Perhaps the following illustration will help clarify the concept of measurement of length.



The input is line segment AB. The machine performs the operation of counting unit segments, and the output, 3 in this example, is the number of unit segments in the input.

Teaching Light Green Module, Unit F

Approximate Time: 6 to 8 days (or longer, depending on extended activities)

Materials

glue or paste
paper clips (6 or 7 per child)
centimetre units (accompanying punchouts are available)
blank ruler (to be made by children with the punchout units)
centimetre rulers if available
square centimetre units
set of strips for each child
various size squares cut from cardboard or stiff paper
yarn or string

One of the most important activities of this module is for the children to make their own centimetre rulers. The punchout units (available separately) should first be used as individual pieces for page f-37 and f-38. Then in the second lesson, these units should be used to make centimetre rulers.

Other activities are suggested for each lesson. You are encouraged to develop as many of these extensions as possible. Although the basic concepts of length, area, and perimeter are developed within the text pages, children should be given the opportunity to experience as many measuring situations as possible. To allow for this type of extension you might choose to work through this module over a long period of time, allowing children to work on measuring projects before continuing through the development of each lesson.

Resources For Active Learning

General Activities:
Balance and weight:

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Weight," pp. 82–87, Herder and Herder
Franklin Series: LEARNING ABOUT MEASUREMENT, pp. 69–73, Lyons and Carnahan
MATH ACTIVITY CARDS, A31–35, Macmillan
MATHEX: Measurement and Estimation No. 5, pp. 37–38, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: BEGINNINGS 1, pp. 58–68; COMPUTATION AND STRUCTURE 2, pp. 23–33, Wiley
THE BALANCE BOOK, Webster, McGraw-Hill

Estimation:

MATHEX: Measurement and Estimation No. 5, pp. 45–48, Encyclopaedia Britannica Publications Ltd.
MATHS MINI-LAB, Card 50, Selective Educational Equipment
WORKJOBS, pp. 232–233, 236–237, Addison-Wesley

Reading and making maps:

ELEMENTARY SCHOOL SCIENCE, Book 2, Unit 1, "Scale Measure," pp. 32–47, Addison-Wesley
MAKING MAPS, Webster, McGraw-Hill
MAPPING GAMES, 1, 10, 16, 23, 24, Webster, McGraw-Hill
MATH ACTIVITY CARDS, A9, Macmillan

Tessellations (tiling):

FREEDOM TO LEARN, p. 130, Addison-Wesley
MATH ACTIVITY CARDS, A15–18, Macmillan
NOTES ON MATHEMATICS IN PRIMARY SCHOOLS, pp. 131–142, Cambridge University Press
Nuffield Project: SHAPE AND SIZE 2, pp. 36–45, Wiley
SIGMA CHIPS (manual), pp. 3–7, Scott Scientific

Manipulative Devices:

Basic shapes set (Educational Teaching Aids, Math Media; Responsive Environments Corp.)
Capacity and weight measures (Childcraft; Educational Teaching Aids; school supplier)
Cuisenaire Cubes, Squares and Rods (Cuisenaire Co.)
Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Height measure (Educational Teaching Aids; Responsive Environments Corp.)
Parquetry pieces (Ideal; school supplier)
Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Trundle wheels (Educational Teaching Aids; Math Media)

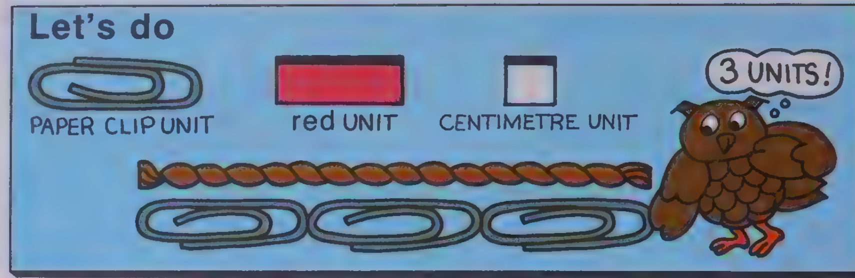
Commercial Games:

Which Is More? (Selective Educational Equipment)

Investigation
Page f-37

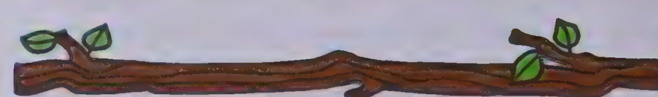
For this investigation children will need the punchout centimetre units sold as separate items to accompany this book. They will also need paper clips, at least six or seven for each child. Call the children's attention to the illustration at the top of the page. Point out that the clips are being used to make a train. Ask them how many paper clips long the piece of rope is. Point out that we can say that the rope is about three paper-clip units long. Introduce the word *unit* informally by talking about the paper clip as a unit or by referring to the measure of the rope as about three paper-clip units long. Then present the children with the investigation question: "Can you make trains as long as each object? Explain that they are to use the units shown at the top and that a space is provided to record how many of each unit they use. They should first measure each object with the paper-clip units, then with the red units, and finally with the centimetre units. Be sure that the children distinguish between the red unit and the centimetre unit. When you introduce the punchout centimetre pieces, stress that the black line shown on the edge of the pieces is the length with which they are comparing their object. That is, the black segment gives the children the unit with which they should measure the object.

Let's do



Can you make "trains" as long as each object?
Use the units shown above.

About 2 paper clip units | About 3 red units | About 8 centimetre units



About 4 paper clip units | About 5 red units | About 13 centimetre units

Purposes

To provide children with experiences related to the basic idea of measurment

To give the children an idea of various situations in which measurement is used

Preparation

Materials

centimetre units (punchouts available separately)
paper clips (6 or 7 per child)

Provide a variety of informal activities involving the basic ideas of measurement. You might challenge the children in pairs to tell how large or long or tall something is by using only their hands and arms. For example, my desk is 8 hands wide or Sharon is 5 arms (elbow to finger tips) tall. Or have one partner guess how far down the hall ten "normal" steps will take the other. The possibilities for exciting measurement-type experiences are limitless. Give the children much freedom in creating their own.

Follow-up

Provide the children with sets of objects which might be used as units of measure. For example, to one group distribute different lengths of string or yarn. To another distribute different length strips of paper. To another a variety of objects such as a chalkboard eraser, a pencil,

Let's talk

Readiness for measurement

Discussion
Page f-38

This page will provide you with a basis for discussing a wide variety of measuring situations. Point out that the two girls are measuring the blackboard in metres, that is, a metre is the unit being used just as in the investigation a paper clip was used or the centimetre and red units were used. Stress that the two flowers being measured are obviously different heights and the boy and girl are using metres to measure them. Show a metre ruler to the class, as well as a tape measure. Point out that both groups at the bottom of the page are using tape measures even though they are of different kinds. Allow for much discussion concerning measuring devices and units.

Also use the fish tank as a basis of discussion since liquid measure will be treated in this module.

a strip of paper and a piece of string. Suggest to the children that they use these objects to measure certain items in the room. Besides measuring things such as window frames, table legs, chalkboards, and other furniture, encourage them to try to measure each others' heights. Some children would benefit from recording their results on a chart which gives their height in different units.

> I am 10 erasers tall.
> This is the same as
> 4 pieces of my string
> 10 pencils
> 56 paper clips
> 12 hands

Resources for Active Learning

Non-standard linear measurement:

ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 1, pp. 21–33, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Length," pp. 59–63, Herder and Herder

MATCH AND MEASURE, Webster, McGraw-Hill

MATH ACTIVITY CARDS, A24–30, Macmillan

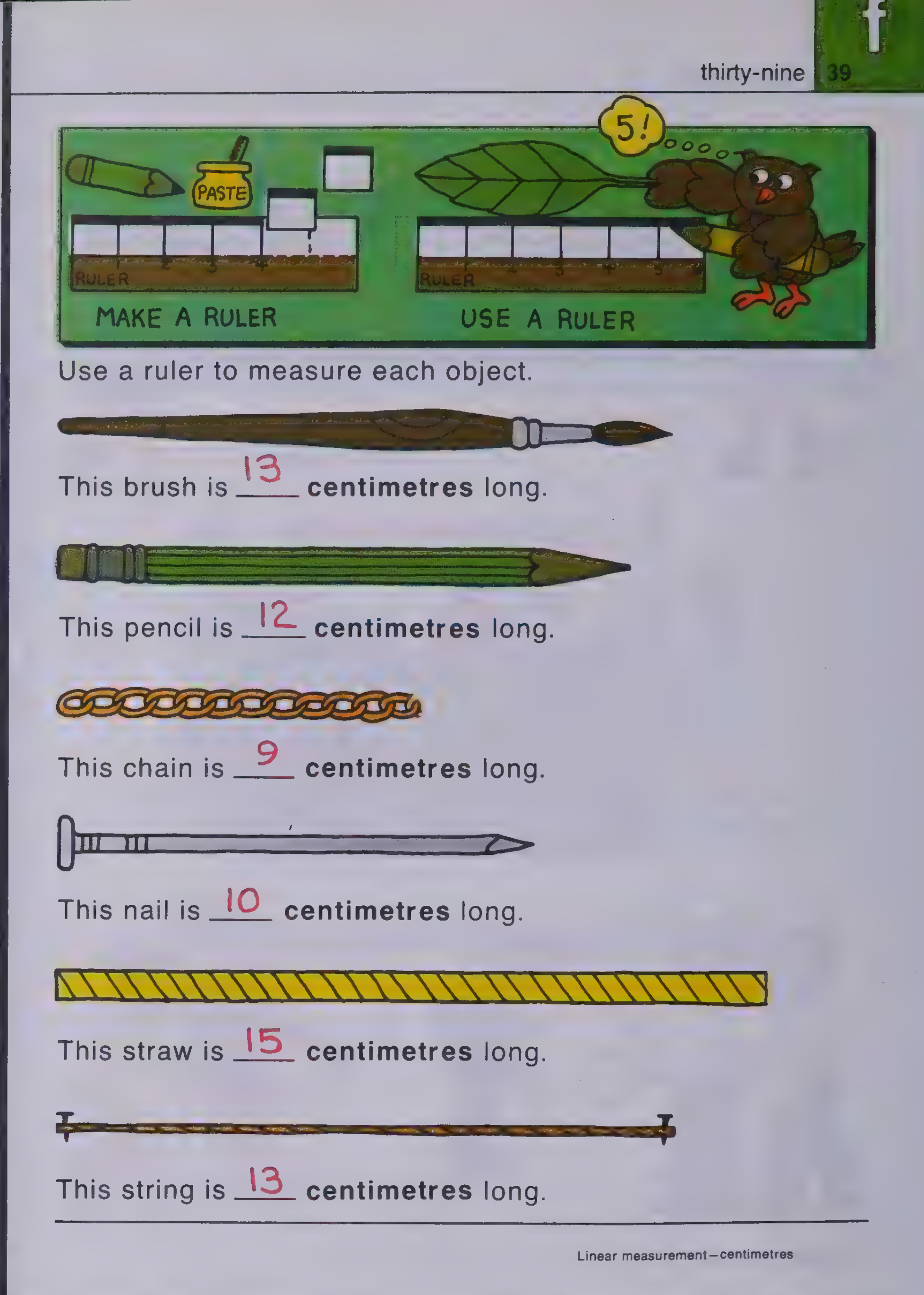
MATHS MINI-LAB, Cards 75–77, Selective Educational Equipment

Nuffield Project: BEGINNINGS 1, pp. 68–81, Wiley

WORKJOBS, pp. 218–221, Addison-Wesley

Teaching
Page f-39

Read the directions at the top of the page with the children. Point out that they just make the ruler as illustrated. Now they may use this ruler to measure each object. You might want to use a transparency on an overhead projector to show the children how the left edge of the ruler should be placed at the left edge of the object to begin measuring it. Then the number of the unit nearest to the right edge of the illustration should be read as the measurement of the object. When you are sure that the children know what to do and can read the word "centimetres," encourage them to complete this page by themselves.


f

thirty-nine 39

Use a ruler to measure each object.

This brush is 13 **centimetres** long.

This pencil is 12 **centimetres** long.

This chain is 9 **centimetres** long.

This nail is 10 **centimetres** long.

This straw is 15 **centimetres** long.

This string is 13 **centimetres** long.

Linear measurement—centimetres

Objective

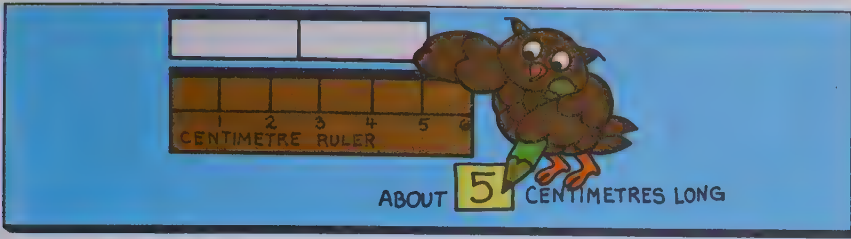
Given a centimetre ruler, the child will be able to measure objects to the nearest centimetre.

Pre-Book Activity

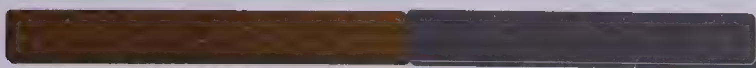
Materials

glue or paste
punchout ruler (which can be purchased to accompany the text)

Have children make the ruler provided with the book. To do this, children should paste the centimetre units in the spaces provided on the punchout ruler. As you direct the children in making their rulers, stress that the black edge of the unit should be placed on the edge of the punchout ruler. Also that the left punchout unit should be placed at the edge of the ruler. Then the next unit should be placed not on top of, but right next to and touching the first unit; similarly the third unit should be placed as if they were making a train and they should be very particular that the units are end to end.



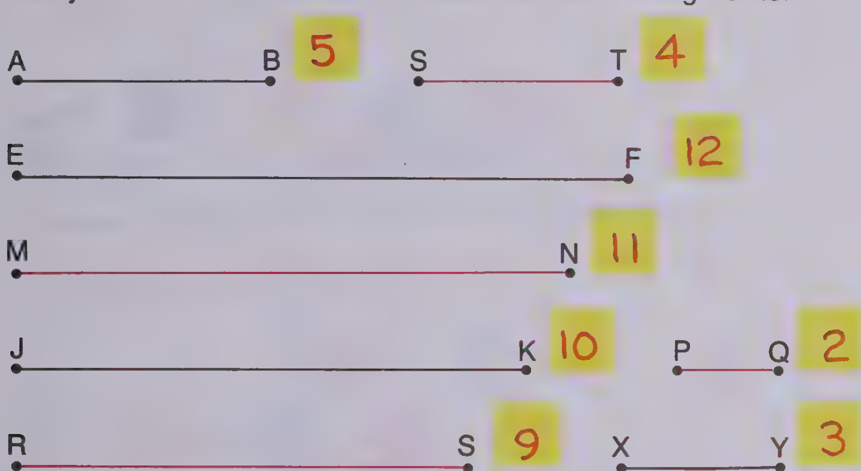
Measure the strip "trains." Use your rulers.



15 centimetres long

13 centimetres long

Use your centimetre ruler to measure the line segments.



Linear measurement – centimetres

Teaching
Page f-40

The illustration introduces the page and reinforces the idea of a centimetre. Point out that the owl is measuring with a centimetre ruler. Help the children realize that since the measurement is not exact we say "about" 5 centimetres. Read the directions at the top of the page with the children. Here they are given strip trains to measure. You might point out that measurement of the strip trains comes out evenly when the centimetre ruler is used. For the bottom half of the page, explain to the children that they are to measure the lengths of these line segments. *Be sure they understand that some of their measurements will not be exact, but they are to record their answers to the closest centimetre mark on their ruler.*

Follow-up

Now that the children have been introduced to the centimetre ruler, you might want to extend discussion to the metre rule. You might also use the metre to measure various objects which the children have already measured in the follow-up for the previous lesson. For example, if they did measure their heights, you might now have them measure their height with a metre stick. Rather than defining metre in terms of centimetres, they may simply say, $1\frac{1}{2}$ metres. Other objects which they might measure are the height of the door, the length of the classroom, the width of the chalkboard. You might also simply draw segments on the chalkboard and have the children measure these.

Resources for Active Learning

Linear measure using standard units:

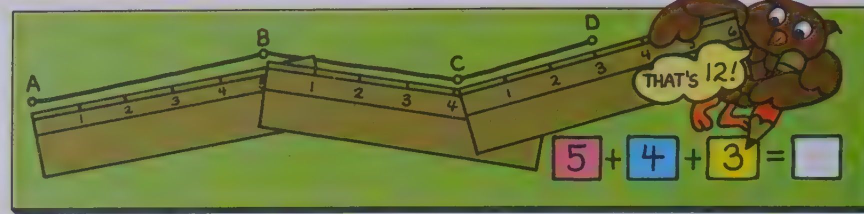
ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 1, pp. 34–39; Book 2, Unit 1, pp. 14–47, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Length," pp. 63–69, Herder and Herder

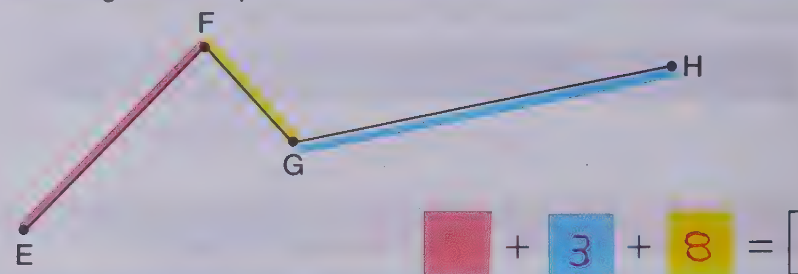
MATHEX: Measurement and Estimation No. 5, pp. 23–27, Encyclopaedia Britannica Publications Ltd.

Teaching
Page f-41

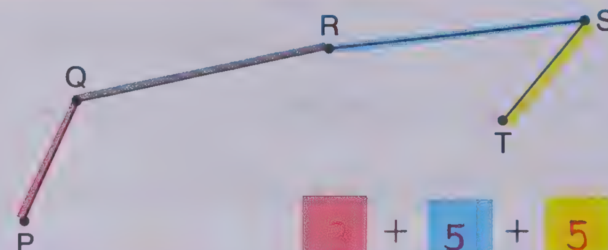
Use the illustration at the top of the page and ask the children how the path from A to B to C to D is being measured. In particular, ask them to explain the numerals written in the equation and to decide which numeral should go in the fourth space provided. Finally ask them how they can find the length of the entire path from A to B to C to D. Again stress that the length of the total path may be found by adding the length of each part of the path. Then ask the children if they can find the length, in centimetres, of each of the paths on this page. Stress that they should use their centimetre rulers for these measurements.



How long is each path?



+ 3 + 8 = 16



+ 5 + 5 + 3 = 16

+ 10 + 13 = 28

Lengths of paths

Objectives

Given a polygonal path a whole number of units long, the child will be able to use a centimetre ruler to find the length of the path.

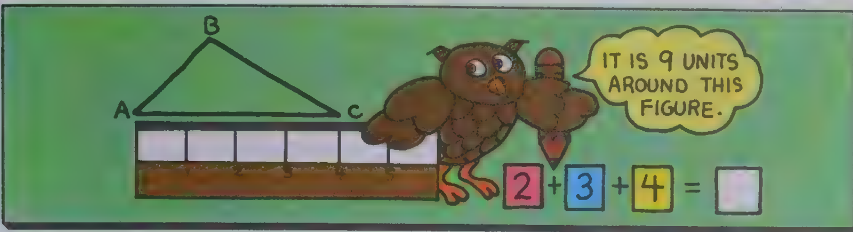
The child will be able to find the perimeter of a triangle or a quadrilateral by using a ruler.

Pre-Book Activity

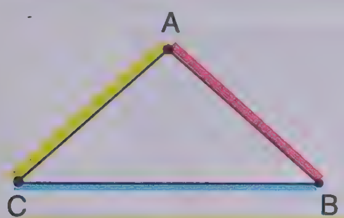
Materials

rulers which the children made in the last lesson or commercial centimetre rulers

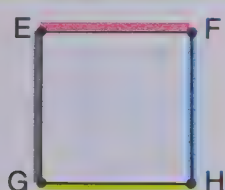
Since this lesson introduces the children to the idea of adding lengths, you might simply have the children stretch out their arms and stand forming a train so that the hand of one child grasps the hand of the next child. You might have the children stretch their arms and make a train alongside the walls of the classroom and then count how many children are needed to go around two walls of the classroom, or across one wall of the classroom, and then ask the children if they can figure out how many children would be needed to make a train which would go completly around all four walls of the classroom. If there are not enough children to extend around all four walls of the classroom, they might have to double the number of children it takes to extend along two walls.



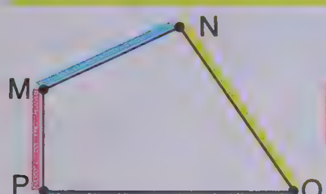
How far is it around each figure?



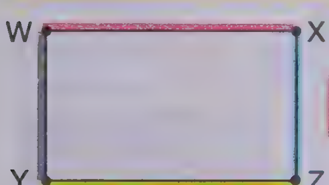
$\square + 6 + 4 = 14$



$\square + 3 + 3 + 3 = 12$



$\square + 3 + 4 + 5 = 14$



$\square + 3 + 5 + 3 = 16$

Perimeter

Teaching
Page f-42

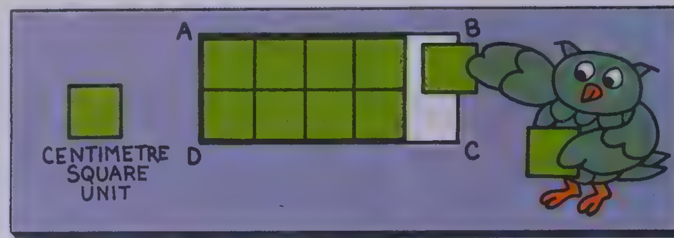
Before giving directions for this page, you might relate it to page f-41 by asking the children how they were able to find the total lengths of the three paths they measured. Then explain that on this page they should do the same thing, again using their centimetre ruler. It would be helpful to use the chalkboard or flannelboard to discuss the length of the paths which form the sides of these closed figures. Use the phrase "distance around the figure." Also point out that the figures may be discussed by referring to the letters which are written at their corners.

Follow-up

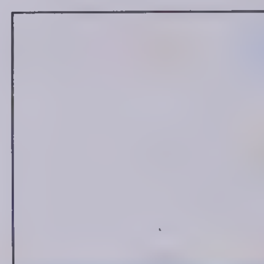
Give the children lengths of string, some 30 cm long, and others 60 cm long. Then suggest that they measure various objects in the room, but that the only thing they can use is this length of string. Suggest different methods of reporting the various findings. For example, some might report lengths in short string or long string. Others might want to "convert" their findings into centimetres.

Teaching
Page f-43

Suggest that the children use their centimetre ruler to measure the sides of the square units shown at the top of the page. Observe with the children that each side of the centimetre square is one centimetre long, thus the name centimetre square. Relate the illustration to the pre-book activity which the children did. Point out that here the centimetre square is being used and that to answer the question: "What is the area of the first figure?" they need only count the number of centimetre square units which it takes to cover the region enclosed by the figure.

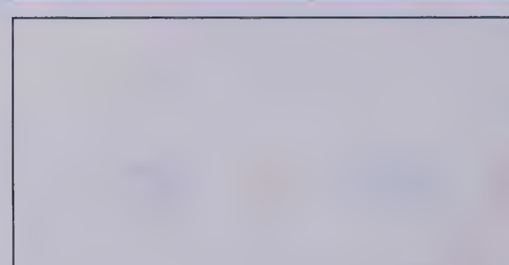


How many square units are needed to completely cover the inside of each figure?



25 centimetre squares

14 centimetre squares



50 centimetre squares

Objective

Given a region which is composed of a whole number of square centimetres, the child will be able to find the area of the figure in the corresponding centimetre square unit.

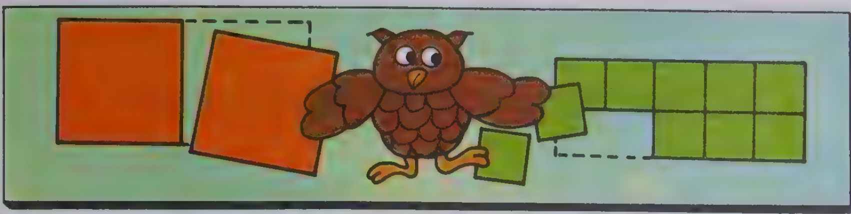
Pre-Book Activity

Materials

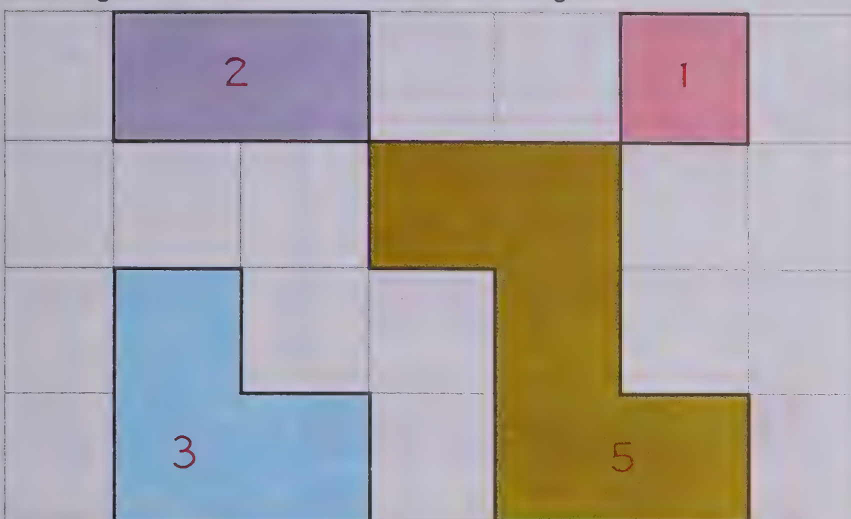
red squares and centimetre squares (available as a separate item)
various sized squares of cardboard (25-by-25 cm, 30-by-30 cm, 15-by-15 cm, etc.)

Give each group of three or four children several of the same size squares. Specify a region for each group to cover with their squares. They might cover the table that they are working on or large pieces of paper which you have measured previously and cut out or certain areas of the floor which you specify with tape. First challenge the children to guess how many of their pieces they will need to cover their area. Then explain that they may use any method they please to find out how close their guesses were. The group should find out how many of their squares are needed to cover the region.

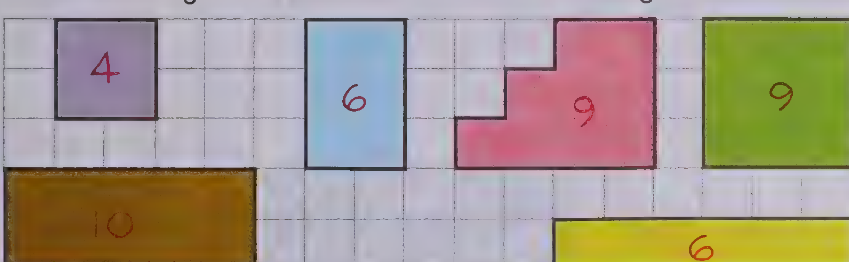
The idea of a centimetre square will be developed in the lesson. This activity simply introduces the question: "What is area?"



How many red squares are needed to cover each figure? Write the number on the figure.



How many centimetre squares are needed to cover each figure? Write the number on the figure.

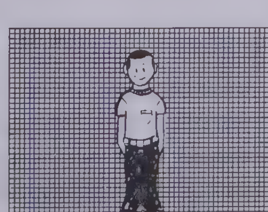


Teaching
Page f-44

Again suggest to the children that they use their centimetre and red squares to find the area of each figure. However, since each figure is placed on an illustration of graph paper, also help them discover that they might figure out the area of each figure without actually covering it with their square units. The use of the overhead projector and a transparency which has been ruled into centimetre squares would be very helpful. You might simply place the transparency over small, rectangular regions to help the children see how to figure out how many square units are contained in the figure. Although all of the figures on this page cover a whole number of red squares or a whole number of square centimetres, you might extend the concepts of this lesson to irregular shapes which require more guess work, as suggested in the follow-up.

Follow-up

Distribute a piece of graph paper ruled into centimetre squares to each child. Then ask them to place one of their feet on this paper, draw its outline, and then try to figure out what the area of their foot is. This may also be done with leaves or hands. You might also have the children tape sheets of graph paper together and measure the area of their bodies.



Resources for Active Learning

Area and perimeter activities:

ACTIVITIES IN GEOMETRY, p. 57 (tessellation activities), Addison-Wesley

DEVELOPMENTAL MATH CARDS, D²17, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, pp. 87–91, Herder and Herder

FREEDOM TO LEARN, pp. 127–131, Addison-Wesley

MATH ACTIVITY CARDS, A40–41, Macmillan

MATHEMATICS IN MODULES, M5 • MEASUREMENT • Plane Shapes: Covering Surfaces, Addison-Wesley

MATHEX: Measurement and Estimation No. 5, pp. 28–34, Encyclopaedia Britannica Publications Ltd.

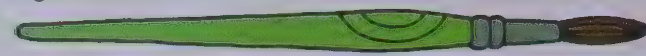
Nuffield Project: SHAPE AND SIZE 2, pp. 61–77, Wiley

TEACHING
Page f-45

Although this page reviews the concepts presented in the module and may be used as an evaluation instrument, you might want to simply use it as a review. Some children will need help with the reading involved, particularly in the bottom section. Observe with the children that three spaces are provided so that they may give the length of the brush using three different units. Be sure that they realize that the first space should show the length in paper clip units, the second in red units, and the third in centimetre units. Similarly, for the second question, they should first measure the sides of the rectangle in red units, and then in centimetres. Encourage children to use their square units for the third and fourth questions. Give as much help with the directions as necessary.

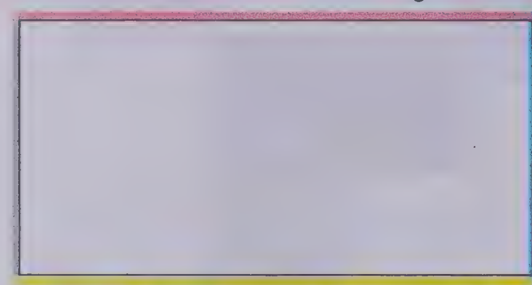
Show you know

How long is the brush?



About	About	About
4	5	13
paper clip units long	red units long	centimetres long

How far is it around this rectangle?

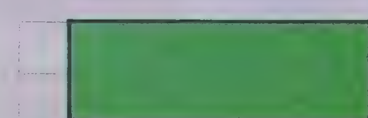


4 + 2 + 4 + 2 = 12 red units

10 + 5 + 10 + 5 = 30 centimetres

How many red square units are needed to cover the inside of the rectangle above? 8

How many centimetre square units are needed to cover the figure below? 12



OBJECTIVE

The child will demonstrate his ability to work with the concepts presented in this module.

PRE-BOOK ACTIVITY

Materials

centimetre ruler
centimetre squares
red unit squares
paper clips

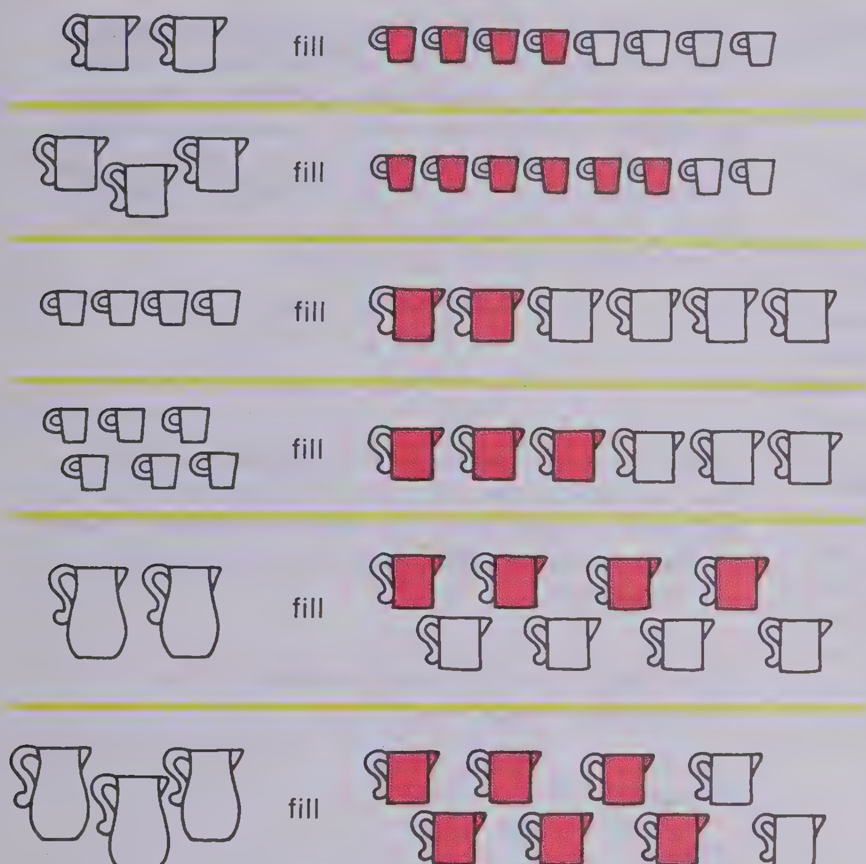
To review the kinds of measurement treated in this module you might display various objects for the children and ask them to describe how they might find the length, height, or area of these objects. For example, show two large sheets of paper which differ slightly in shape and size, and ask the children to explain how they might be able to find out which piece would cover the most area. Or, you might show a piece of yarn and ask how they might find out if it is long enough to be used as a border for a bulletin board. To review the use of centimetres, you might also ask the children to measure the length of one of their books in centimetres.

FOLLOW-UP

Task cards may be used to encourage the children to further investigate liquid measure. Divide the children into small groups and have them use the liquid containers to find the answers to the questions on the cards.

Let's have fun

Color the correct number of containers.



Liquid measure

Teaching
Page f-46

Although this page is presented as a change of pace, you might wish to develop it into a full day's lesson or even extend it into a project. Provide the class with as many liquid measure containers as possible. Children should be encouraged to actually pour from one container to another to compare the quantities that they hold. The text page may be handled by assigning children into groups and having some children do the pouring while others record the results.

Sample tasks:

1) Find the largest and the smallest pouring containers in the classroom. How many small containers do you need to fill the large container?
2) Find a tall narrow glass and a short fat glass. Which one holds more liquid?
3) How many cups do you need to fill two half-litre pitchers? How many of these cups are needed to fill one litre?
4) Take three glasses of the same size and shape. Fill each with a different amount of water. Tap each one with a pencil or scissors. Which glass makes the lowest tone? Which glass makes the highest tone?
5) Find six containers of different size and shape. Arrange them in order by guessing how much water each one can hold. Now check your guesses.
6) Find four containers of different size and shape. Without measuring try to pour the same amount into each container. Now measure how much is in each container. How close were your guesses?

Resources for Active Learning

Volume and capacity:

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Capacity," pp. 77–82, Herder and Herder

FREEDOM TO LEARN, pp. 131–134, Addison-Wesley

MATHEMATICS IN MODULES: SK1 • SPATIAL KNOWLEDGE • Face and Plane Shapes, pp. 2-3, Addison-Wesley

MATHEX: Geometry No. 5, pp. 9–10; Measurement and Estimation No. 5, pp. 7–14, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: SHAPE AND SIZE 2, pp. 16–25; COMPUTATION AND STRUCTURE 2, pp. 33–41, Wiley

Two-digit Addition and Subtraction— Without Regrouping

Pages f-47 to f-62

General Objectives

To develop an understanding of the addition of two-digit numbers

To develop an understanding of subtraction of two-digit numbers

To maintain mastery of addition and subtraction combinations of 10 or less

Although the basic order and grouping principles are essential in developing the addition and subtraction algorithms for two-digit numbers, the fact that addends may be rearranged in any order is only indirectly stressed in this module. Again the children are given problems that are related to illustrations of concrete objects and they should also be given opportunity to use concrete objects themselves. Since this module treats addition and subtraction of two-digit numbers *without regrouping,* children need only understand the meaning of place value of two-digit numbers and know or be able to find the basic addition facts through sums of nine. After a thorough and separate development of addition, the subtraction module provides some mixed practice using both operations. The module concludes with the usual review and change of pace pages.

Mathematics

In order to complete this unit successfully, the children should know or be able to find sums of nine or less. They should understand facts about place value such as: $16 = 10 + 6$, $25 = 20 + 5$, $83 = 80 + 3$.

For a sum such as $23 + 5$, the children need to know the grouping principle of addition and the combination of $3 + 5$. In detail, this sum is found as follows:

$$\begin{aligned} 23 + 5 &= (20 + 3) + 5 \\ &= 20 + (3 + 5) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

Sums of multiples of 10 are found by referring to ideas about sets. Abstractly, however, finding such a sum involves the distributive principle. For example:

$$\begin{aligned} 20 + 30 &= (10 \times 2) + (10 \times 3) \\ &= 10 \times (2 + 3) \\ &= 10 \times 5 \\ &= 50 \end{aligned}$$

In developing addition of two two-digit numbers, the rearranging of the addends is most important. If all order and grouping ideas used in such a problem were shown, the number would be impractical. However, by utilizing the idea of rearranging addends, the number of steps can be reduced to only a few. Of course, rearranging depends on the order and grouping principles. Below are the shortened steps for two-digit addition.

$$\begin{aligned} 23 + 64 &= (20 + 3) + (60 + 4) \\ &= (20 + 60) + (3 + 4) \\ &= 80 + 7 \\ &= 87 \end{aligned}$$

Because of the relation between addition and subtraction, we assume that the subtraction algorithm can be performed in columns just as addition. However, there are different mathematical ideas involved which we will not explore here.

Teaching Dark Green Module, Unit F

Approximate Time: 8 to 12 days

Materials

counters

flannelboard, felt objects to show tens and ones (2-cm square pieces of felt may be used to show ones and strips 2-by-20-cm may be used to show tens)

objects which can be grouped in tens

one set of strips for each child

play-money coins from Unit E (dimes and pennies) for each child

Although a few children will still benefit from the use of individual counters, encourage those children who want to use concrete materials to use more mathematically structured manipulatives, such as their orange and white strips or, if you have them, number blocks such as Dienes base-ten blocks. Objects which can easily be grouped into tens are also helpful; pencils, pipe cleaners, sticks, are some suitable objects. Suggestions are given throughout the teacher comments to aid you in guiding children with the use of the strips. You might suggest to the children that when they use the orange strip to represent a two-digit number they turn the uncolored, marked side up to reinforce the idea of thinking of 10 ones as 1 ten.

Evaluation of Progress

Evaluation of the children's achievement in this unit should be based on their mastery of skills in adding and subtracting two-digit numbers and on their understanding of the concepts involved in finding these sums and differences. Your observation of their method of solving the exercises should help you evaluate their understanding, and you might also ask children individually to think through the solution of a problem aloud for you.

Resources for Active Learning

General Activities:
Basic facts games:

DEVELOPMENTAL MATH CARDS, C-4, Addison-Wesley

MATH ACTIVITIES, Games 3/31–79, pp. 97–119, Allyn and Bacon

MATHEMATICS IN MODULES: WHOLE NUMBERS—WN1 • Tallying and Addition; WN2 • Difference and "Take Away"; WN3 • Number Facts; WN6 • Addition: Tens and Units, Addison-Wesley

MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Publications Ltd.

Manipulative Devices:
Multi-base arithmetic blocks (Educational Teaching Aids; Herder and Herder)

Commercial Games:
For games to develop competence with the basic facts, refer to the Introduction of the blue module for Unit E.

Investigation
Page f-47

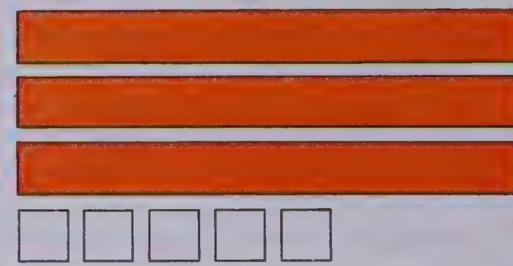
Ask the children to show the strip representation of the number 35 by placing their strips over those illustrated on the page. Then read the directions with them. First help them see that they should make two sets of strips as shown. They should place 1 orange strip and 2 white strips over the illustration on the left and then place the remaining 2 orange and 3 white strips on the space at the right and record these two sets in the spaces provided. Finally, read the main investigation question with them and ask them to find and record other ways to make two sets out of the 3 orange and 5 white strips shown at the top. It would be helpful to write on the chalkboard the phrases they might use to record their other sets such as:

1) ____ tens and ____, ____ tens and ____

2) ____ tens and ____, ____ tens and ____

If you choose, you might have them simply record the two two-digit numbers for each different split they find.

Let's do

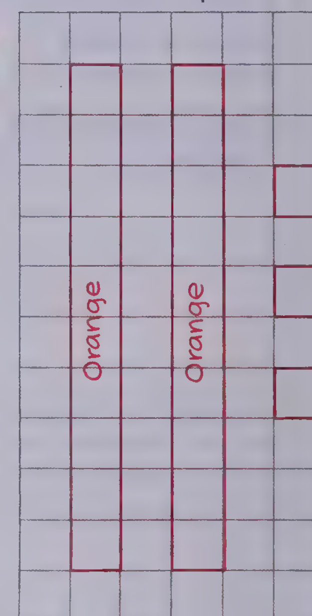


3 tens and 5

35

Put 1 ten and 2 in this set.

Draw a picture of the other strips here.



1 ten and 2

12

2 tens and 3

23

Can you find and record other ways to make two sets out of three orange and five white strips? Answers will vary.

Readiness for two-digit addition without regrouping

Purposes

To provide readiness for two-digit addition and subtraction

To review the meaning of two-digit numerals

Since the orange and white strips are such suitable materials for a study of two-digit numbers, provide many activities such as those suggested in the Investigation and the Follow-Up.

Preparation

Materials

1 set of strips per child

play-money coins from Unit E (at least 4 dimes and 7 pennies for each child)

After a brief free play period with the strips, give the children various two-digit numbers to display with the strips. For example, ask them to show the number 36 with their strips. They should show 3 orange strips and 6 white strips or 1 dark green strip. You might want the children to work in pairs if they do not have enough strips to work by themselves. After working through several examples, direct them to the investigation on page f-47.

Let's talk

You have this much money.

Readiness for two-digit addition without regrouping

Discussion
Page f-48

The use of dimes and pennies is very helpful when studying addition and subtraction with two-digit numbers. First ask the children how much money is pictured at the top of the page. Then explain that the girl pictured wants to purchase some of the items. Discuss items she might buy using just the coins she has without getting any change, such as a mask, a balloon, and a rubber snake. Or the jack o'lantern she might buy with 2 dimes and 1 penny. Talk about how much money she would have left after buying various single items. Then, discuss the price of pairs of items and which pairs could or could not be purchased with her 47¢. For example, ask questions such as: "Would she be able to buy the 12¢ item and the 32¢ item?" "Would she be able to buy both a hat and a jack o'lantern?" "a hat and a face mask?" Many children will benefit most from this discussion if they are able to work with the play-money dimes and pennies used in Unit E.

Follow-up

Extended use of either the strips or coins would be an excellent follow-up for this introductory lesson. For example, suggest to the children other numbers which they might show using their strips as they did in the investigation. You might put the following chart on the chalkboard or on duplicated worksheets to help children with their recording.

In how many ways can you show each number below?

Number	
28	___ tens and ___ + ___ tens and ___ ________
	___ tens and ___ + ___ tens and ___ ________
	___ tens and ___ + ___ tens and ___ ________
42	___ tens and ___ + ___ tens and ___ ________ ________

Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN1 • Tallying and Addition, Addison-Wesley

Teaching
Page f-49

Relate the illustrations in the top frames to the demonstration art and the pre-book activity. Help the children realize that 2 tens and 3 equal 23 and may be written $20 + 3 = 23$. Work through the top four frames displaying actual objects that can be grouped by tens. It would also be helpful to write a few of these sums in vertical notation. For example, write

$$\begin{array}{r} 70 \\ +4 \\ \hline \end{array}$$

and point out that the 4 is written under the zero, that is, in the ones' place. Encourage children to solve the remaining equations independently.

1 TEN AND 5 → 15
10 + 5 = 15

Find the missing numbers. Then solve the equation.

2 tens and 3 → 23

$20 + 3 = \boxed{23}$

3 tens and 1 → 31

$30 + 1 = \boxed{31}$

1 ten and 4 → 14

$10 + 4 = \boxed{14}$

2 tens and 4 → 24

$20 + 4 = \boxed{24}$

Solve the equations.

$30 + 4 = \boxed{34}$ $80 + 2 = \boxed{82}$

$10 + 6 = \boxed{16}$ $20 + 9 = \boxed{29}$

$40 + 2 = \boxed{42}$ $90 + 2 = \boxed{92}$

Place value

Objective

Given an addition equation such as $40 + 50 = \square$ or $40 + 6 = \square$, the child will be able to find the sum.

Pre-Book Activity

Materials

sets of objects to be grouped in tens (pencils, sticks, pipe cleaners)

Provide sets of objects that can be grouped by tens, such as pencils, which can be held together in sets of ten with rubber bands. First, review the idea of grouping the objects in tens. For example, select 4 groups with ten in each group and write the numeral 40 on the chalkboard. Then, review the idea of grouping the objects in tens with some left over. For example, count out 2 groups of ten and 4 left over and write the numeral 24 on the chalkboard. Display a group of tens only, such as 30, and present the example of adding some number of objects less than ten to the 30. Demonstrate the joining of the sets and ask a child to write the total number of objects on the chalkboard. Use statements such as: "Thirty plus six more gives us a total of 36." or "If we have thirty and add six, we have the sum 36." Work through other examples similarly. Also, combine groups of ten. For example, combine 4 tens with 3 tens. Finally, relate these demonstrations to equations such as $30 + 6 = 36$ and $40 + 30 = 70$.

Give the number of tens.
Then solve the equations.

3 tens and 2 tens ⟶ 5 tens in all.

$30 + 20 = \underline{50}$

4 tens and 3 tens ⟶ 7 tens in all.

$40 + 30 = \underline{70}$

4 tens and 2 tens 6 tens

$4 + 2 = \boxed{6} \longrightarrow 40 + 20 = \boxed{60}$

5 tens and 3 tens 8 tens

$5 + 3 = \boxed{8} \longrightarrow 50 + 30 = \boxed{80}$

$20 + 50\ \boxed{70}$ $60 + 20\ \boxed{80}$

$40 + 10\ \boxed{50}$ $70 + 10\ \boxed{80}$

$30 + 30\ \boxed{60}$ $50 + 40\ \boxed{90}$

Sums of multiples of 10

Teaching
Page f-50

You might choose to work through parts of this page before assigning the equations on page f-49. Again, relate the illustrated examples to demonstration objects. The examples on this page are intended to help the child relate the phrase "3 tens and 2 tens → 5 tens" to the expression $30 + 20 = 50$. This utilizes the idea that to add 30 and 20 we simply think $2 + 3$. Discuss the equations in the centre of the page which develop this concept. Assign the remaining equations for independent work but, if children want to use concrete objects as aids, allow them to do so.

Follow-up

To extend the ideas of this lesson to include adding a one-digit number to a two-digit number, devise a "basketball game." Draw a "basket" on tagboard or on the chalkboard. Print a rule such as "+4" on the backboard. On 10-cm circles of tagboard, representing basketballs, print two-digit numbers, such as 33, 62, 70, 24, and so on. Make one "basketball" for each child. For the present, be sure that the number in the rule is small so that the children will not have to regroup to find the sums. Each child chooses a "basketball" and "makes a basket" by naming the correct sum for the number on his basketball and the number on the basket. To continue, change the rule on the backboard and give each child a turn.

For written practice in adding a two-digit number and a one-digit number (without regrouping), duplicate a worksheet similar to the following:

Find the sums.

5 +53	22 + 7	13 + 6	4 +15	41 + 7	13 + 6
35 + 2	53 + 3	3 +53	1 +48	48 + 1	65 + 2

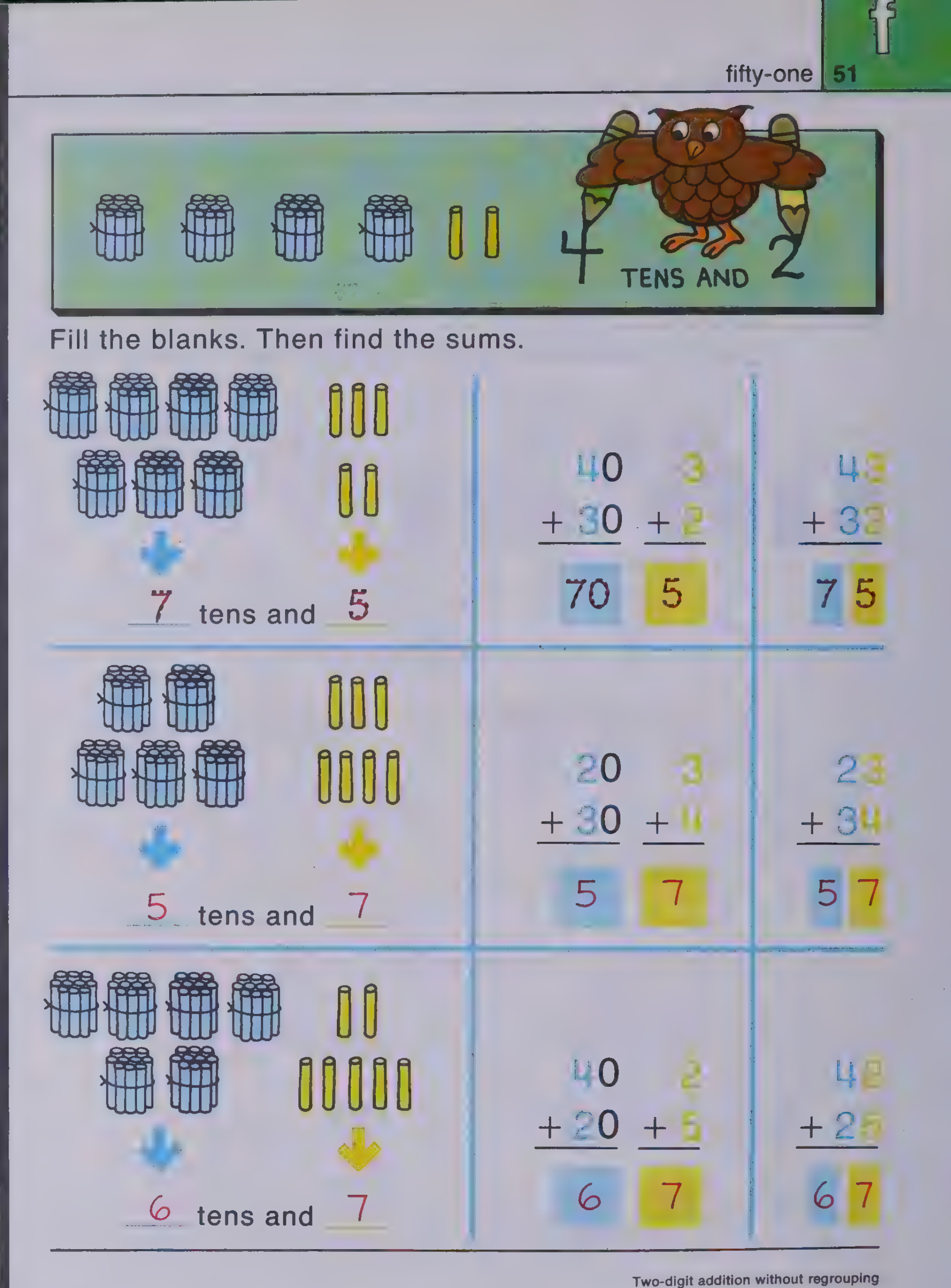
Teaching
Page f-51

Ask the children to read the directions. Display 43 + 32 in vertical notation on the flannelboard and say: "Suppose you want to find the sum of these two numbers." Call attention to the sets pictured, and ask someone to identify them as representing 43 and 32. Then ask, "How many tens all together?" When a child answers "seven," ask how many single sticks there are. Then, direct the children to trace the numerals 7 and 5.

Next, point out that instead of showing the bundles, they could write 4 tens as 40, and 3 tens as 30, and that they could add 40 and 30, then 3 and 2, to get 70 + 5 or 75.

Go on to the last frame in the right and comment that the shortest way to solve the problem is to remember that they are adding tens and ones.

Work through the next two exercises with the children, asking for their explanations and answers as much as possible.



4 TENS AND 2

Fill the blanks. Then find the sums.

7 tens and 5

$$\begin{array}{r}40\\+30\\\hline 70\end{array}\quad\begin{array}{r}3\\+2\\\hline 5\end{array}\quad\begin{array}{r}43\\+32\\\hline 75\end{array}$$

5 tens and 7

$$\begin{array}{r}20\\+30\\\hline 5\end{array}\quad\begin{array}{r}3\\+4\\\hline 7\end{array}\quad\begin{array}{r}23\\+34\\\hline 57\end{array}$$

6 tens and 7

$$\begin{array}{r}40\\+20\\\hline 6\end{array}\quad\begin{array}{r}2\\+5\\\hline 7\end{array}\quad\begin{array}{r}42\\+25\\\hline 67\end{array}$$

Two-digit addition without regrouping

Objective

Given two two-digit numbers with the number of tens and the number of ones shown separately, the child will be able to find the sum.

Pre-Book Activity

Materials

1 set of strips for each child

Have the children work in groups of two so that they will have enough strips between them to form any two-digit number. First, ask them to show a two-digit number such as 35 with their strips; then, in a different section of their work area, to show the number 22. Explain that you want them to use the strips to solve the equation 35 + 22 = □. Ask them to explain which kinds of strips they combined or counted together, and to give the numeral for the sum. Then write:

$$\begin{array}{r}30\\+20\\\hline\end{array}\qquad\begin{array}{r}5\\+2\\\hline\end{array}$$

and ask children to explain these expressions in terms of their strips, that is, that they grouped the white or one strips with each other, and then the orange or ten strips with each other. Show how this may be written in the single expression:

$$\begin{array}{r}35\\+22\\\hline\end{array}$$

Guide the children in working through several examples of this kind with their strips.

f 52 fifty-two

Solve.

$\begin{array}{r} 30 \\ +20 \\ \hline 50 \end{array}$ $\begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$ → $\begin{array}{r} 34 \\ +23 \\ \hline 57 \end{array}$	$\begin{array}{r} 50 \\ +10 \\ \hline 60 \end{array}$ $\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$ → $\begin{array}{r} 53 \\ +12 \\ \hline 65 \end{array}$
$\begin{array}{r} 60 \\ +20 \\ \hline 80 \end{array}$ $\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$ → $\begin{array}{r} 61 \\ +24 \\ \hline 85 \end{array}$	$\begin{array}{r} 30 \\ +10 \\ \hline 40 \end{array}$ $\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$ → $\begin{array}{r} 36 \\ +13 \\ \hline 49 \end{array}$
$\begin{array}{r} 20 \\ +30 \\ \hline 50 \end{array}$ $\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$ → $\begin{array}{r} 24 \\ +32 \\ \hline 56 \end{array}$	$\begin{array}{r} 40 \\ +20 \\ \hline 60 \end{array}$ $\begin{array}{r} 1 \\ +6 \\ \hline 7 \end{array}$ → $\begin{array}{r} 41 \\ +26 \\ \hline 67 \end{array}$
$\begin{array}{r} 50 \\ +10 \\ \hline 60 \end{array}$ $\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$ → $\begin{array}{r} 54 \\ +14 \\ \hline 68 \end{array}$	$\begin{array}{r} 60 \\ +30 \\ \hline 90 \end{array}$ $\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$ → $\begin{array}{r} 65 \\ +32 \\ \hline 97 \end{array}$
$\begin{array}{r} 10 \\ +30 \\ \hline 40 \end{array}$ $\begin{array}{r} 7 \\ +1 \\ \hline 8 \end{array}$ → $\begin{array}{r} 17 \\ +31 \\ \hline 48 \end{array}$	$\begin{array}{r} 50 \\ +30 \\ \hline 80 \end{array}$ $\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$ → $\begin{array}{r} 54 \\ +34 \\ \hline 88 \end{array}$

Two-digit addition without regrouping

Teaching
Page f-52

Explain to the children that on this page they are to find the sums. Point out that the first expressions in each exercise show each numeral written as tens and ones to help them add two-digit numbers. Tell the children that if they think of 34 as 30 + 4 and of 23 as 20 + 3, they can add the ones and the tens separately. Call on a child to do this. Then, direct all the children to trace the dashed numerals for both types of notation.

Next, give all the children an opportunity to complete the second exercise. When they have finished, show the answers and ask each child to check his paper. Have the children complete the page by themselves.

Follow-Up

Many children would benefit from a worksheet such as the following which emphasizes patterns.

10 + 6 = □	13 + 4 = □
20 + 6 = □	23 + 4 = □
30 + 6 = □	33 + 4 = □
40 + 6 = □	43 + 4 = □
50 + 6 = □	53 + 4 = □
60 + 6 = □	53 + 4 = □
70 + 6 = □	73 + 4 = □
80 + 6 = □	83 + 4 = □
90 + 6 = □	93 + 4 = □

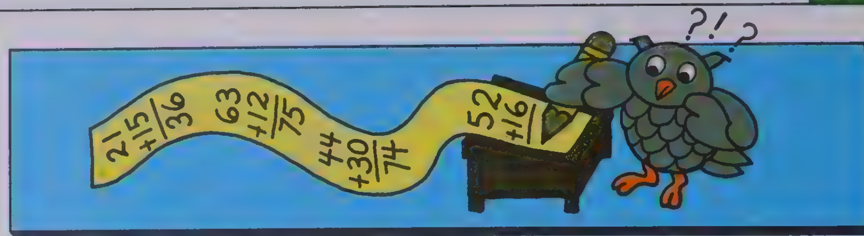
As an enrichment page for more capable children you might develop a worksheet of reconstruction problems similar to the following ones. Note that none of them require regrouping.

Find the missing digits.

$\begin{array}{r} 7\square \\ 24 \\ \hline \square 9 \end{array}$	$\begin{array}{r} 36 \\ \square\square \\ \hline 78 \end{array}$	$\begin{array}{r} \square 4 \\ 6\square \\ \hline 88 \end{array}$	$\begin{array}{r} 55 \\ 3\square \\ \hline \square 7 \end{array}$	$\begin{array}{r} \square 1 \\ 5\square \\ \hline 95 \end{array}$	$\begin{array}{r} \square 7 \\ 8\square \\ \hline 99 \end{array}$
$\begin{array}{r} 3\square \\ \square 2 \\ \hline 58 \end{array}$	$\begin{array}{r} 27 \\ 4\square \\ \hline \square 9 \end{array}$	$\begin{array}{r} 8\square \\ 15 \\ \hline \square 7 \end{array}$	$\begin{array}{r} 43 \\ \square 6 \\ \hline 7\square \end{array}$	$\begin{array}{r} 19 \\ 5\square \\ \hline \square 9 \end{array}$	$\begin{array}{r} 11 \\ 6\square \\ \hline \square 5 \end{array}$

Teaching
Page f-53

Explain to the children that for each of these exercises they should try to find the sums. Suggest that they might use their strips or objects which can be bundled into tens, although most children will be able to work without these aids. Encourage the children to work this page independently. Note that each section contains some particular kinds of problems: the problems in set A are relatively easy; those in set B contain larger numbers; set C contains some special cases of the type 40 + 23 wherein zero is in the ones' place; and set D contains some special cases of the type 43 + 4.



Find the sums.

A

23 + 12 = 35	41 + 13 = 54	22 + 14 = 36
32 + 11 = 43	12 + 13 = 25	24 + 31 = 55

B

56 + 22 = 78	13 + 74 = 87	55 + 32 = 87
76 + 21 = 97	64 + 33 = 97	17 + 81 = 98

C

46 + 32 = 78	37 + 20 = 57	40 + 23 = 63
43 + 25 = 68	56 + 10 = 66	34 + 52 = 86
50 + 29 = 79	65 + 30 = 95	34 + 43 = 77

D

62 + 17 = 79	43 + 4 = 47	56 + 12 = 68
24 + 5 = 29	7 + 61 = 68	3 + 44 = 47
62 + 15 = 77	74 + 5 = 79	14 + 81 = 95

Two-digit addition without regrouping

Objective

Given an addition problem with two two-digit numbers written in vertical notation, the child will be able to find the sum.

Pre-Book Activity

Make up several story problems which require two-digit addition without regrouping. For example:

1) The photographer came to school. He took individual pictures of 13 boys and 14 girls. How many pictures did he take in that class?
2) The boys and girls in Room 1 each got a comb as they lined up for the photographer. If 18 boys and 20 girls got combs, how many combs were given out?
3) In the three first-grade rooms 42 boys and 46 girls had their picture taken. How many first-graders had their picture taken?
4) Kenny had 11 cars. Kirk had 28. How many cars altogether?
5) There were 12 girls and 15 boys who had bikes. How many bikes?
6) Jan had 31 jacks, and Kathy had 17 jacks. How many jacks did the girls have in all?

Have some children write the problems in vertical notation on the chalkboard and have others solve them. As a short practice session on two-digit addition, write five or six exercises on the chalkboard and have one child from each row or table write a sum. Then, have the rest of the class judge their correctness.

Short Stories

1 Marty had 30 cents. She earned 15 cents. How much does she have now? 45¢

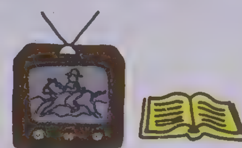
2 Willy had 75 baseball cards. He bought 12 more. How many does he have now? 87

3 Football team: 11 players. Baseball team: 9 players. How many more on a football team? 2

4 Model cars cost 75 cents. The tax is 4 cents. How much in all? 79¢

5 The Tigers scored 6 runs. The Twins scored only 5. How many runs were scored? 11

6 Ann has 50 cents. Mary has 25 cents. How much in all? 75¢



7 Frank watched TV for 30 minutes. Then he read for 45. How long for both? 75 minutes

8 15 children came to Sue's party. 8 were girls. How many were boys? 7

9 The Bucks scored 84 points. The Lakers scored 15 more than the Bucks. How many did the Lakers score? 99

10 Pam is 13 years old. Her sister is 8. How much older is Pam? 5 years

Teaching
Page f-54

You will want to read through these problems with most children. Ask them what method should be used to solve each problem. Problems 3, 8, and 10 are not addition problems and warrant extra discussion. If your class is at a mature reading level, an alternative manner of treating this page would be to divide children into groups and have them discuss the solutions to each problem. Then, have each group explain one or two of the problems they solved. In any event, be sure you check the problems with the children and give them ample opportunity to discuss those which caused difficulty.

Follow-up

Activities using dimes and pennies may help some children make the transition from sets of objects to sets of tens and ones, and ultimately, to more abstract symbolic representation such as

$$\begin{array}{r} 42 \\ +23 \\ \hline 65 \end{array}$$

Give each child an envelope containing 9 dimes and 9 pennies. Allow the children to handle the coins and make up their own problems.

You might also prepare a set of cards, showing two-digit addition problems without regrouping so that there is one for each child in your class. Have five or six children each choose a card from the pack and copy the problem on the chalkboard. Each child should then choose a classmate to solve the problem. If this child solves the problem correctly, he may choose a card, copy the problem on the chalkboard, and then pick someone else to solve it. If the children are quick, make some of the problems reconstruction problems with missing addends.

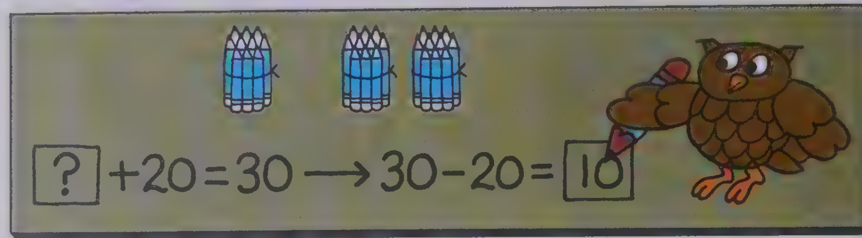
Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN3 • Number Facts; WN6 • Addition: Tens and Units, Addison-Wesley

Teaching
Page f-55

Call attention to the sets in the demonstration art. Ask the children to identify the number of each set, and the total number of pencils. Ask the children what number should complete the equation $\square + 20 = 30$. Discuss the relationship of the equations, then work through the first frame with the children. Ask them to fill in the numeral that is missing in the first equation. Discuss that 30 solves both the addition and the subtraction equation. To simplify, they can think of the pencils in sets of ten: $3 + 2 = 5 \rightarrow 5 - 2 = 3$.

Remind the children that they may find a difference by thinking about a missing addend. Stress again that they need only think of the basic facts regarding sums and differences of ten or less in order to subtract multiples of ten. That is, to solve $50 - 20$, they need only think $5 - 2 = \square$, or $\square + 2 = 5$. Relationships such as these are shown in the remaining frames. You might provide similar exercises on the chalkboard.



Solve the equations.

$\boxed{30} + 20 = 50 \longrightarrow 50 - 20 = \boxed{30}$

$\boxed{10} + 30 = 40 \longrightarrow 40 - 30 = \boxed{10}$

$\boxed{3} + 4 = 7 \longrightarrow 7 - 4 = \boxed{3}$

$\boxed{30} + 40 = 70 \longrightarrow 70 - 40 = \boxed{30}$

$\boxed{3} + 6 = 9 \longrightarrow 9 - 6 = \boxed{3}$

$\boxed{30} + 60 = 90 \longrightarrow 90 - 60 = \boxed{30}$

Readiness for two-digit subtraction without regrouping

Objective

Given a subtraction equation such as $60 - 20 = \square$, the child will be able to find the difference.

Pre-Book Activity

Materials

objects which can be grouped in tens

Use a brief oral warm-up to review sums and differences of 10 or less. For example, you might say: "I'm thinking of a number which when added to 4 gives me 9." "What's my number?" Examples which review missing addends would be most appropriate. Gradually introduce examples of the same form which use multiples of ten. With these you might also use demonstration objects bundled in tens. For example, put some tens in a bag but do not let the children see how many bundles you use. Then put in some bundles which you count with the children. Finally, ask a child to count the total number of bundles in the bag and challenge the children to figure out how many were in the bag to begin with. Work through several examples of this kind. Stress that knowing the basic facts regarding sums and differences of 10 or less enables them to work with these multiples of ten without having to learn new facts.

Find the differences.

7 − 2 = 5	70 − 20 = 50	6 − 4 = 2	60 − 40 = 20	9 − 2 = 7	90 − 20 = 70
8 − 5 = 3	80 − 50 = 30	5 − 2 = 3	50 − 20 = 30	7 − 5 = 2	70 − 50 = 20
40 − 10 = 30	60 − 20 = 40	70 − 30 = 40	50 − 30 = 20	90 − 30 = 60	80 − 60 = 20

Complete the matching.

40 − 30	30
80 − 60	20
60 − 30	10
50 − 10	50
70 − 20	40

Subtraction – multiples of 10

Teaching
Page f-56

Call the children's attention to the first frame. Observe the pattern in the problems and ask for volunteers to complete the two exercises in that frame.

Point out that at the bottom of the page, they are to match differences. Ask a child to give the difference 40 − 30, then direct all the children to trace the matching line. Ask the children to try to finish the exercises independently.

Follow-up

An exercise using a patterned approach to subtracting multiples of ten may prove both helpful and necessary for many children. Duplicate problems such as the following:

Work across. Find the pattern and then the differences.

6 − 5 = [1]	60 − 50 = [10]
9 − 3 = [ ]	90 − 30 = [ ]
8 − 6 = [ ]	80 − 60 = [ ]
7 − 2 = [ ]	70 − 20 = [ ]
5 − 4 = [ ]	50 − 40 = [ ]
8 − 5 = [ ]	80 − 50 = [ ]
4 − 4 = [ ]	40 − 40 = [ ]

You might also play "What's My Rule." For example, ask the children to give you two multiples of ten that are less than 100. Suppose they respond 60 and 20. You then answer 40. Remind the children to fold their arms over their chests when they know your rule. Give several examples, and then call on a child with his arms folded. He should then ask you to give him two multiples of ten less than 100. If he gives the correct response, that is, the difference between these multiples, tell the children that he knows the rule, and should keep it a secret. Continue the game until everyone knows your rule.

Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN2 • Difference and "Take Away", Addison-Wesley

Teaching
Page f-57

Help the children identify the number of the set in the first frame of the page as 48. Tell the class that the x marks show that a set of 2 tens and a set of 3 are to be removed. Ask the children to tell the total number of objects being removed and, then, to tell the number of objects that will be left. Point out that the problem in the first frame below the set picture shows this subtraction problem, 48 − 23 = □.

Observe that the middle frame in each row shows the multiples of ten and one being subtracted separately to help the children understand that they are not subtracting 2 from 4, but 2 tens from 4 tens. You might want to have the children show 48 with their strips and then remove 2 ten strips and 3 units. Ask the children to check the examples carefully and to trace over the dashed numerals. Then instruct the children to complete the page by finding the differences. Read the correct answer when they complete the exercises.

$$\begin{array}{r} 56 \\ -14 \\ \hline \end{array} \quad \Bigg| \quad \begin{array}{r} 50 \\ -10 \\ \hline 40 \end{array} \; \begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array} \quad \Bigg| \quad \begin{array}{r} 56 \\ -14 \\ \hline 42 \end{array}$$

Find the differences.

$\begin{array}{r} 48 \\ -23 \\ \hline \end{array}$	$\begin{array}{r} 40 \\ -20 \\ \hline 20 \end{array} \; \begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array}$	$\begin{array}{r} 48 \\ -23 \\ \hline 25 \end{array}$
$\begin{array}{r} 65 \\ -21 \\ \hline \end{array}$	$\begin{array}{r} 60 \\ -20 \\ \hline 40 \end{array} \; \begin{array}{r} 5 \\ -1 \\ \hline 4 \end{array}$	$\begin{array}{r} 65 \\ -21 \\ \hline 44 \end{array}$
$\begin{array}{r} 68 \\ -2 \\ \hline \end{array}$	$\begin{array}{r} 60 \\ -0 \\ \hline 60 \end{array} \; \begin{array}{r} 8 \\ -2 \\ \hline 6 \end{array}$	$\begin{array}{r} 68 \\ -2 \\ \hline 66 \end{array}$
$\begin{array}{r} 85 \\ -34 \\ \hline \end{array}$	$\begin{array}{r} 80 \\ -30 \\ \hline 50 \end{array} \; \begin{array}{r} 5 \\ -4 \\ \hline 1 \end{array}$	$\begin{array}{r} 85 \\ -34 \\ \hline 51 \end{array}$

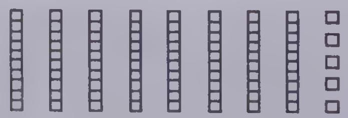
Two-digit subtraction without regrouping

Objective

Given two-digit subtraction problems that do not require regrouping, the child will be able to find the differences.

Pre-Book Activity

Many children would benefit from individual or small group practice with the strips or with number blocks. For example, give them a number to build such as 85.



Ask them to record this number. Then, ask them to remove 3 units, that is, 3 one-strips. Again, they should record their action. Then, ask them to remove 4 ten-strips or rods and record. Finally, ask them to record both of their take away actions together:

$$\begin{array}{r} 85 \\ -43 \\ \hline \end{array} \qquad \begin{array}{r} 80 \\ -40 \\ \hline 40 \end{array} \qquad \begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \qquad \begin{array}{r} 85 \\ -43 \\ \hline 42 \end{array}$$

Encourage them to work through several examples. Familiarity with using the ten strips and one strips in this way will help them when they study subtraction with regrouping.

Find the differences.

60 − 20 40	5 − 3 2	65 − 23 42	70 − 20 50	8 − 6 2	78 − 26 52
50 − 10 40	6 − 3 3	56 − 13 43	40 − 10 30	9 − 7 2	49 − 17 32
90 − 60 30	9 − 4 5	99 − 64 35	70 − 20 50	6 − 5 1	76 − 25 51
87 − 34 53	36 − 15 21	52 − 30 22	75 − 32 43	68 − 51 17	47 − 25 22
29 − 5 24	34 − 10 24	77 − 33 44	96 − 80 16	84 − 3 81	67 − 23 44

Two-digit subtraction without regrouping

Teaching
Page f-58

Explain to the children that in order to find the difference for each exercise in the shaded box, they should first work the examples for the multiples of ten and for the ones. Suggest that they build the top number in the shaded box with their strips. (You might have them work in pairs in order to provide enough strips.) Then, they can remove the tens, record their answer, and repeat the same steps for the ones. Finally, they should record their answer in the shaded box. Have them complete the examples independently or with a partner using their materials as they choose. When they have finished, help the children check their answers and review the terminology by stressing that they have found the difference in each exercise. Note: Children should discontinue using the strips as aids whenever they are ready. An indication of this readiness is when the use of the materials becomes tedious or bothersome.

Follow-up

Worksheets of various kinds will provide the children with important reinforcement of skills. Here are some sample worksheets.

Find the sums and differences.

52 +32	84 −32	35 +64	99 −64	11 +36	47 −36
10 +14	24 −14	22 +36	58 −36	13 +51	64 −51

Find the differences.

−20		−50		−30		−40	
47		92		84		59	
62		57		91		73	
95		76		75		86	
38		64		36		67	
56		81		62		95	

Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "Take-away," $D^1$12, Addison-Wesley

ENRICHMENT OF ARITHMETIC, (subtracting on a chunk number line), pp. 2/6–7, Webster, McGraw-Hill

Teaching
Page f-59

The exercises on this page provide the children with both addition and subtraction problems. Explain to them that frame A contains addition problems, and frame B subtraction problems. Since frames C and D contain both addition and subtraction problems, warn the children to be particularly careful in reading the addition and subtraction signs. Notice that the exercises in frame C contain smaller numbers compared with those used in frame D. Be sure to help the children check their work when they have completed the page.

99	88	77
−63	−34	+11

Solve.

A

56	23	17
+ 13	+ 42	+ 42
69	65	59
54	63	75
+ 20	+ 14	+ 3
74	77	78

B

68	73	49
− 25	− 50	− 36
43	23	13
76	95	87
− 3	− 52	− 34
73	43	53

C

24	44	36
+ 11	− 21	+ 21
35	23	57
56	17	32
− 20	− 5	+ 24
36	12	56
12	43	51
+ 12	− 12	+ 13
24	31	64

D

62	67	85
+ 27	− 35	− 24
89	32	61
54	36	94
+ 33	+ 53	− 72
87	89	22
76	43	59
− 43	+ 45	− 18
33	88	41

Two-digit addition and subtraction without regrouping

Objective

Given exercises or problems involving addition or subtraction of two-digit numbers that do not require regrouping, the child will be able to find the sums or differences.

Adapt the oral activities suggested in this lesson to your classroom situation. Keep in mind that development of mental facility with those problems is an ongoing objective.

Pre-Book Activity

Introduce word problems for the children to solve orally. For example:

1) Jane collects bubble-gum wrappers. She needs 55 wrappers for a string of beads. She has 30 wrappers. How many more does she need?
2) Mary Jane had 63 stones in a display case. Jay had 78 stones in his collection. Who had more stones? How many more?
3) Tom had 24 unpainted model cars. He painted 3. How many still unpainted?
4) Cynthia brought 36 candy bars to share. Counting Cynthia, there were 32 children. How many candy bars would be left over?
5) There were 18 girls who each made a paper-maché doll, and 5 girls who each made a clothespin doll. How many dolls did the girls make?

Short Stories

1 Jan read 23 pages. Then she read 15. How many pages? 38

2 Doug lives 12 blocks from school. How far does he walk going both ways? 24

3 Norma had 50 cents. She spent 30 cents. How much left? 20¢

4 Laura has a butterfly collection. She has 15 butterflies in one case and 12 in another. How many? 27

5 The Jets scored 38 points. The Bears scored 24. How many more did the Jets score? 14

6 There are 28 children in Marsha's class. 14 are girls. How many boys? 14

7 Kathy's father drove 52 miles the first hour. He drove only 46 the second. How far in all? 98

8 Joe had 78 cents. He spent a quarter. How much left? 53¢

9 Sam's team scored 9 runs. The other team scored 7. How many runs in all? 16

10 Helen scored 98 in spelling. Tom scored 93. How much higher is Helen's score? 5

Story problems

Teaching
Page f-60

Although many children will be able to read these problems independently you might want them to read the problems together. Encourage them to discuss what they are to find and what they have been given. For example, read the first problem with the children. Say something such as: "What information are we given about the pages Jan has read?" (How many has she read at the two different times.) "What are we asked to find out about these pages?" (The total she read.) After you have worked through the first two or three problems, encourage the children to solve the rest of the problems with a partner or in a small group. Remind them that the problem may be either addition or subtraction. When the children are finished, discuss the solution for each problem so the children can check their own work.

Follow-up

Problems using dimes and pennies may help the children develop problem-solving skills as well as reinforce their skill with using two-digit numbers. Have each child write on an index card one addition or subtraction problem involving money (less than one dollar). Then, children can exchange cards to see if they can work their classmate's problem. Encourage the children to use the play-money coins as aids. If you prefer, you might prepare several task cards of this type yourself. For example:

1) The following five items were shown in store windows. Which ones can you buy if you have 82¢?

tablet – 52¢	pencil case – 30¢
binder – 70¢	pencil sharpener – 20¢
2 erasers – 10¢	

2) Tom earned 25¢ on Monday, 30¢ on Tuesday and 42¢ on Wednesday. On Thursday he bought a toy car for 45¢. How much money did he have left?

Teaching
Page f-61

Explain to the children that they should try to work these problems by themselves, but that they might use the strips if they wish, perhaps to check their answers. Warn them to be careful in reading the addition and subtraction signs. Help any child who needs assistance in reading the word problems, but encourage all children to do their own thinking to find the answers.

Show you know

Solve.

34 + 21 55	56 + 32 88	15 + 23 38	56 − 36 20	78 − 3 75	95 − 52 43
38 + 20 58	56 + 3 59	17 + 32 49	62 − 40 22	59 − 16 43	84 − 32 52
24 + 24 48	67 − 32 35	33 + 46 79	56 − 40 16	43 + 35 78	37 − 7 30
48 − 16 32	12 + 75 87	65 − 12 53	38 + 50 88	99 − 88 11	75 + 4 79

Fred had 45 cents.
He earned 20 cents.
How much does he have now? 65¢

The record costs 78 cents.
Linda has only 65 cents.
How much more does she need? 13¢

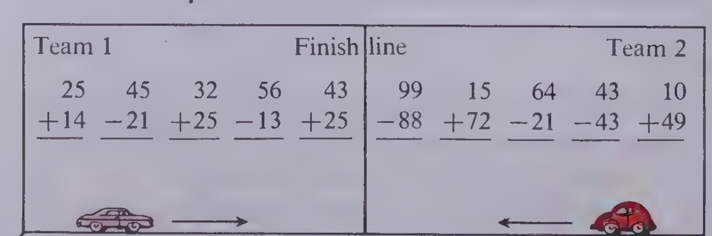
Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

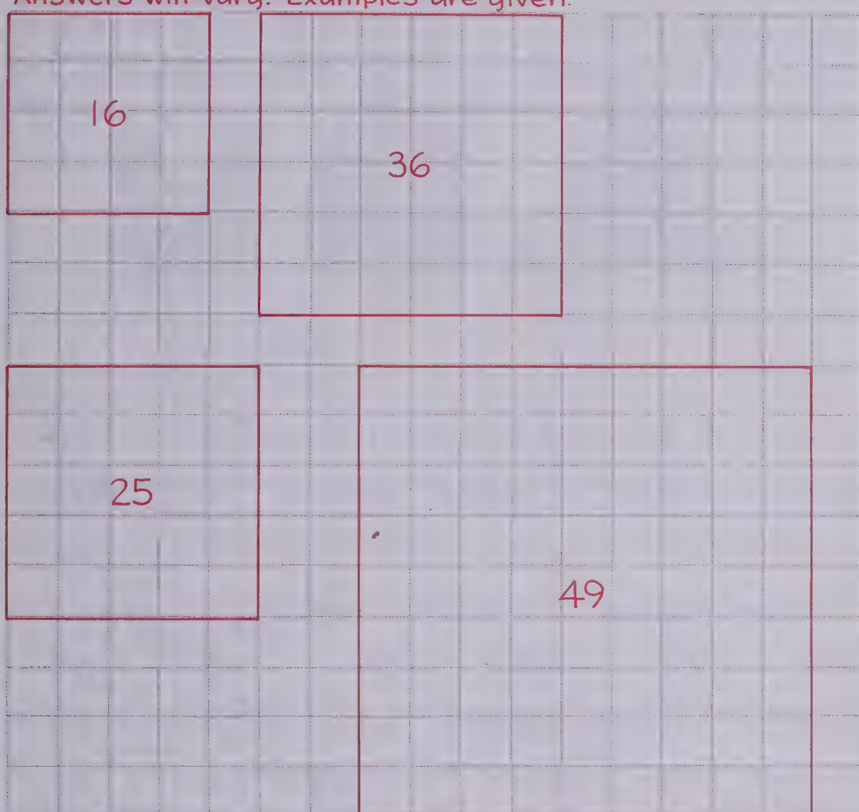
To review addition and subtraction of two-digit numbers, play a game such as the following: Divide the children into two groups for a chalkboard auto race. Give each team a car to place on opposite ends of the chalkboard. Write addition and subtraction examples on the chalkboard, if possible, one for each child on the team. You might have as many as six teams if you have three chalkboards. Explain to the children that when you give the starting signal the first member of each team comes forward and works the problem; if it is correct he moves the car forward and the second member may do the next problem, and so on until all the problems have been correctly worked. If a child makes a mistake, the next child must work that same problem over and try to correct the mistake. It is helpful to have the problems and answers written on a card so that you may check the answers immediately. The team which gets to the finish line first wins.



Let's have fun

The numbers 1, 4, and 9 are square numbers.

Can you show some other square numbers below?
Answers will vary. Examples are given.



Square numbers

Teaching
Page f-62

Children will benefit most from this change of pace page if it is accompanied by a short review of the term *square*. Show examples of square shapes and explain that the three illustrations at the top of the page are squares. Also, help the children realize that the numbers 1, 4, and 9, come from counting the number of small squares or square units in each square. It would also be helpful to explain why 3, for example, is *not* a square number: there is no way in which you can arrange 3 square units to form a square. Encourage the children to try to find some other square numbers. Many children will need several sheets of graph paper to find other square numbers. Others, would work better with several individual small squares such as the white strips which they can try to arrange into the shape of a square.

Follow-up

Children still having difficulty with two-digit numbers would benefit from extended use of the ones and tens strips.

> Show 45 with your strips. Remove 2 tens and 3 ones. Record your work:
>
> $$\begin{array}{r} 45 \\ -23 \\ \hline \end{array}$$
>
> Show 87 with your strips. Remove 4 tens. Record your work.
>
> $$\begin{array}{r} 87 \\ -40 \\ \hline \end{array}$$

Resources for Active Learning

Number patterns-square numbers:

DEVELOPMENTAL MATH CARDS, D²5, Addison-Wesley

FREEDOM TO LEARN, pp. 129–130, Addison-Wesley

MATH WORKSHOP: Games and Enrichment Activities, pp. 22–23, Encyclopaedia Britannica Educational Corp.

Teaching
Page f-63

Read the directions with the children. Explain that the top two frames review the fact that numbers may be ordered and grouped differently without affecting the sum. Be sure they realize that for all equations except those in the bottom frame they should write the sum in the space provided. The last section contains subtraction equations. Also point out the number line which is shown so that it may be used as an aid by those who choose to use it. Strips should also be available for those who prefer these as an aid.

Looking back

Solve the equations.

$4 + 5 = \boxed{9}$

$5 + 4 = \boxed{9}$

$8 + 2 = \boxed{10}$

$2 + 8 = \boxed{10}$

$(2 + 4) + 3 = \boxed{9}$

$2 + (4 + 3) = \boxed{9}$

$(6 + 3) + 1 = \boxed{10}$

$6 + (3 + 1) = \boxed{10}$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$6 + 5 = \boxed{11}$ $\quad 7 + 6 = \boxed{13}$

$8 + 8 = \boxed{16}$ $\quad 9 + 5 = \boxed{14}$

$8 + 6 = \boxed{14}$ $\quad 9 + 8 = \boxed{17}$

$9 + 6 = \boxed{15}$ $\quad 4 + 7 = \boxed{11}$

$12 - 8 = \boxed{4}$ $\quad 17 - 8 = \boxed{9}$

$13 - 6 = \boxed{7}$ $\quad 14 - 5 = \boxed{9}$

$15 - 9 = \boxed{6}$ $\quad 11 - 6 = \boxed{5}$

$18 - 9 = \boxed{9}$ $\quad 10 - 7 = \boxed{3}$

Cumulative review

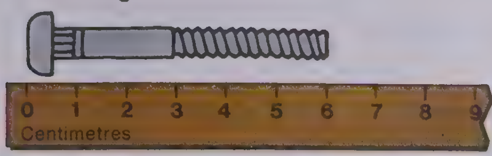
Objective

The child will demonstrate his ability to work with the concepts presented throughout Unit F.

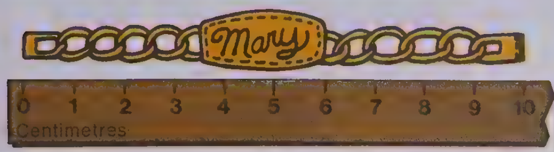
Pre-Book Activity

A review of basic facts and principles would be a suitable introduction for this lesson. Remind children also of how the number line might be used as an aid in solving both addition and subtraction equations. Use of the demonstration number line to solve equations similar to those on page f-63 would be appropriate. You might also want to review the concepts of measurement with centimetres, and the concept of area as preparation for page f-64.

How long?

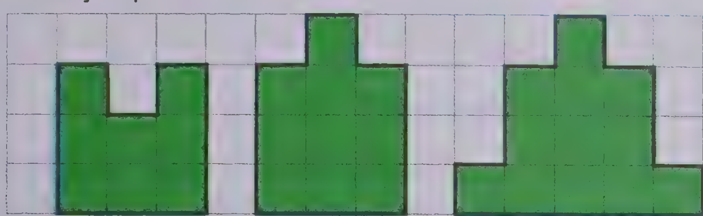


6 Centimetres



10 Centimetres

How many square centimetres?



8 10 12

Find the sums.

51	37	46	23	12	56
+ 26	+ 12	+ 52	+ 23	+ 84	+ 32
77	49	98	46	96	88

Find the differences.

76	87	68	74	59	44
− 34	− 52	− 17	− 50	− 39	− 13
42	35	51	24	20	31

Cumulative review

Teaching
Page f-64

You might treat this page as a review page which you work with the children, or you may prefer that children demonstrate their ability to work on it independently. In either case, provide them with a brief review of measurement of length and area.

Also provide sets of strips for children to use with the problems in the bottom sections. Many children, however, will successfully find the sums and differences without any manipulative aid.

Follow-up

The children will need large sheets of paper and crayons. Instruct them to fold their paper into four sections. Then tell them to copy one of these sets of numbers at the top of each section:

1) 4,6,7; 2) 5,8,3;
2) 9,2,8; 4) 7,3,6.

Next, ask them to find the sum of each set of numbers by grouping the numbers any way they choose. After they have found the sums, tell them to write down as many ways as they can find to regroup, or reorder, the three numbers to get the same sum.

Three-digit Numbers

Pages g-1 to g-14

General Objectives

To introduce the names for the numbers from 100 to 999

To extend the concept of place value to include hundreds

To use the concept of order to develop a greater understanding of place value

Although children will have been exposed to the numeral for one hundred, the first lessons of the module carefully treat its development. After a lesson on ordering numbers less than one hundred, the place value meaning of three-digit numerals is stressed. Activities with strips and objects grouped in tens are suggested. Finally the topics of order and inequality are extended to the three-digit numerals. After a module review, the change of pace lesson provides further experiences with three-digit numerals by utilizing them in dollar and cents notation for amounts of money.

Mathematics

This unit stresses place value for hundreds and inequalities involving such numbers. The idea of one number being greater than another is intuitively obvious for the set of whole numbers. For a precise mathematical definition, you may wish to review the Mathematics of the orange module of Unit E.

Set demonstrations are not practical when comparing numbers in the hundreds or thousands. For example, suppose we wanted to compare 4764 with 4564. Because in 4764 the 4 stands for 4000 (four groups of 100 tens), 7 stands for 700 (seven groups of 10 tens), 6 stands for 60 (six groups of 10), and 4 stands for four, the grouping procedure for this number alone would be tedious. However, when comparing large numbers, such as 4764 with 4564, we can observe that both contain the same number of thousands, tens, and ones. But, 4764 contains more hundreds than 4564, so $4764 > 4564$. Likewise, we can conclude that $4564 < 4764$. It is not always necessary to examine all the places in the numbers being compared (when comparing 5625 with 3426, we need only observe that 5625 contains more thousands than 3426).

Teaching Yellow Module, Unit G

Approximate Time: 7 to 10 days

Materials

abacus
felt hundred squares, ten strips, and units for demonstrations
flannelboard
hundred board (optional)
individual place value charts (optional)
objects which can easily be bundled into tens, such as sticks, pipe cleaners, tongue depressors, and so on
set of strips for each child
10-by-10 centimetre squares

Vocabulary

hundreds   place value

Since the use of discrete objects to give set demonstrations of place value concepts for the three-digit numbers is often not practical, a 10-by-10 centimetre square should be provided for use with the strips each child should have. The white unit strips, the orange ten strips, and these 10-by-10 hundred squares provide excellent materials for developing place-value concepts. If children work in small groups, they will have enough materials to represent any three-digit number. If you prefer that the children work individually, you might distribute extra duplicated copies of the 10-by-10 centimetre squares. Activities with these materials may then be related to demonstrations on the flannelboard in which you use felt units, ten strips, and hundred squares. Much work with actual objects such as beans or sticks will also be important for developing the children's ability to estimate one hundred, and get a feel for numbers less than 100. Such materials might be made available on a table or shelf with task cards asking children to estimate numbers between 100 and 1000.

Evaluation of Progress

Although you will want to observe the children's ability to tell how many hundreds, tens, and ones are in a given three-digit numeral, the real test of understanding comes when they are asked to compare two three-digit numbers. To be successful in ordering three-digit numbers, children must be skilled in counting by ones and be able to order the numbers 0 through 9. If they can do this, a knowledge of place value should enable them to order the numbers through 999.

Resources For Active Learning

General Activities:
Place value-base ten:
CHIP TRADING ACTIVITIES – SET II, Cards 1–3, Scott Scientific

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN11 • Addition: Hundreds, Tens and Units, Addison-Wesley

Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 70–74, Wiley

Manipulative Devices:

Abacus or abacus board (Educational Teaching Aids; school supplier)

Chips with hole in centre (Educational Teaching Aids; Selective Educational Equipment)

Grid Kit (Scott Scientific)

Hundred number board (Hammett, Learning Research Assoc.; Mafex Assoc.)

Hundred peg board and cylinders (Educational Teaching Aids; Responsive Environments Corp.)

Multi-base arithmetic blocks (Educational Teaching Aids; Herder and Herder)

Unifix materials (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Commercial Games:

Place Value I (Creative Publications; Creative Teaching Assoc.)

Investigation
Page g-1

Read the investigation question with the children. Give them an opportunity to discuss occasions when they have counted to one hundred. Then explain that the activity for this page is to count to one hundred. Point out the numbered squares and the rows of ten. You might encourage them to count with a partner.

For the bottom of the page you might provide the children with pieces of newsprint on which to draw 100 things. If a child does not know what to draw, you might suggest geometric shapes or letters. When the children start to count their items, ask questions such as: "Can you think of a way that will make your counting easier?" "Can you group your objects in some way for this purpose?" "How were the numbers at the top of the page grouped?" "Can you show your objects in groups of ten?" When the children have completed their drawing, suggest that they exchange papers with a classmate and count to see if their partner has drawn one hundred. Again point out how grouping may help them do the counting. Before continuing on to page g-2, it would be helpful to write the numeral for one hundred, 100, on the chalkboard and be sure children can identify it as the counting number which comes after 99. The place value meaning of this numeral will be developed in the next lesson.

Let's do

Can you count to one hundred?
Try counting these squares.

ten →	1	2	3	4	5	6	7	8	9	10
ten →	11	12	13	14	15	16	17	18	19	20
ten →	21	22	23	24	25	26	27	28	29	30
ten →	31	32	33	34	35	36	37	38	39	40
ten →	41	42	43	44	45	46	47	48	49	50
ten →	51	52	53	54	55	56	57	58	59	60
ten →	61	62	63	64	65	66	67	68	69	70
ten →	71	72	73	74	75	76	77	78	79	80
ten →	81	82	83	84	85	86	87	88	89	90
ten →	91	92	93	94	95	96	97	89	99	100

Can you draw 100 of something?

Purposes

To provide readiness for work with three-digit numbers
To give the child an intuitive feel for the number 100
To provide practice in counting to 100

Preparation

To prepare for this lesson, encourage the children to talk briefly about the numbers that they have been studying. Ask questions such as: "Can you name a number that you have worked with recently?" "Write a numeral that has two digits on the chalkboard?" "Can you name a larger number?" "Can you name a number larger than 99?"

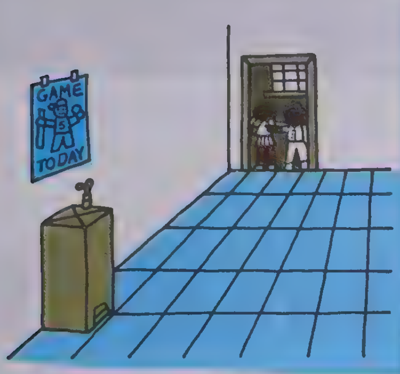
If children do not mention 100, you might say: "I'm thinking of a number that is larger than 99. Can you guess my number?" If no one mentions 100 after several guesses, explain that your number is exactly one more than 99. When 100 is introduced into the discussion, explain to the children that their investigation is to find out if they can count to 100. Then introduce the investigation page.

Follow-up

1. a) Fill various size containers with 100 beans. Ask the children to guess which of the containers hold 100 beans.
   b) Fill various containers of the same size with different sets of beans. That is, put 50 beans in one jar; 100 in another; 500 in a third and so on. Again ask the children to guess which of the containers holds 100 beans. When everyone has had a turn to guess,

Let's talk

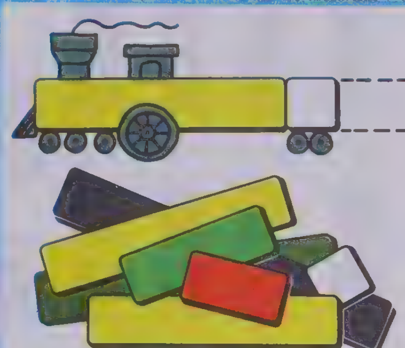
Try one of these activities.
Report your findings to the class.



How far is 100 steps?

How long does it take you to breathe 100 times?

Can you find 100 of anything in your room?



Can you make a 100-train with strips?

How long is a chain of 100 paper clips?

Preparation for three-digit place value

Discussion
Page g-2

To introduce the "Let's talk" page, ask the children how long they think 100 seconds are. For example, ask them to put their hands up when they think 100 seconds have passed. Of course, they will be influenced by each other's responses, but such a beginning should stir up interest for the type of activities suggested on page g-2. Although this page is a discussion page, it actually gives the children investigative type of activities to explore. Since there are five activities, you might section your class into five groups and have each group try to complete at least one activity. Then when each group has at least one activity on which to report, encourage the children to share their findings.

Note that most of the activities have a variety of responses. For example, 100 steps of a very small child will not remove him as far from the room as 100 steps of a tall child. Also if children are aware that their breaths are being counted they will usually breathe differently than they would if someone could count their breaths without their being aware of it. Various sets of strips may be used to make a train which, if the white unit is 1, may be thought to represent the number 100. For example, 4 ten strips, 2 seven strips, 2 three strips, 1 nine strip, 1 one strip, and 3 eight strips and 3 two strips may be used to make a train of 100.

give the children the opportunity to verify their guesses by actually counting the beans in the various containers. (Kernels of unpopped popcorn are also suitable for such an activity.)

2. To practice oral counting, direct a child to start counting, then stop and ask another child to continue the sequence. The second child should count for a short time, stop and call on another child, and so on, until each child has had a turn.
3. It would be helpful to give children more practice in ordering numbers less than 100. A duplicated activity such as the one in the following column would be suitable:

> Rewrite each row in order from the least to the greatest.
>
> 51, 93, 39, 53, 95, 35, 19, 15, 91, 59,
> **15, 19, . . .**
> 74, 38, 43, 84, 47, 65, 56, 16, 61, 15,
> **15, 16, . . .**
> 98, 21, 89, 40, 76, 12, 67, 14, 33, 93
>
> Rewrite each row above in order from the greatest to the least.
> **95, 93, . . .**

Teaching
Page g-3

Ask how many tens there are in the first set. When a child answers seven, direct the children to trace the dashed numeral 7. Then ask the children what numeral is used to show seven tens. When someone answers seventy, or seven, zero, tell the children to trace over the dashed numeral. Continue by asking how many tens there are in the second set. Direct the children to write the proper numerals in the blanks. As you work through the page, it would be helpful to demonstrate each example with the materials which the children used in the pre-book activity so that they may relate the illustrated bundles to the objects which they bundled. As you work through this page, stress that 100 represents 10 tens.

Give the numbers of tens.
Then write the numeral.

7 tens

We write 70.

8 tens

We write 80.

9 tens

We write 90.

10 tens

We write 100.

Objective

Given the number 100, the child will realize that it is the multiple of 10 following 90 and the counting number following 99.

Pre-Book Activity

Materials

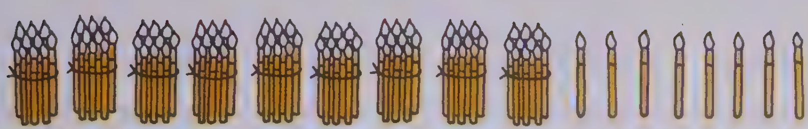
objects which can easily be bundled into tens, such as: sticks, pipe cleaners, tongue depressors, and so on

Give a group of children 100 objects that have not yet been bundled. Ask them to see if they can find out how many groups of ten they can make and to record the bundles as they count them.

In order to develop the idea that 100 is the number following 99, you might take eight bundles of ten and count the rest of the objects singly, having the children record their numerals. Or challenge them with the idea of making 9 bundles and then counting one by one to 100. Elicit from them what they will do when they come to 9 tens and 10 single objects. If sticks for this type of an activity are not available, you might work through a similar development on the flannelboard with felt strips which represent ten, and smaller squares of felt which represent one. Stress both that ten tens are one hundred and that one hundred is one more than nine tens and nine, that is, one more than 99. Also, be sure children know how to read and write the symbol 100 (one hundred).

Fill the blanks.

9 tens and 7 We write 97.



9 tens and 8 We write 98.

9 tens and 9 We write 99.

10 tens and 0 We write 100.

Teaching
Page g-4

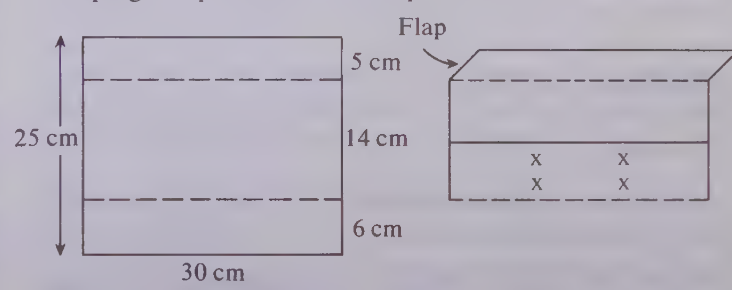
Call attention to the first set on the page, and ask how many tens there are and how many ones. When a child says "nine tens and seven," direct the class to trace over the dashed numerals 9 and 7. Then ask how we write 9 tens and 7. When a child answers "ninety-seven," direct the children to trace the dashed 97.

Work through each frame with the children showing actual objects which children can relate to the illustrations. Stress that each set is one more than the last, and finally, that 10 tens is 1 more than 9 tens and 9, and that the symbol for 10 tens is 100. It would also be helpful to count the numbers 90 through 100 orally.

Follow-up

The Hundred Board can be used to emphasize that 100 may be thought of as 10 tens or one more than 99.

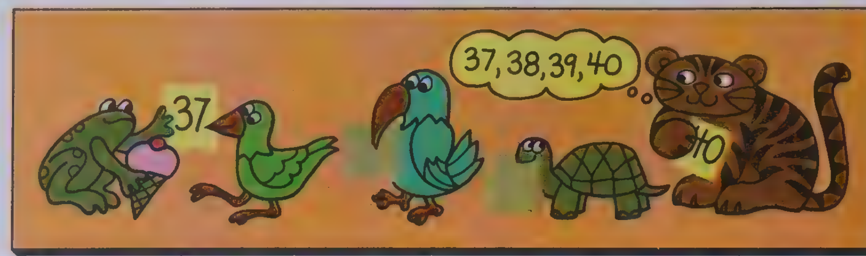
It would also be helpful at this time to help the children make individual place-value charts as suggested on page e-8 if you have not already done so. Instruct the children to mark a third set of index cards 0 to 9. Label the pockets on the children's charts 100's, 10's, 1's. These will be used in later lessons. At this time you might simply provide practice in building two-digit numbers on the place-value chart. Dictate one number at a time. Ask the children to find the proper numeral from each deck of cards and place it in its respective pocket. At your signal, all the children should show their completed charts. A quick visual check should be sufficient to reveal any children having difficulty. Any child still having difficulty making two-digit numbers should receive special attention before continuing with the development of place value meaning for three-digit numbers. Color coding the cards and labelling the pockets should prove helpful in keeping the packs of cards separated.



Note: Fold flap forward to secure chart and cards for storage. Fold flap back when chart is in use.

Teaching
Page g-5

Ask the children to tell you the number that comes just after 29. When a child answers 30, tell the class to trace the dashed numeral in the first blank. Then ask what comes just after 39. When another child answers 40, tell the class to trace over that numeral in the second blank. Instruct the children to complete the top section of the page by writing the numerals that come just after the given numeral. After the children have had time to finish that section, call their attention to the directions for the middle of the page. Some children may be able to continue the page independently, but you will want to work through it with many. Point out that each line is a short sequence and they are to count by ones and write in the missing numerals. Finally, read the last direction lines. Explain to the children that on the first line they are to count by tens and on the second line they are to count by fives.



Give the next number.

29, 30	39, 40	49, 50	59, 60
69, 70	79, 80	89, 90	99, 100

Give the missing numbers.

26	27	28	29	30	31	32	33	34	35
64	65	66	67	68	69	70	71	72	73
83	84	85	86	87	88	89	90	91	92
91	92	93	94	95	96	97	98	99	100

Complete the counting by tens.

10	20	30	40	50	60	70	80	90	100

Complete the counting by fives.

55	60	65	70	75	80	85	90	95	100

Objective

Given an incomplete sequence of numbers less than 100, the child will be able to complete the sequence in order.

Pre-Book Activity

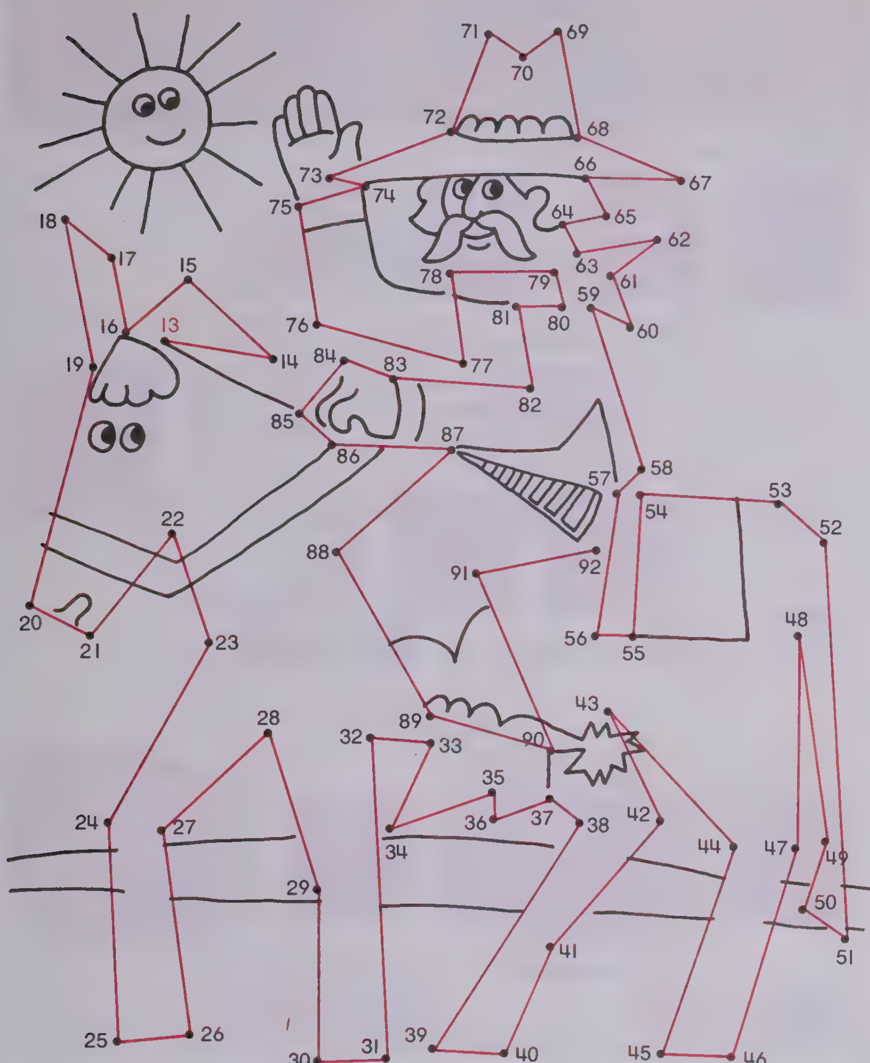
Materials

a 100-square (10 cm by 10 cm)
one set of strips for each child

Give the children practice in counting and building two-digit numbers with their strips. Then introduce the 10-by-10-cm square as a representation for the number 100. Ask the children to represent a number such as 78 with their strips, then ask them to add one more to that number to show 79 and continue, showing 80, 81, and so on. Work up to the sequence 97, 98, 99, 100. Children will probably have to work in groups of three in order to have enough orange strips to do this. Ask them to explain what they must do when they reach 99. Here, after they have changed their 10 single units for a 10-strip, ask them to examine the 100-square. Ask them to place their 10 orange strips on top of the 10-by-10 square. Elicit from the children that this single 10-by-10 square may be thought to represent the number 100. It may be thought of as 100 single units as indicated on the grid side or it may be thought of as 10 tens as shown on the colored side. If time permits, work orally through some sequences of numbers less than 100. For example, begin with a number such as 35 and ask children to continue counting for four or five numbers. Then give the

g 6 six

Connect the dots. Start at 13.



Practice counting

Teaching
Page g-6

Explain to the children that in this dot picture they are to start with the number 13. Tell them to find the red 13 and then find the dot beside 14 and draw a line between these two dots. Ask them to complete the picture by continuing from 13 until they have reached 100. Urge the children to draw the lines carefully so that the completed picture will be neat and recognizable.

next child another number, such as 18 or 56, and again, have him give the correct number in sequence.

Follow-up

To review both inequalities and order, play "Unscramble 100." Remove most of the tags from a hundred board and shake them up in a shoe box. Have two children each choose a tag from the box and show it to the rest of the class. Ask the two children to tell which number is smaller or which is larger and then hang the tags in their proper place on the hundred board. Call on the children until each child has had a turn. Observe the children who have difficulty either in comparing numbers or in determining the proper order and give them individual help.

You might also write large numerals 0 to 100 on index cards. Shuffle the cards and hold them so that the numerals are not visible. Ask six children to come up, draw a card and then order themselves smallest to largest. The class should judge whether the ordering is accurate. Allow these six children to choose six others to continue the game. Play until everyone gets a turn, then increase the number of cards to be ordered to be eight or ten, thus giving each child one more chance to participate with a larger set to be ordered.

Resources for Active Learning

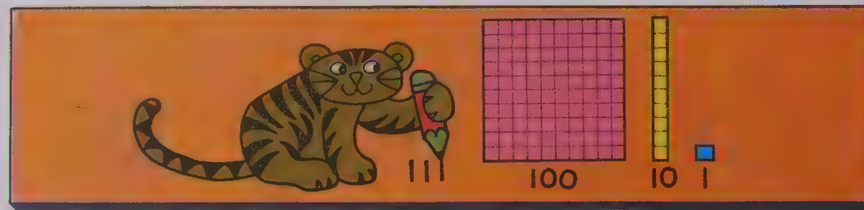
Connecting dots:

A CLOUDBURST, Vol. 1, No. 8231, Midwest publications

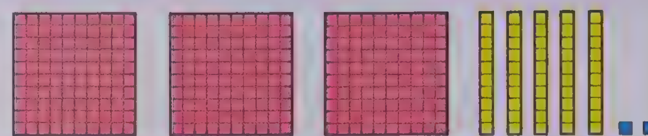
ENRICHMENT OF ARITHMETIC, pp. 1/35–38, Webster, McGraw-Hill

Teaching
Page g-7

If you have used the pre-book activity suggested, children should not have difficulty relating their work with the strips to the illustration in the text. Encourage them to work directly from the illustration. You might work through the first frame showing that there are 3 hundred squares or 3 hundreds, 5 ten strips, that is, 5 tens, and finally, 2 single strips or 2. Then ask the children to trace over the three-digit numeral, 352, that we write to show this number. Encourage the children to complete the page independently. As they work you might ask the children to explain a digit in one of the numerals. For example, write the numeral 352 on the chalkboard and ask them to explain what the 5 represents, or ask them to explain what the 3 represents.

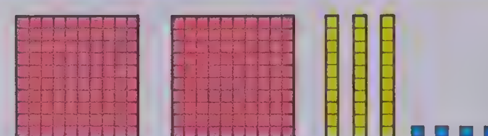


Fill the blanks.



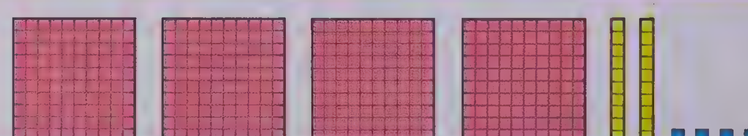
__3__ hundreds, __5__ tens, and __2__

We write __352__.



__2__ hundreds, __3__ tens, and __4__

We write __234__.



__4__ hundreds, __2__ tens, and __4__

We write __424__.

Objective

Given illustrations showing hundreds, tens, and ones, the child will be able to write the corresponding three-digit numeral.

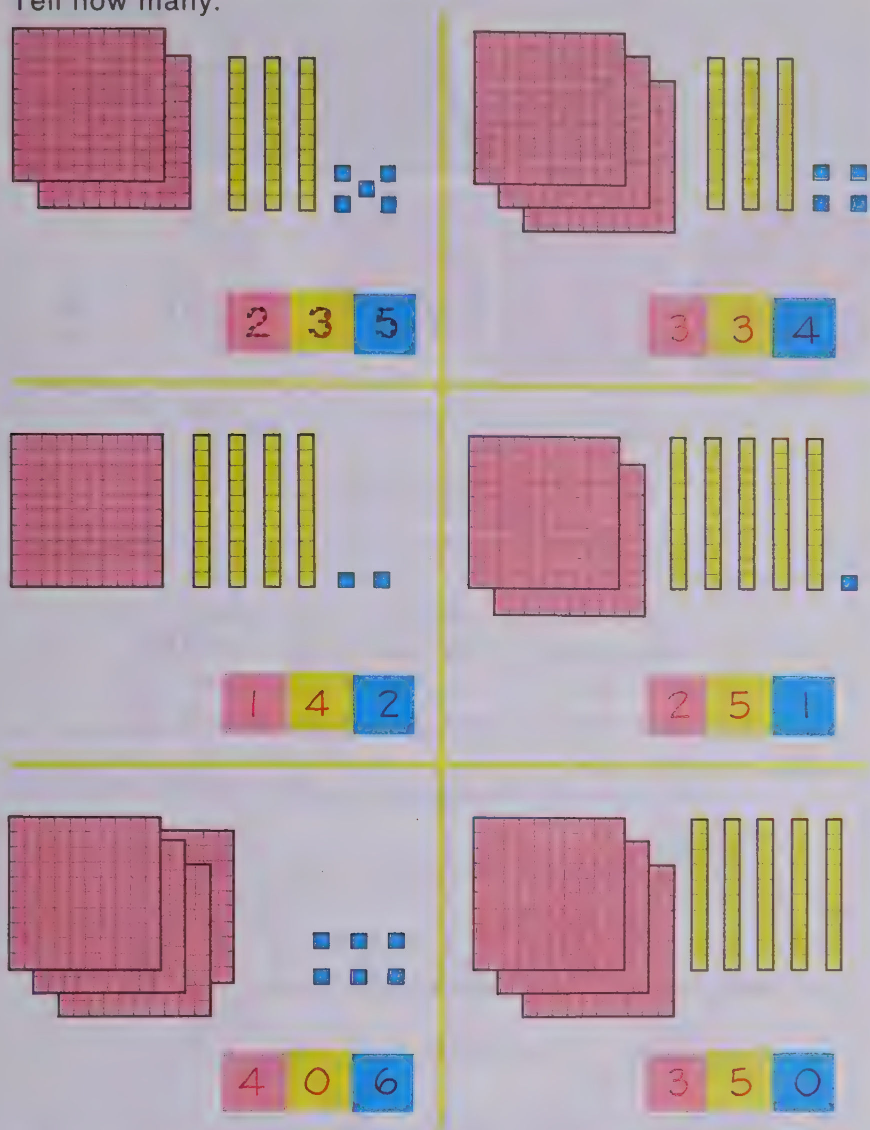
Pre-Book Activity

Materials

duplicated copies of a 100 square (10 cm by 10 cm) so that each child has five or six hundred squares

For the pre-book activity, help the children use their squares, strips, and units to build three-digit numbers. For example, ask them if they can work with a partner and show the number 235. Ask a volunteer to explain what the strips represent. You might want to use your demonstration flannelboard and felt objects to show 2 hundred squares, 3 tens, and 5 units, then write on the chalkboard 2 hundreds, 3 tens, and 5, explaining to the children that they write 235 to represent this number. Then give the children a variety of other three-digit numbers depending on the number of hundred squares which they have available. Include numbers which have zero 10's and zero 1's. A hundred square grid may easily be duplicated simply by drawing a 10-by-10-centimetre grid on a duplicating master, running it off on stiff paper, then cutting it to form as many 10-by-10-centimetre grids as possible. Ask the children to represent a number by saying: "Show with your strips and squares 1 hundred, 9 tens, and 2. Now record this number."

Tell how many.



Teaching
Page g-8

Point out the similarity between the illustrated squares and strips and those used by the children. Explain that they might use the strips if they choose or they may work directly from the illustration on the text page. Ask them to write the three-digit numeral that shows how many squares are pictured in each frame. As children complete this page, it would be helpful to use these numbers to give practice in reading the numerals. Also stress the meaning of the digits in the hundreds' and tens' places.

Follow-up

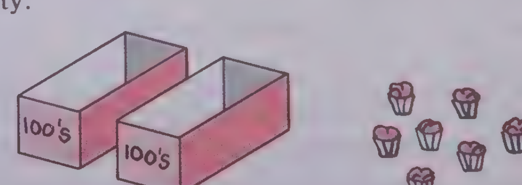
Write a group of three-digit numerals on the chalkboard. Ask the children to show each of these with their hundred squares, ten strips, and units. Encourage children to work with a partner in such an activity so that they may check each other's work.

A duplicating master such as the following may be used to accompany such an activity.

Show each number with your strips.
Record what pieces you used.

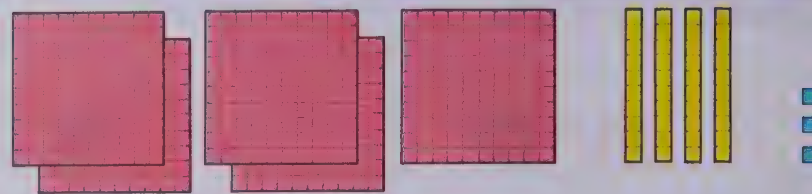
271	______ hundreds	______ tens and	______
502	______ hundreds	______ tens and	______
780	______ hundreds	______ tens and	______

To help children get a better feel for the number 100, you might also have objects such as beans or unpopped kernels of popcorn in small containers such as nut cups labelled 10 and also have larger containers such as shoe boxes, each of which might hold ten nut cups. Then ask the children to count the beans or kernels by using the nut cups and the boxes to group the beans into tens and hundreds. Children who have difficulty in understanding place value might be helped by such a counting activity.



Teaching
Page g-9

Discuss with the children how the hundred squares, ten strips, and units are used to represent the number 543. Ask a child to explain the meaning of the digits that are written in the hundreds', tens', and ones' places in the numeral 543. Direct them to trace over the numerals and to complete the phrase. For the second frame, ask the children what numeral they would write for 3 hundreds, 7 tens, and 6. After a child answers correctly, instruct all the children to trace the dashed numerals and to do the next exercises in the same way. Then continue with the bottom of the page and ask how many hundreds, tens, and ones the numeral 483 shows. Instruct the children to fill in the blanks so that the sentence reads "483 means 4 hundreds, 8 tens, and 3." Direct them to complete the remaining exercises in the same manner. You may wish to extend this lesson by using the ideas suggested in the follow-up or simply by presenting other phrases and numerals such as these on the chalkboard.



For 5 hundreds, 4 tens, and 3,

we write 543.

Fill the blanks.

For 3 hundreds, 7 tens, and 6, we write 376.

For 6 hundreds, 3 tens, and 5, we write 635.

For 4 hundreds, 9 tens, and 0, we write 490.

For 8 hundreds, 0 tens, and 2, we write 802.

483 means 4 hundreds, 8 tens, and 3.

280 means 2 hundreds, 8 tens, and 0.

306 means 3 hundreds, 0 tens, and 6.

Objectives

Given some number of hundreds, tens, and ones, the child will be able to express it as a three-digit numeral.

Given a three-digit numeral the child will be able to give the place value of each digit.

Pre-Book Activity

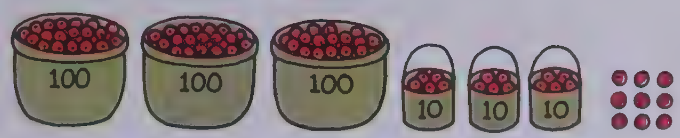
Materials

objects which may be bundled in groups of ten bundles of ten which may be placed in groups of one hundred

It would be helpful to use other materials besides the strips and squares to show numbers in the hundreds. For example, if some children have been counting beans or kernels of popcorn and grouping them in nut cups and boxes, you might wish to use these to show some numbers. If you have bundles of sticks available, use them to show numbers in the hundreds. If you do not have such materials, you might simply use three different size bags and label the smallest bag *ones*, the next size bag *tens*, and the largest bag *hundreds*. These might then be used to express a number in the hundreds even though you might not have actual items in them.

Use the materials available to display a number in the hundreds. Ask the children how many hundreds, how many tens, and how many ones are shown and record this information on the chalkboard. Then ask the children to write a numeral for this number. When a volunteer has written a numeral, such as 648, on the chalkboard have the children explain the meaning of each digit.

Fill the blanks.



3 hundreds, 3 tens, and 9.

We write 339.

971 means 9 hundreds, 7 tens, and 1.

503 means 5 hundreds, 0 tens, and 3.

826 means 8 hundreds, 2 tens, and 6.

347 means 3 hundreds, 4 tens, and 7.

719 means 7 hundreds, 1 tens, and 9.

Ring the correct word.

681 The 8 means 8 hundreds tens ones	**309** The 3 means 3 hundreds tens ones
495 The 4 means 4 hundreds tens ones	**827** The 7 means 7 hundreds tens ones

Three-digit place value

Teaching
Page g-10

Let the children read the directions at the top of the page. Mention that the items illustrated are grouped in groups of hundreds, tens, and ones. In the first frame, ask the children how many hundreds, how many tens, and how many ones there are. Ask them to fill in the blanks and then to write the three-digit numeral which is used to express this number of berries. Next explain that in the second section, they should fill in the blanks to show the meaning of each digit in the numerals. Finally, explain that they should complete the phrases at the bottom of the page by ringing the word which explains the place value meaning of the digit indicated.

Follow-up

Prepare a new set of 8-by-15-cm tagboard cards for the matching games suggested on page e-11. On nine of these cards, print three-digit numerals using the expression *hundreds, tens,* and *ones*. On the other nine cards, write the corresponding numerals. Shuffle the one set of cards and place them face down in the pockets of either poster. Then shuffle the other set and place the cards in the remaining pockets. To play the game, hang the boards where everyone can see them. Direct a child to choose the letter of a pocket on the left side and then the numeral of a pocket on the right. The teacher removes a pair of cards and shows them to the class. If they match, a point is scored. If they do not match, they are returned to their pockets face down and the other team gets a turn.

Resources for Active Learning

Place-value games and activities:

DEVELOPMENTAL MATH CARDS, C-17, 13, Addison-Wesley

FREEDOM TO LEARN, pp. 116–117, Addison-Wesley

MATHEX: Operations No. 3, pp. 9–16, Encyclopaedia Britannica Publications Ltd.

MATH WORKSHOP: Games and Enrichment Activities, pp. 19–20, Encyclopaedia Britannica Educational Corp.

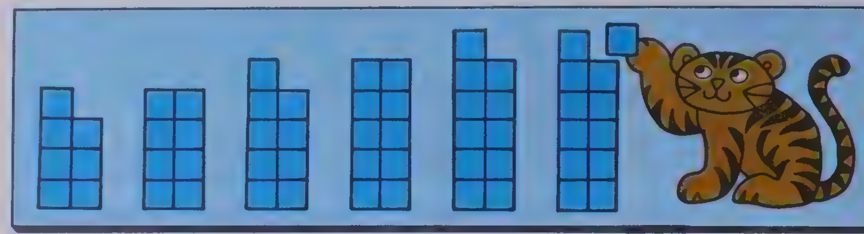
Nuffield Project: COMPUTATION AND STRUCTURE 2, p. 71, Wiley

TEACHING AIDS FOR ELEMENTARY MATHEMATICS, pp. 24–25, Holt, Rinehart and Winston

Teaching
Page g-11

Tell the children that they can complete each row by counting by ones and writing in the missing numerals. Call attention to the first row and begin by saying: "7, 8, what comes next?" When a child answers, "9," instruct all the children to trace that numeral. Continue by saying: "Now what comes next?" Then point out the sixth line and begin counting: "97, 98, what comes next?" When a child responds correctly, tell all the children to write 99. Again, ask: "What comes next?" and have the children write 100.

Let the children complete the page independently. Note the patterned development of the page, and use it as a basis of discussion.



Complete the counting.

7	8	9	10	11	12	13
17	18	19	20	21	22	23
47	48	49	50	51	52	53
147	148	149	150	151	152	153
347	348	349	350	351	352	353

97	98	99	100	101	102	103
197	198	199	200	201	202	203
397	398	399	400	401	402	403
497	498	499	500	501	502	503
597	598	599	600	601	602	603

Counting, one- to three-digit numerals

Objectives

Given an incomplete counting sequence of two or three-digit numbers, the child will be able to complete it.

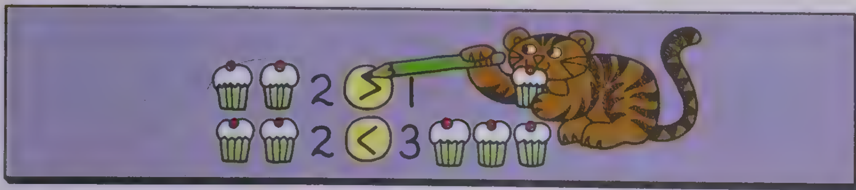
Given a pair of numbers, both less than 999, the child will be able to indicate which is greater by using the inequality sign.

Pre-Book Activity

A brief pre-book activity for this lesson would be a review of counting with large numbers. You might use an oral game such as "Chain Reaction." Begin by explaining that you will give two numbers out loud and call on someone to give the next two numbers. If this child does so correctly, he may call on another child to give the next two numbers in the sequence. Before starting the game, tell the children that you will interrupt and change the sequence after 10 or 20 whole numbers and establish rules such as calling on those who have not had a turn and so on. After first using simple sequences of numbers less than 100, work hard on spots that usually prove troublesome, such as 198, 199, 200, 201; or 209, 210, 211 and so on.

You might also review inequalities. Prepare two sets of cards on which you have written the numerals 0–9. Divide the class into two teams. Have three children from each team each draw a card from one of the piles. Ask each group of children to form the highest number possible with their three numerals and face the class. Ask a volunteer to place an inequality sign correctly to show which number is larger. Then ask three other children from each team to come up and this time to form the smallest number which they can with their three nu-

g 12 twelve



Put > or < in each

7 > 4	3 < 6
70 > 40	30 < 60
700 > 400	300 < 600
5 < 8	7 > 2
45 < 48	67 > 62
245 < 248	567 > 562
9 > 4	1 < 8
92 > 42	15 < 85
392 > 342	715 < 785
5 < 6	9 > 8
52 < 62	96 > 86
523 < 623	963 > 863

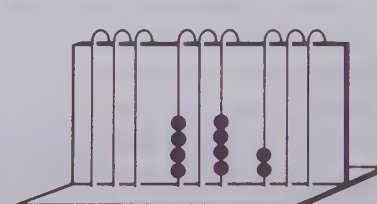
Reasoning inequalities

Teaching
Page g-12

Read the directions with the children. Review with them the greater and less than symbols. Ask whether 7 is greater or less than 4. Direct the children to trace the symbol after someone has responded correctly. Continue by asking whether 70 is greater or less than 40. When a child responds correctly, direct all the children to trace the symbol. Then tell the children to finish the page.

merals. Again have them stand so that a child can hold the inequality statement. Stress the correct use of the inequality signs to express the comparison of the two numbers. Vary the directions so that the children have a chance to work with both large and small numbers.

Follow-up
Use of an abacus with looped wires having a set of beads or washers in different colors to represent each place may help children attain a clearer understanding of order and place value. Manipulating the beads as they count sometimes helps some children visualize the idea of *one more* or *one less* than a given number. If possible, give each child in the group an abacus to handle, or place several of the large ones in a convenient spot for a free-time activity. The children may practice reading and using the abacus by copying a pattern shown in a sketch of an abacus on an idex card and then writing the numeral represented in standard symbols. Another form of an abacus is suggested on page 183.

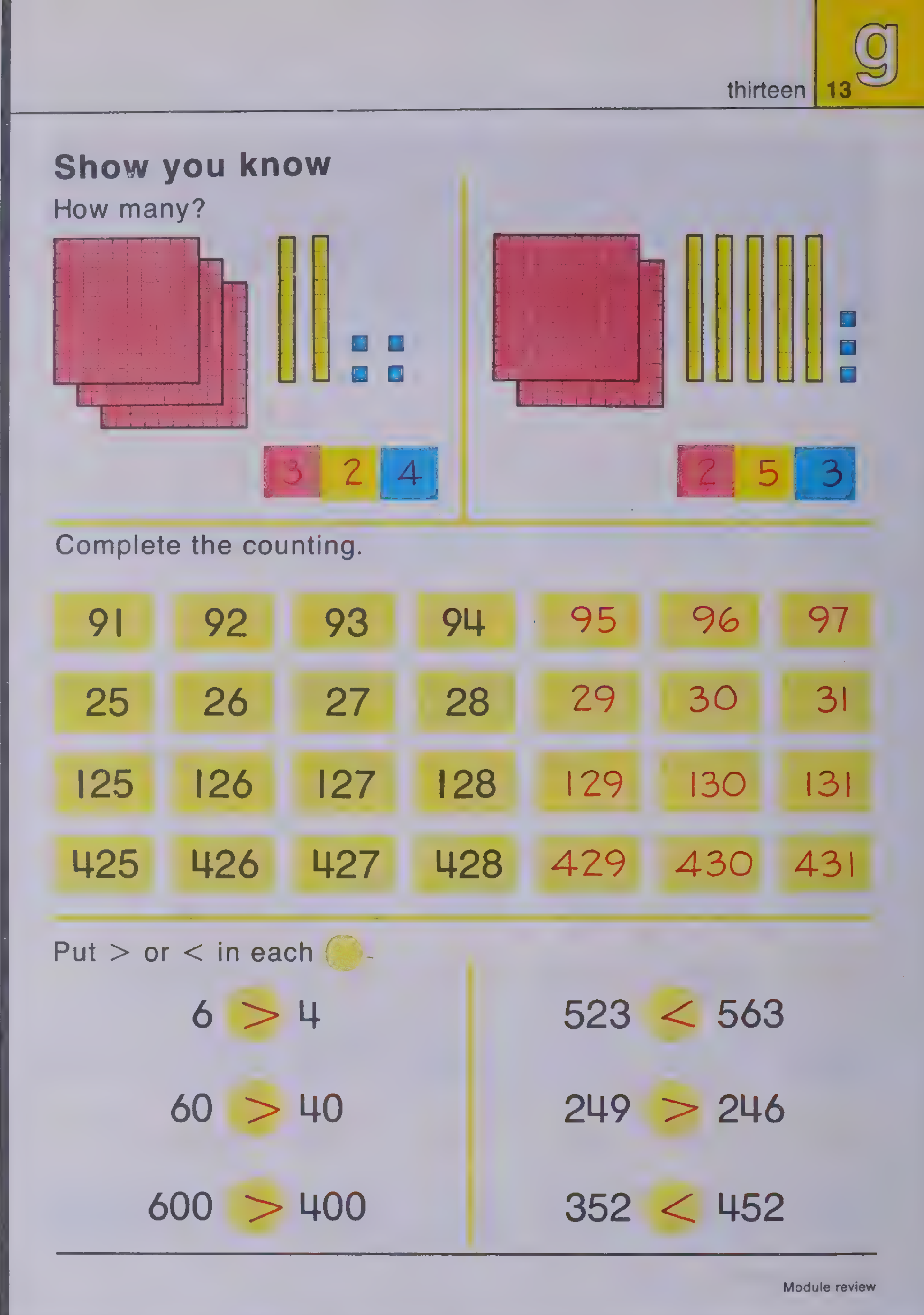


Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN11 • Addition: Hundreds, Tens and Units, pp. 2–9, Addison-Wesley

Teaching
Page g-13

Since each section of this page has different directions, you will want to be sure the children understand what they are to do in each section before you assign the exercises. Explain that in the first section they simply put down how many small unit squares there are. Notice that the color coding will again help them relate the place values to the hundred squares, the ten strips, and units. In the second section, they should complete the counting. Here, point out that they need simply count by ones and then fill in the boxes. Finally in the last section, they should put in the greater than or less than sign in order to show which number is greater. When children have finished with these exercises, it would be helpful to review them together.



Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Materials

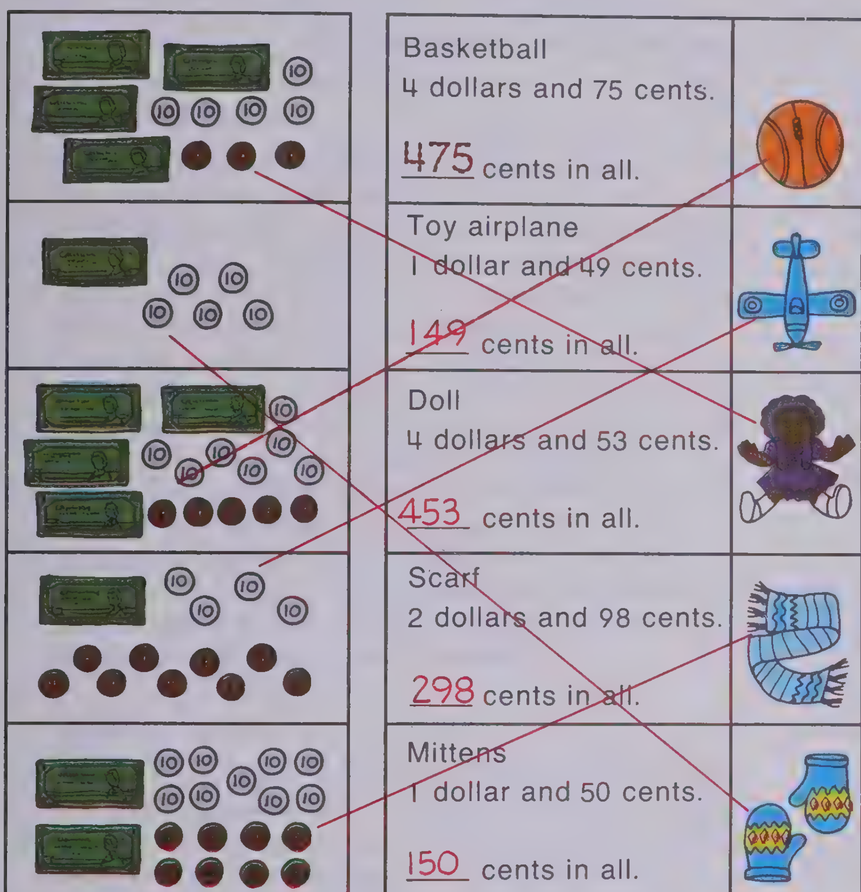
felt hundred squares
flannel board
ten strips and units

Since this lesson contains a review of the concepts presented in this module, it would be helpful to use a demonstration to review the meaning of three-digit numerals. For example, write on the chalkboard a three-digit numeral, such as 426. Ask the children to explain the meaning of each digit. Then ask a child to show a representation of this number on the flannelboard using hundred squares, ten strips, and units. To review inequalities, you might also ask the children to suggest a number greater than the one you have introduced. Again ask them to explain the meaning of each digit in the numeral and to show the representation of this number on the flannelboard. Continue with several examples.

write down four digits at the top of the paper. Then ask them to write as many different three-digit numbers as they can, using those four digits. After a few minutes, ask them to select the numeral representing the greatest number on their list and circle it, and then choose

Let's have fun

Match the money with the picture.
Then write the number of cents in the blank.



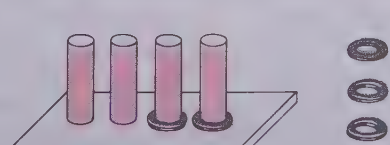
Dollars and cents

Teaching
Page g-14

This change of pace page should be treated with a light touch, although you may wish to use it to discuss dollars and cents. For example, use play money or transparencies on the overhead projector to talk about dollars and sets of coins. Be sure the children understand that one dollar is the same as 100 cents and may be thought of as ten dimes. Point out that so many dollars and cents may be thought of as some number of cents. For example, you might discuss some things that someone might buy if they had 345 pennies, or you might discuss the way someone who had 345 pennies would be able to change them so that they wouldn't have as many coins to carry and relate this to the purpose of the dollar bill. Finally, explain that on this page they should match the illustrated money with the picture, then write the number of cents in the blank. Discuss the first one as an example. Point out how 4 dollars and 75 cents matches the third picture on the left and that this amount may be thought of as 475 cents in all.

Follow-up

Some children may need other concrete aids to help them gain more understanding of place value. Four small dowel rods approximately 15 centimetres long, may be pushed through a 7-by-7-by-25-centimetre piece of plastic foam to make a type of abacus. You may wish to have the children paint the rods different colors or label each one with an initial to remind them of the value of each place. Flat colored beads, or faucet washers from the hardware store will make counters to fit on the dowel rods.



To further review place value and inequalities, give the children large sheets of lined paper, and ask them to write down four digits at the top of the paper. Then ask them to write as many different three-digit numbers as they can, using those four digits. After a few minutes, ask them to select the numeral representing the greatest number on their list and circle it, and then choose the numeral representing the least number and draw a box around it. More capable children might be able to put their resulting list in order from least to greatest.

Resources for Active Learning

Money and place value:
Franklin Series: LEARNING ABOUT MEASUREMENT, pp. 96–97, Lyons and Carnahan

Three-digit Addition and Subtraction Without Regrouping

Pages g-15 to g-22

General Objectives

To develop an understanding of the addition and subtraction of three-digit numbers (without regrouping)

To maintain skills with addition and subtraction combinations

The emphasis in this module is directed toward teaching children to understand addition and subtraction involving three-digit numbers. A later module will develop the idea of addition and subtraction with regrouping. Since regrouping is not introduced in this module, it is not anticipated that children will have much difficulty. The strips and hundred squares as well as activities with concrete materials are again suggested for those children who need them; however, children should be able to do most of the work in this module without concrete materials. Concepts such as adding multiples of ten and multiples of one hundred are introduced. The notation of horizontal equations and of vertical exercises both are presented. A set of word problems is also included so the children have an opportunity to identify the operations of addition and subtraction in context.

Mathematics

The basic underlying mathematical concepts for addition and subtraction of three-digit numbers are identical to those for two-digit numbers; that is, the basic ordering and grouping principles are involved in the rearranging of addends in column addition.

Teaching Orange Module, Unit G

Approximate Time: 4 to 6 days

Materials

bean bags, three (optional)
felt hundreds squares, ten strips, and units
flannelboard
heavy cardboard or a primary chart book (optional)
orange strips
sticks bundled in groups of tens and hundreds (optional)
10-by-10-centimetre squares

Vocabulary

addition differences subtraction sums

Since the nature of this module is primarily algorithmic, most children will not need as many concrete materials as previously suggested. It is helpful to relate the addition and subtraction of three-digit numbers to the addition and subtraction of two-digit numbers. Stress subtracting hundreds and tens, since this module develops both power skills and speed skills for addition and subtraction of three-digit numbers without regrouping. Although the children might not need many concrete materials to manipulate, many demonstrations with materials that can be grouped by tens will be important to help the children easily see the combinations of hundreds, tens, and ones. Since most of the combinations involved deal only with the basic facts of nine or less, most children should be expected to do the algorithmic work with ease.

Evaluation of Progress

Your evaluation of the children's achievement in this module should be based on their mastery of skills in adding and subtracting two-digit numbers, adding and subtracting three-digit numbers, and on their understanding of the concepts involved in finding these sums and differences. If you are uncertain about a child's understanding, you might want to ask him to work through a problem for you with the strips and hundred square materials. You can often test the children's understanding more effectively by testing them orally and having them tell and show you how to add and subtract three-digit numbers than you can by simply evaluating their performance with written exercises.

Resources for Active Learning

General Activities:
Basic facts games:

DEVELOPMENTAL MATH CARDS, C-4, Addison-Wesley

MATH ACTIVITIES, Games 3/31–79, pp. 97–119, Allyn and Bacon

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN7 • Addition and Difference (A); WN11 • Addition: Hundreds, Tens and Units, Addison-Wesley

MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Publications Ltd.

NUMBER-BLOX, Creative Publications

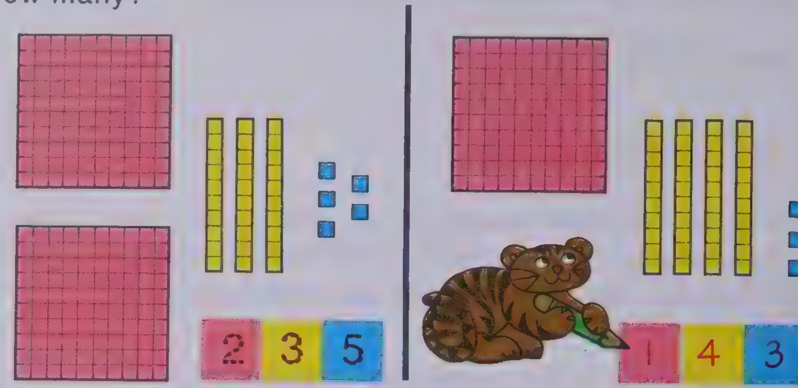
Manipulative Devices:
Number-Blox (Creative Publications)

Commercial Games:
For games to develop competence with the basic facts, refer to the Introduction of the blue module in Unit E.

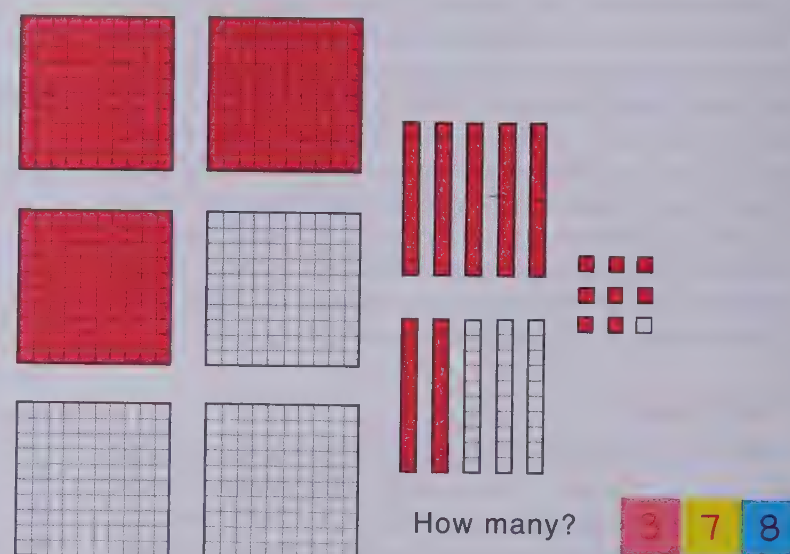
Investigation
Page g-15

Explain to the children that the illustrations on this page correspond to the materials that they have been using. If you choose, you might have the children continue to work in groups so that they may actually construct the sets suggested by the illustration. However, many children will be able to work directly from the text. Point out that the first frame shows 2 hundred squares so the 2 is written in the hundred's place. Have a child explain the meaning of the numerals in the tens' and ones' places. Direct the children to trace over the dashed numerals written in the hundreds', tens', and ones' places. Then ask that they figure out what number is represented by the strips shown in the top right frame. Explain that their main challenge is to color enough of the figures at the bottom to show the combination of the sets illustrated at the top. Then they should write the total amount in the space provided at the bottom. Depending upon how well children work with this investigation, you will either have to develop this lesson further or allow them to work independently on other sets with the strips. With children who are having more difficulty in using the materials and combining the sets, you might use demonstration squares, strips, and units on the flannelboard and have the children work through several examples with you.

Let's do
How many?



Can you color enough of these figures to show the two sets together?



Readiness for addition, subtraction, and subtraction without regrouping

Purposes

To introduce addition and subtraction of three-digit numerals

To provide experience with concrete materials in grouping hundreds, tens, and ones

Preparation

Materials

10-by-10-centimetre squares
orange strips
white strips

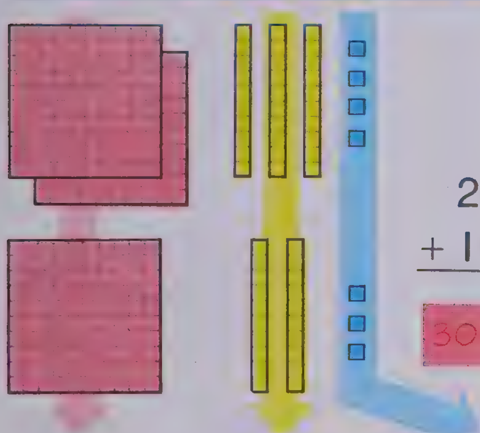
In order for children to have enough materials for the preparation, you might suggest that they work together in groups of three or four. Give them a variety of three-digit numbers to show with their strips. For example, ask, "Can you show the number 531 with your strips?" Or, you might write a list of three-digit numbers on the chalkboard and have various groups show different numbers in this list. After the children have worked on this, you might ask them to explain the numbers that their group built and to show them on the flannelboard, using flannel or felt·hundred squares, ten strips, and units.

Let's talk

Fill in the blanks. Then find the sums.

40	3	43
+ 20	+ 2	+ 22
60	5	65

6 tens and 5



200	30	4	234
+ 100	+ 20	+ 3	+ 123
300	50	7	357

3 hundreds, 5 tens, and 7

Can you find these differences?

500	60	7	567
− 200	− 30	− 5	− 235
300	30	2	332

Readiness for addition, subtraction, and subtraction without regrouping

Discussion
Page g-16

Since this module is algorithmic in nature, this discussion page is different from previous discussion pages. It is recommended that you work through the exercises with the children. Be sure to relate addition and subtraction of three-digit numbers to addition and subtraction of two-digit numbers. For example, stress the idea that when they add the numbers for the two sets at the top of the page, they combine the three ones with the two ones and the four tens with the two tens. Then, in the next frame, they again combine the four ones with the three ones and the three tens with the two tens and, similarly, they will combine the two hundreds with the one hundred. Even though children need only think of a basic fact in order to work out an algorithm, at this time, it would be helpful to stress the fact that they are adding hundreds and tens. It is recommended that you work through the problem at the bottom of the page using demonstration materials with the flannelboard, or three various-sized bags or, whatever materials you have which show groupings of tens, hundreds, and ones. Note that the purpose of the page is to develop the power skill or understanding of addition and subtraction of three-digit numerals. The development of the speed skill will follow in the next two lessons.

Follow-up

A duplicating master such as the following would be helpful, particularly, for those children who have difficulty in relating the work with an algorithm to concrete materials.

Show each pair of numbers with your strips. Then write the total.	
341 and 628	TOTAL ______
250 and 125	TOTAL ______
109 and 340	TOTAL ______
632 and 256	TOTAL ______

It would also be helpful to review the basic addition and subtraction facts to nine since the children will need these with the addition of larger numbers. A game such as the Combo game suggested on page 83 would be appropriate for such a review.

Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN7 • Addition and Difference (A), pp. 2–8, Addison-Wesley

TEACHING
Page g-17

Explain to the children that on this page they should simply find the sums. Point out that the problems at the top of the left section of the page show the problem on the right side in separated form. Thus, after they have added the hundreds, tens, and ones on the left section of the page, they will have actually done the work for the problem on the right. If necessary, guide the class through the first two problems to help them see the relationship between the separated problems and the problem on the right. Note that the exercises on the bottom move from the power skill of working separately with hundreds, tens, and ones to the speed skill. Children should have no difficulty with this, but if necessary, guide them through this exercise with the aid of demonstration materials.

$$\begin{array}{r}200\\+100\\\hline 300\end{array}\quad\begin{array}{r}30\\+40\\\hline 70\end{array}\quad\begin{array}{r}6\\+2\\\hline 8\end{array}\qquad\begin{array}{r}236\\+142\\\hline\end{array}$$

Find the sums.

300 + 200 = 500	50 + 30 = 80	4 + 5 = 9	354 + 235 = 589
500 + 100 = 600	60 + 20 = 80	3 + 4 = 7	563 + 124 = 687
100 + 400 = 500	60 + 30 = 90	8 + 0 = 8	168 + 430 = 598

826 + 142 = 968	214 + 571 = 785	105 + 492 = 597	382 + 403 = 785	71 + 325 = 396
402 + 296 = 698	134 + 63 = 197	763 + 215 = 978	403 + 194 = 597	461 + 238 = 699

Three-digit addition without regrouping

OBJECTIVE

Given an addition or subtraction problem involving three-digit numbers and no regrouping, the child will be able to find the sum or the difference.

PRE-BOOK ACTIVITY

As a preparation for these pages, it would be helpful to use the chalkboard or the flannelboard to demonstrate how to split numbers into hundreds, tens, and ones and then to add or subtract these numbers as three separate problems. Exhibit the following exercise on the chalkboard in vertical notation:

$$\begin{array}{r}526\\+372\\\hline\end{array}$$

Explain that you are going to illustrate the ideas in working such an addition problem by writing each number in its expanded form, 500 + 20 + 6 and 300 + 70 + 2. Suggest that now you can write three separate problems.

$$\begin{array}{r}500\\+300\\\hline\end{array}\qquad\begin{array}{r}20\\+70\\\hline\end{array}\qquad\begin{array}{r}6\\+2\\\hline\end{array}$$

Ask the children to solve each of these three problems and then explain that the sums put together are like the expanded form of the numeral; that is, 800 + 90 + 8 can be written as a single numeral, 898. Repeat this idea with other examples until the children understand it. Also write a subtraction example such as 758 minus 346 on the chalkboard. Explain to the children that to help them understand the ideas in subtracting large numbers, they can first write the given numbers in expanded form to show place value as in 700 + 50 + 8 and 300 + 40 + 6,

Find the differences.

500	60	9	569
− 200	− 10	− 3	− 213
300	50	6	356
800	40	8	848
− 300	− 30	− 2	− 332
500	10	6	516
600	90	7	697
− 500	− 20	− 5	− 525
100	70	2	172

936	854	746	495	894
− 213	− 422	− 615	− 321	− 680
723	432	131	174	214
853	469	936	757	356
− 702	− 214	− 332	− 34	− 341
151	255	604	723	15
589	675	647	493	865
− 587	− 222	− 30	− 272	− 315
2	453	617	221	550

Three-digit subtraction without regrouping

Teaching
Page g-18

After the children read the directions, help them realize that each separated problem on the left is related to the problem on the right. Thus, after they have subtracted the first three problems, they should see the relationship between these problems and the problem on the right. If necessary, discuss the first three problems at the top and then encourage the children to work the remaining exercises independently.

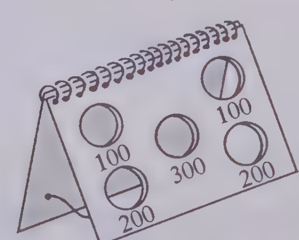
and then find the differences for three simple problems.

$$\begin{array}{r} 700 \\ -300 \\ \hline \end{array} \qquad \begin{array}{r} 50 \\ -40 \\ \hline \end{array} \qquad \begin{array}{r} 8 \\ -6 \\ \hline \end{array}$$

These differences can be written together as the expanded form of a single numeral; that is, as 400 + 10 + 2 = 412. Work through other examples of this kind until you are sure children can add and subtract three-digit numerals by treating them as separate problems.

Follow-up

The covers from a primary chart book can be used to make a bean-bag toss game. (Or substitute heavy cardboard, bound with cloth tape on both sides of one edge.) Attach a 45-cm strip of twill tape to both leaves at the bottom to hold them in place when the book is being used as a target. Cut five 15-cm holes in one side and label them as shown in the figure.



Bean-bag toss

Every child should have a chance to toss three bean-bags at the target from a distance of from one to two metres, depending upon the weight of the bags. Children can take turns holding the chart if it is not stable. Each child should keep his own score, thus practicing addition with hundreds.

Teaching
Page g-19

On this page children work both addition and subtraction problems involving three-digit numbers. Encourage the children to work the problems independently, but help any who are having difficulty. Before they begin, be sure to point out that the frames at the bottom of the page contain both addition and subtraction problems. Stress that they must be careful to observe the signs in order to perform the correct operation. When the children are finished with the exercises, check their answers with them.

423 +156 579	854 -532 322	742 +116 858	789 -650

Find the sums and differences.

123 + 714 837	473 + 125 598	595 + 300 895	932 − 121 811	677 − 342 335	749 − 625 124
340 + 128 468	296 + 402 698	351 + 26 377	856 − 752 104	297 − 120 177	348 − 105 243
526 + 143 669	657 − 134 523	405 + 293 698	893 − 251 642	214 + 574 788	476 − 420 56
523 − 413 110	740 − 340 400	164 + 332 496	356 + 402 758	306 + 431 737	865 − 362 503
268 + 430 698	658 − 244 414	343 + 215 558	935 − 530 405	535 + 254 789	726 − 405 321

Three-digit addition and subtraction without regrouping

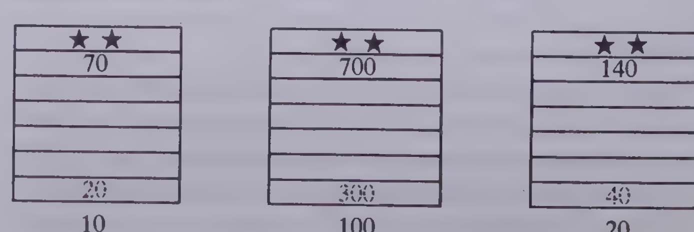
Objective

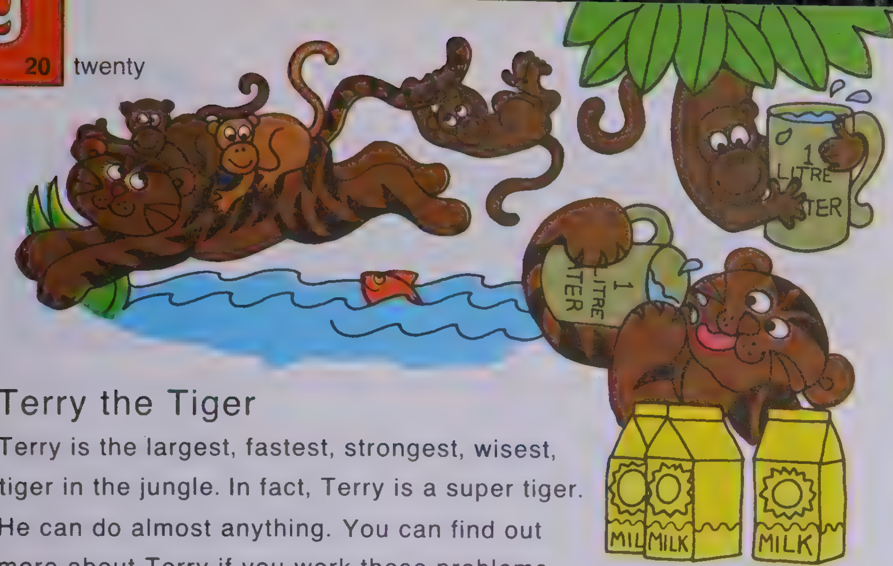
Given addition or subtraction problems involving three-digit numbers which do not require regrouping, the child will be able to find the sums or differences.

Pre-Book Activity

An oral review of the basic addition and subtraction facts to nine would be helpful for this lesson. It would also be helpful to review addition and subtraction with multiples of 10 and 100. For example, after a brief review of the basic addition and subtraction facts to nine, you might play "What's My Number?" by asking such questions as: "I'm thinking of a number one hundred more than 800. What's my number?" or "I'm thinking of a number 20 more than 30. What's my number?" You might also practice reading a sequence of numbers by saying: 10, 20, 30, 40, and asking a child to tell you what comes next. Begin other sequences with different numbers, all multiples of 10 or multiples of 100. To stimulate children in such a game, you might make "Ladder to the Stars," such as those shown in the illustration on the chalkboard and have children complete the sequences to see if they can reach the top of the ladders.

Ladder to the Stars





Terry the Tiger

Terry is the largest, fastest, strongest, wisest, tiger in the jungle. In fact, Terry is a super tiger. He can do almost anything. You can find out more about Terry if you work these problems.

1. Terry jumped across a river that was 645 metres wide. Then he jumped 214 metres. How far in all? 859 metres
2. Terry drank 9 litres of milk and 8 litres of water. How much did he drink? 17 litres
3. Terry once ran 134 kilometres, rested and ran 23 more. How far? 157 kilometres
4. Terry weighs 216 kilograms. He ate a big meal and gained 23 kilograms. How much does he weigh now? 239 kilograms
5. Terry is 299 years old. His friend is only 132. How much older is Terry? 167 years
6. Terry got tired. He slept for 36 hours. Then he turned over and slept 12 more hours. How long? 48 hours
7. Terry watched 46 monkeys playing tag. 21 more monkeys came. How many in all? 67

Teaching
Page g-20

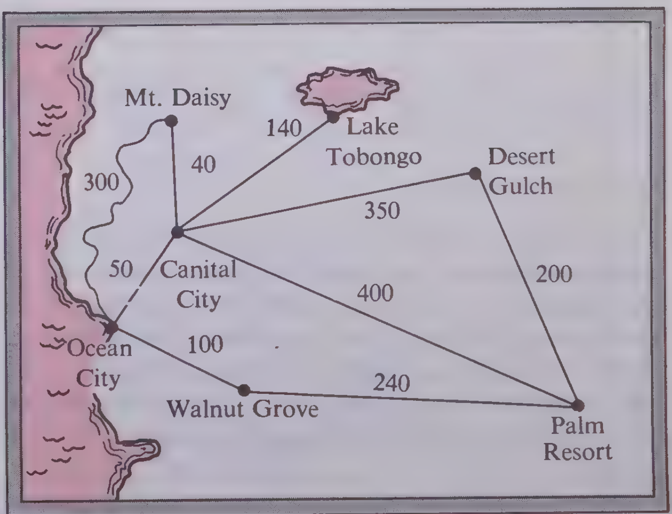
You will wish to read the problems together with many children; however, encourage them to think through the problems independently. You may wish to have the children write an equation for each of the problems although this is not necessary. It might prove advantageous if you are working the problems with children. Alternatively the children may work in small groups with this page. You might encourage those who finish the problems quickly to make comparisons with the measurements indicated in the problems. For example, they might try to find a distance comparable to the number of metres actually jumped, or figure out how many of them it would take to match Terry's weight, or how long Terry read and sang in terms of days. Such problems will challenge the capable children, particularly, if they are encouraged to think of their own method of solving them.

Follow-up

You may wish to devise an exercise using a map like the following one to give children practice in map reading. Sample questions might be:

1) Sam took a trip from Ocean City to Mt. Daisy, and then to Capital City. How far did Sam travel?

2) Sharon and her family, who lived in Capital City, went to Palm Resort for a vacation. If they came home by way of Desert Gulch, how long was their trip? Was this trip longer than if they had gone to Palm Resort and directly back home?



Teaching
Page g-21

Encourage the children to work these problems independently. However, stress that they should be careful in watching the signs so that they perform the correct operation. When they are finished, read the correct answers and help them check their papers. If you wish, you might have volunteers work out a few problems on the chalkboard, explaining them as they work through them. Also, the problems at the bottom should be handled according to the reading ability of the children. You might wish the children to try to work them independently and then as you check the papers, have volunteers explain how they solved them.

Show you know

Solve.

232	215	203	736	584	396
+ 514	+ 580	+ 724	− 122	− 231	− 105
746	795	927	614	353	291
413	466	365	237	764	835
+ 354	+ 232	+ 24	− 33	− 524	− 610
767	698	389	204	240	225

623	786	563	518	234	925
+ 146	− 123	− 113	+ 230	+ 104	− 403
769	663	450	748	338	522
842	452	656	129	615	749
− 641	+ 300	− 305	+ 620	+ 43	− 622
201	752	351	749	658	127

One frog jumped 69 centimetres The second frog jumped 42 centimetres How much farther did the first frog jump? 27 centimetres

The green grasshopper jumped 235 centimetres. The brown one only jumped 212 centimetres. How much farther for the green one? 23 centimetres

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

According to the children's need you might wish to review adding and subtracting three-digit numbers without regrouping. For example, with some children you will want to work through a few examples on the chalkboard before assigning these pages. With other children, however, you might simply want to use an oral review such as a Combo game and then assign the pages.

Follow-up

For more capable children provide an activity by reproducing reconstruction problems, as suggested below.

Find the missing numbers.			
□□□	3 4 2	□ 0 □	2 □ 3
+ 6 2 5	+ □□□	+ □ 5	+ □ 1 □
9 5 9	5 8 7	8 6 5	9 9 9
9 0 0	2 □ 3	□ 5 6	□ 4 0
+ □□□	+ □ 4 1	+ 5 □ 0	+ 4 □ 7
1 0 0 0	8 9 4	7 8 □	5 6 □

Let's have fun

Can you find which animals are inside the blue fence?

Maze

snail turtle
duck rabbit

Teaching
Page g-22

Explain to the children that on this change of pace page they are to figure out which animals are inside the blue fence. Encourage them to figure out their way of finding out; however, one way is for them to start coloring inside the fence and never cross over a blue line. Then any animal which is colored will be inside the fence. Another way is to draw an escape path for each animal, finding out which animals may escape, that is, which animals are on the outside of the fence.

Give children who had difficulty with the algorithmic work of this module guidance in using their strips to complete a worksheet such as the following.

Build with your strips:		
2 hundreds		3 hundreds
5 tens	and	4 tens
2 ones		7 ones
Combine your strips and solve:		252
		+347

Build with your strips:		
3 hundreds		4 hundreds
7 tens	and	1 ten
9 ones		0 ones
Combine your strips and solve:		
	379	
	+410	

Resources for Active Learning

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN11 • Adding: Hundreds, Tens and Units pp. 4–9, Addison-Wesley

RED MODULE, UNIT G

Geometry

Pages g-23 to g-34

General Objectives

To introduce basic geometric concepts such as point, line, line segment

To review basic geometric shapes, such as square, circle, triangle, rectangle

To develop an understanding of the difference between straight and curved line segments

To develop understanding of attributes such as size and shape by work with attribute pieces

The material presented in this module might be thought of as an introduction to topics whose treatment could be extended over a long period of time and accompanied by many activities. For example, the text develops the use of the attribute pieces which can be purchased separately as punchout pieces or made from the patterns provided on the facing page. These pieces give rise not only to the study of similarities and differences with regard to attributes such as size and shape, but also to a review of the basic geometric shapes of triangle, square, rectangle, and circle. Although geoboards are not essential to the development of this module, many children would benefit from an opportunity to work with them. The physical activity of sliding, turning and flipping provides valuable background for later work with geometric transformations (see mathematics section). Although basic topics of geometry are developed, it is important that a module such as this be treated with a light touch and be an enjoyable experience for the children. Supplement the suggested activities with favorites of your own to help the children participate more fully in studying these topics.

Mathematics

The mathematics of this module can be separated into three basic areas: 1. recognition of basic geometric shapes, 2. sorting and classification, 3. transformations. In this module, the children are not only given an opportunity to work with and recognize a variety of geometric shapes, they are also given an opportunity to work with and express some of the basic characteristics of the shapes. In addition to this, they are provided with introductory work with segments and paths. By working with some of the basic characteristics of the geometric figures, the children are given an opportunity to sort and classify these objects into various sets that have particular properties. This classification and sorting of various kinds of objects is a most important mathematical skill to be developed since much of the future work in mathematics involves recognizing various characteristics of mathematical objects and then sorting them into special sets. Some of the basic underlying ideas of transformations are introduced on a very informal basis in these materials. For example, the children are exposed to such concepts as similarity of figures, rotations, flipping, sliding. Of course, no formal mention need be made of these very basic transformational ideas since they are treated on a very simple and informal level.

Teaching Red Module, Unit G

Approximate Time: 6 to 9 days

Materials

attribute pieces (see facing page)
dot paper
geoboards (optional)
models of the basic geometric shapes (square, circle, triangle, rectangle)

Vocabulary

attribute pieces	line	simple closed curve
circle	line segment	straight
closed curve	outside	square
curve	point	triangle
inside	rectangle	

Although the topics in the text have been carefully developed, it is important for you to feel free to stress various topics according to the interest and need of the children. For example, if children become very enthused about working with the attribute pieces, develop further activities with them, or if geoboards are available, you might wish to spend more time developing the concepts treated in the text with the use of the geoboard. Since line segments are studied in this module, consider giving the children conservation of length tasks. Some children at this level will not yet be able to understand that lengths are conserved. In order to find out if a child has arrived at this level of reasoning, take two pipe cleaners and place them in a parallel position in front of the child. Then ask him if he agrees that these two pipe cleaners are the same length; that is, are their endpoints the same distance apart? Then move one of the pipe cleaners forward and to the right about five or six centimetres. Then ask the child which of the pipe clean-

ers is longer now. The child who is able to understand conservation of length will realize that even though the position of the pipe cleaner has been changed, its size or length has not. A child who has not yet achieved this level of maturation will think that the pipe cleaner extending further is longer. For further development of such conservation tasks, see *How Children Learn Mathematics* by Richard Copeland.

Evaluation of Progress

Since this module deals with non-metric geometry, you might find it hard to judge the children's understanding of the material. A child's use of the vocabulary suggested is not always a good criterion for judging his understanding of the concepts; however, if he uses the correct vocabulary when referring to the concepts and figures studied, you can be fairly certain that he is understanding the ideas. The child's performance on the pages will also indicate his understanding of the concepts whether or not he has been able to master the vocabulary; however, very little emphasis should be put on *mastery* of the concepts presented in this module. Keep in mind that the lessons of this module should be thought of as an introduction to geometric concepts which will be extended throughout the remainder of the child's schooling.

Resources for Active Learning

General Activities:

BOXES, SQUARES AND OTHER THINGS, National Council of Teachers of Mathematics

ACTIVITIES IN GEOMETRY, pp. 39, 42, 56, 62 and 63, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Geometry," pp. 35–55, Herder and Herder

MATHEMATICS IN MODULES: SPATIAL KNOWLEDGE • SK1 • Faces and Plane Shapes, Addison-Wesley

Nuffield Project: ENVIRONMENTAL GEOMETRY, Wiley

Tangrams:

IT'S A TANGRAM WORLD, pp. 5–17, Educational Science Consultants

MAPPING GAMES, 8, Webster, McGraw-Hill

TANGRAMS (cards and pieces), Webster, McGraw-Hill

TANGRAMS: 330 PUZZLES, Dover Publications

Manipulative Devices:

Attribute games (Teaching Resources; Webster, McGraw-Hill; school supplier)

Attribute stickers (Webster, McGraw-Hill)

Basic shapes set (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Dienes Logical Blocks (Herder and Herder)

Geo-boards (Addison-Wesley)

Shape Matching Cubes (Childcraft; Selective Educational Equipment)

Tangram cards and pieces (Selective Educational Equipment; Webster, McGraw-Hill)

Commercial Games:

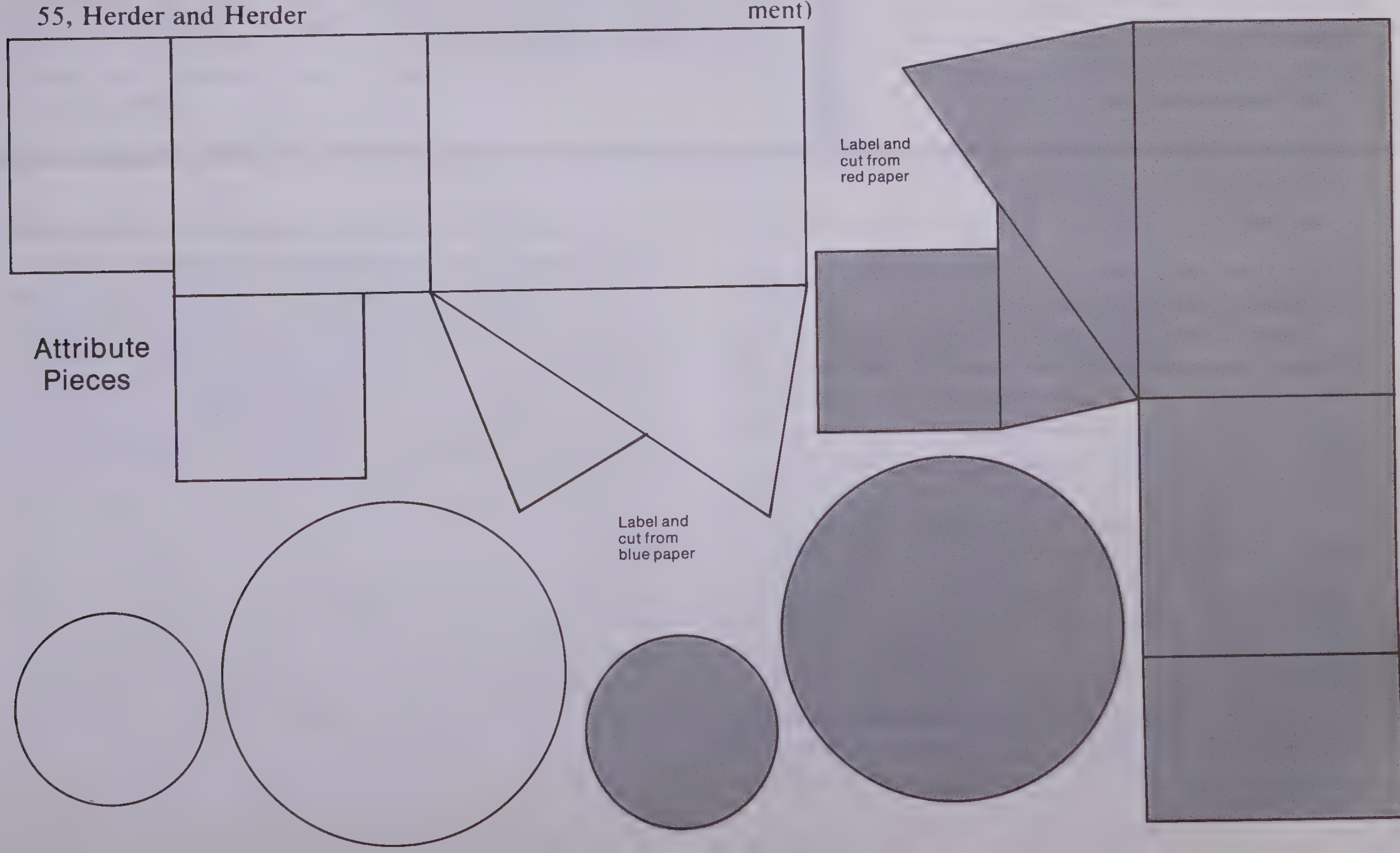
Geometric dominoes games (Childcraft; Selective Educational Equipment)

One, Two, Three, Think! (Selective Educational Equipment)

Shape Analysis Matching Game (Math Media; Responsive Environments Corp.)

Shape-Up Game (Lakeshore)

Spot the Set (Childcraft; Selective Educational Equipment)

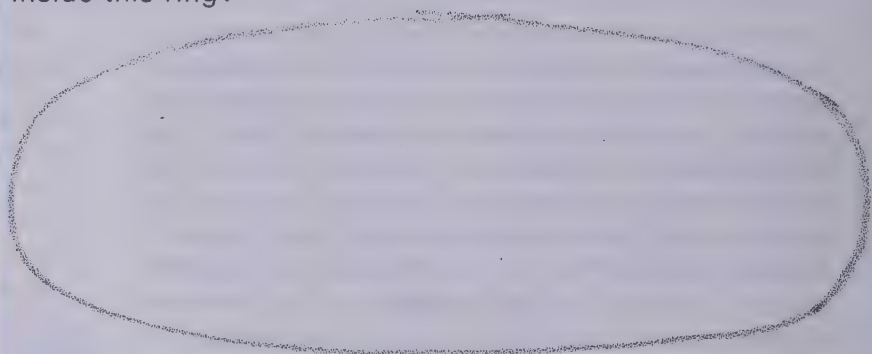


Investigation
Page g-23

Direct the children to look at the illustration at the top of page g-23. Ask them how the figures that the tiger is putting into the loop are alike. Acceptable answers would be "They all have three corners." "They all have three sides." or "All are triangles." Point out the first row in the chart at the bottom of the page. Discuss that the phrase "all have 3 corners" describes the four figures shown at the top (which they should put into their ring on the page). Then point out the dashed letters in the column labelled punchouts. Discuss how these letters match the labels on the attribute pieces or punchouts and ask the children to trace over them. Then work through the next phrases in a similar manner. Ask the children to find all the pieces which are round, or which are circles. They should place these in their ring at the top of the page. Children should then print the letters B, F, K, and O in the second row under "Punchouts." Complete the page with the children. It would also be helpful to continue with other examples. Ask children to choose "All small figures," or "all blue figures," or "all large figures that have four corners." Then ask the children to suggest some "alike sets" and have the others place the correct attribute pieces in the ring on their page. It is important for the children to experience sorting the figures into groups of like sets according to descriptions that you give them.

Let's do

Can you put punchout figures that are "alike" inside this ring?



How are they alike?	Punchouts
ALL HAVE 3 CORNERS	red blue
ALL CIRCLES	red blue
ALL BLUE SQUARES	
ALL LARGE AND RED	
ALL SMALL CIRCLES	red blue

Geometric attributes—introduction

Purposes

To introduce students to the similarities and differences among the attribute pieces

To provide an opportunity for the student to classify attribute pieces according to some of their similarities

To introduce the student to the geometric concepts which will be studied in this module, such as: point, line segment, square, circle, triangle, rectangle

Preparation

Materials

attribute pieces (see p. 195 or the punchout figures that can be purchased as a separate item)

It is essential to this lesson and other lessons throughout the module that the children have the attribute pieces. For the preparation period, you might simply have the children punch out their attribute pieces. It would be helpful to supply each child with an envelope in which he can store his pieces. You might encourage the children to discuss the shapes and sizes of the pieces as they punch them out, but these topics will be covered throughout this module.

Let's talk

Geometric attributes—introduction

Discussion
Page g-24

After the main ideas of the investigation have been discussed, use the illustration on page g-24 as a basis for the introduction of terms and for a review of familiar geometric shapes. For example, ask the children to point out shapes in the picture that are rectangular, such as sides of the wagon, or of the boxes. Or to point out parts of the picture which look like circles, such as the target on the dart board, or the wheels of the wagon. Also point out use of curved lines, as in the slide and the cord of the tetherball. A discussion of this type will not only review these concepts for the children, but will also give you an opportunity to assess their knowledge of these basic ideas, particularly, whether or not they understand the concepts of square, circle, triangle, and rectangle.

Follow-up

Games with the attribute pieces are very helpful, not only in helping children learn the basic geometric shapes, but also in developing their ability to classify and sort these figures. One such game involves having one child hide one of the attribute pieces from his partner who then tries to decide which piece has been hidden by asking questions. A game such as this might be made more interesting if the number of questions asked is limited to five or six; however, many children will need more questions, particularly, until they become more familiar with the pieces. Another game which is similar to "20 Questions" starts with one child saying, "I'm thinking of an attribute piece." By asking as few questions are possible the rest of the children then try to decide which attribute piece the first child is thinking about.

Resources for Active Learning

Attributes and classification:

ATTRIBUTE GAMES AND PROBLEMS, Webster, McGraw-Hill

DEVELOPING NUMBER EXPERIENCES, Kit A, Holt, Rinehart and Winston

DEVELOPMENTAL MATH CARDS, C#15, Addison-Wesley

EARLY NUMBER MULTI-GROUP LAB, Cards 1–4, 17–19, Responsive Environments Corp.

ELEMENTARY SCHOOL SCIENCE, Book 1, Unit 4, "Solids," pp. 100–115, Addison-Wesley

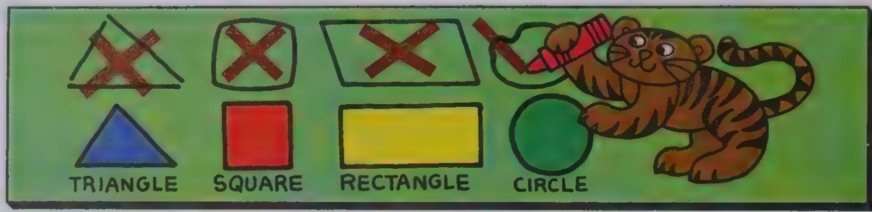
LEARNING LOGIC AND LOGICAL GAMES, Herder and Herder

MAPPING GAMES, 2, Webster, McGraw-Hill

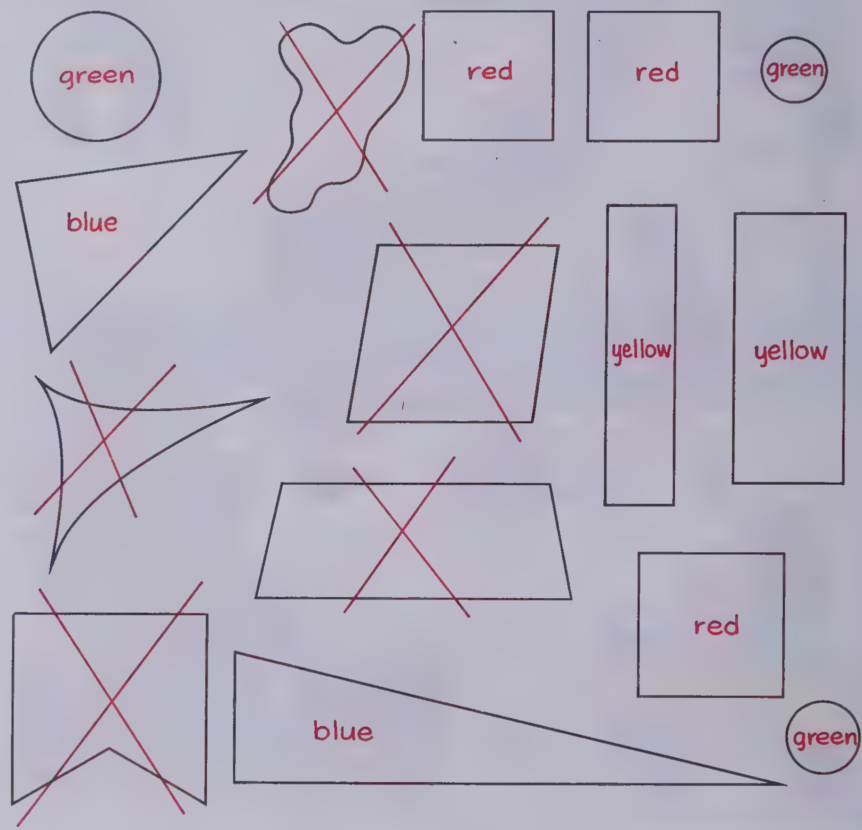
MATHEMATICS IN MODULES: SPATIAL KNOWLEDGE • SK1 • Faces and Plane Shapes pp. 8—11, Addison-Wesley

Teaching
Page g-25

Relate the illustration at the top of the page to the piles of attribute pieces which the children prepared in the pre-book activity. Point out that at the top of each figure a shape which looks something like their geometric shape has been crossed out. Explain that on the page they should first mark an "X" on each figure that is not either a triangle, a square, a rectangle, or a circle. After they have done this, they should color each triangle, square, rectangle, and circle according to the color-code given at the top of the page. It would also be helpful at this time to help children recognize not only the oral words *triangle, rectangle, square,* and *circle;* but to teach them to read the words for these terms. You might have the words and figures displayed together in a prominent place so that the child will be able to associate the word with the figure more readily.



Mark an X on each figure that is not one of the figures named above.
Color each triangle, square, rectangle, and circle the same color as shown above.



Objectives

Given a set of geometric figures, the child will be able to identify squares, rectangles, triangles, and circles.

Given dot paper, the child will be able to draw a triangle, a square, and a rectangle.

Pre-Book Activity

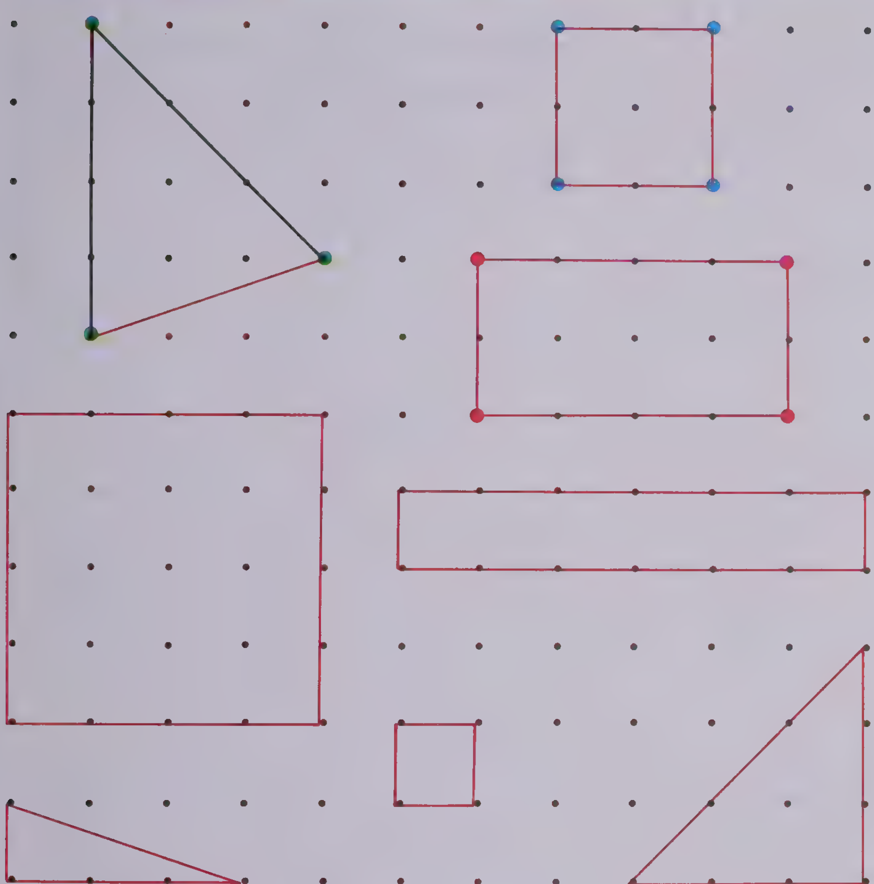
Materials

attribute pieces
geoboards (optional)

Ask the children to separate their attribute or punchout pieces into four piles so that the pieces in each pile are alike in a certain way. Some children may need further guidance to do this. For example, introduce a pile of triangles, and say: "All of the pieces in this pile are alike in a special way. Every piece that is alike in this way has been included in the pile. Make a pile like this." "Can you separate the rest of your pieces into three piles so that all the pieces in each pile are alike in a certain way?" Give as much guidance as is necessary to help the children build a pile of triangles, a pile of squares, a pile of rectangles and a pile of circles. When they have successfully done this, help them review the term for each piece and talk about what is alike about the pieces in each pile. For example, the triangles all have three corners; the squares all have four corners and the sides are all the same length; the rectangles have four corners, but only opposite sides are the same length.

Use the green dots to draw a triangle.
Use the blue dots to draw a square.
Use the red dots to draw a rectangle.
Draw some more triangles,
rectangles, and squares.

Answers will vary. Examples are given.



Triangles, squares, rectangles, circles

Teaching
Page g-26

The page simply provides an opportunity for children to draw triangles, rectangles, and squares freehand on dot paper. Help them read the directions. Explain the purpose of the colored dots. Notice that their last direction is simply to draw some more triangles, rectangles, and squares. It you wish, you might ask them to draw all the triangles with green sides, all the squares with blue sides, and all the rectangles with red sides.

If geoboards are available, children would benefit from the opportunity to make these figures with rubberbands on the geoboards.

Follow-up

Further work with dot paper and the geoboard in constructing triangles, squares, and rectangles would be helpful. Ask children to make three different size triangles on the geoboard or to see if they can make five different-sized squares on the geoboard.

You might also put various triangles, rectangles, squares, and circles made out of cardboard into a bag and ask the children to reach in and retrieve a specified type of figure. For example, ask them if they can reach in and by feeling the figures find and pull out a circle.

Another activity would be to distribute to each child two large pieces of construction paper. On one have them use templates to draw triangles, squares, rectangles, and circles. Have the children cut these shapes out, then tell them to place the shapes in any way that they choose on the other piece of construction paper to form a figure such as a clown, a boat, or a house. Stress that they may only use these geometric figures.

Resources for Active Learning

ACTIVITIES IN GEOMETRY, pp. 39, 42 and 56, Addison-Wesley

INQUIRY IN MATHEMATICS VIA THE GEO-BOARD, Geo-card 2, Walker

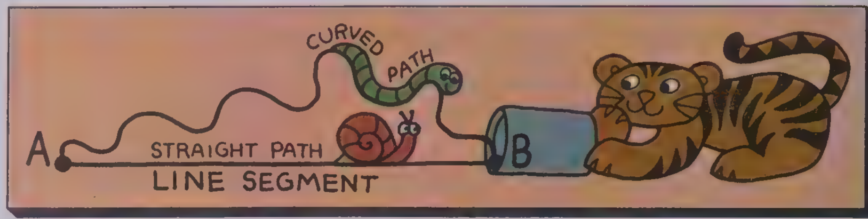
MATH ACTIVITY CARDS, "Nail Boards," A 11–13, Macmillan

MATHEX: Geometry No. 4, "Games with Plane Shapes," pp. 18–19, Encyclopaedia Britannica Publications Ltd.

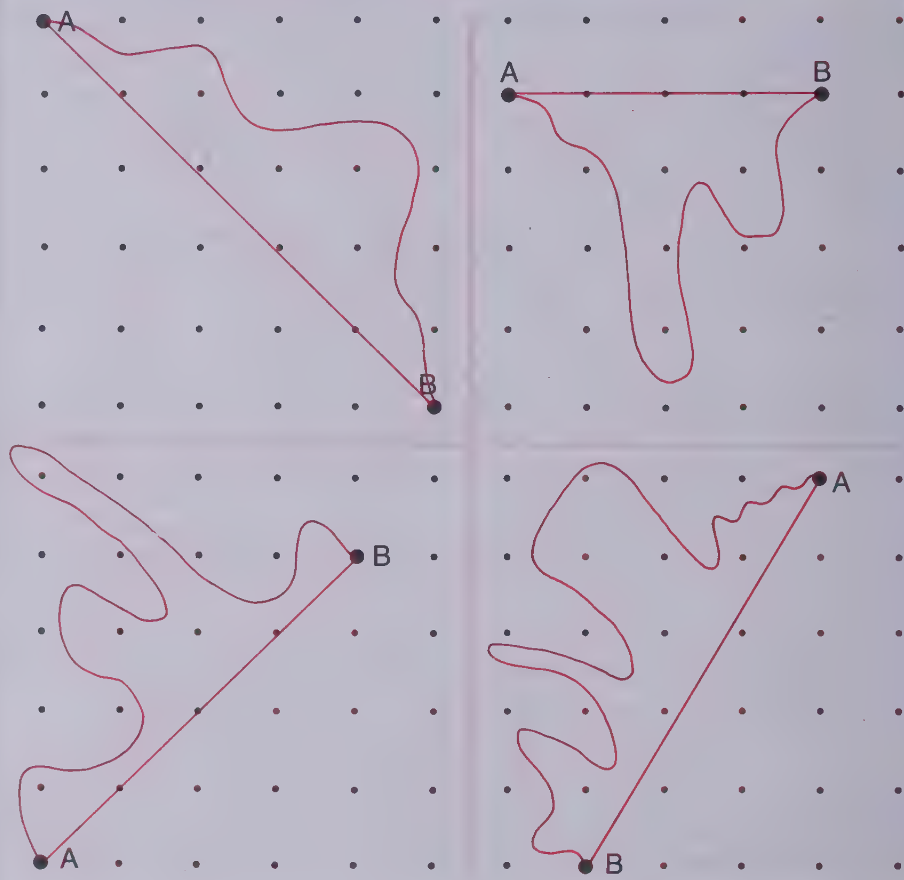
MATHS MINI-LAB, Cards 100–102, Selective Educational Equipment

Teaching
Page g-27

Discuss the illustration at the top of the page and relate it to the pre-book activity. Again emphasize that the straight path is known as a *line segment* or just *segment* and the curved path is simply called a *curve*. When you are sure children understand the meaning of straight and curved paths, encourage them to complete the page. They should draw straight and curved paths from A to B in each frame. It would also be helpful to talk about a path, which we might call a broken path or a polygonal path, that is made up of a union of line segments. For example, show such a broken path between two points A and B on the chalkboard or overhead projector. Explain that when two points are connected by one or more segments we say that a polygonal path is formed by the union of the segments. However, it is not necessary to use the term polygonal path with all of the children. You might encourage children to draw such paths using many line segments to connect the points A and B. Then, again using the overhead projector, children might compare the distance covered by the straight path and the path made up of the union of many segments. If you wish children to compare the distance covered by the curved path, and a path of line segments, you might have them use small pieces of string to compare the distances.



Draw straight and curved paths from A to B. Answers will vary. Examples are given.



Segments

Objective

Given two points on a grid, the child will be able to connect the points with either a curved path or a straight path as specified.

Given a line segment pictured on dot paper, the child will be able to draw a segment that has the same size.

Pre-Book Activity

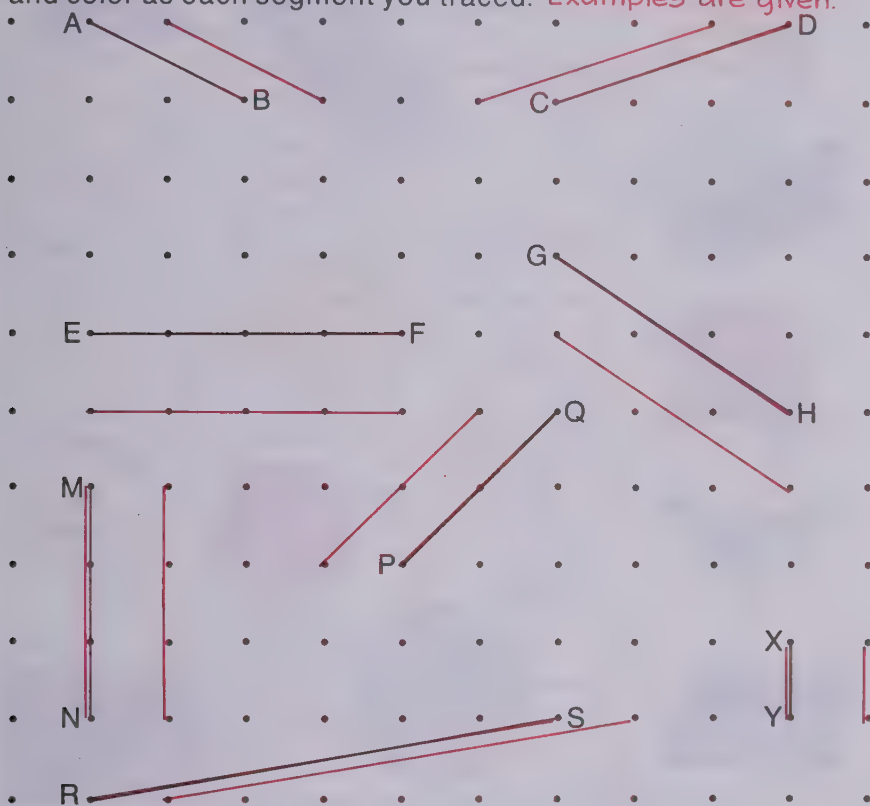
Materials

dot paper

Distribute to each child a page of dot paper on which you have labelled two dots, A and B. Ask the children if they can draw the shortest path from point A to point B. Have points A and B situated on the paper so that the shortest path will go through a diagonal of dots.

Use the overhead projector and a transparency of dot paper to discuss with the children which path is shortest. If no one made a curved path, you might do so and ask the children to compare it with the paths that they made. Encourage them to discuss the differences between the curved path and the straight line. Emphasize that the straight line path is recognized as the shortest path between the two points. You might use analogies, such as having people travel from point A to point B along different paths. Ask: "If three travellers start at point A and are travelling the same speed, which one will get to point B first?" Point out three different ways in which the travellers might go and help the children in this way to recognize how the line segment may be used as a straight path between two points.

Trace over each of the line segments with
a different color crayon.
Make another line segment the same size
and color as each segment you traced. Examples are given.



Segments—congruence

Teaching
Page g-28

Although the activity on this page appears to be rather simple, it actually involves the beginning of the concept of congruence and should be developed *carefully*. First explain to the children that they should trace over each of the line segments with a different color crayon. Point out that each of the line segments has a different set of letters at the two endpoints so that the segments may be named when being talked about. Then talk about ways they might use to draw another line segment the same size, that is, just as long as a given segment. This may be done in a variety of ways. Some children might benefit most from simply tracing a line segment and then placing that line segment on another section of the dot paper to find out where their new endpoints should be. Give children small pieces of yarn to use. Notice that in drawing a segment of the same size they are actually comparing lengths. Be sure children realize that when we talk about the size of a segment, we are not talking about how thick or thin a mark their crayon makes, but we are talking about the distance between the two endpoints.

Follow-up

Activities with string may be used to help children compare lengths of line segments, curved paths, and paths made up of many line segments. For example, you might pick two points in the classroom, then ask the children if they can find the shortest path between these two points. Note that they have to go around any furniture that is in the way of the straight path between the two points. Then the children can use their string and tape it to the ground to show various paths. You might also encourage more capable children to draw a map showing their home, school, and other buildings in their community on a piece of dot paper. Then they might draw various paths which they travel and compare which paths might be the shortest to go from school to home or from home to the store and which paths might be the longest. Another material that might be very helpful in working with broken line segments and paths made up of many line segments is toothpicks. Children might enjoy making three or four different paths with toothpicks and then asking a partner if all their paths are the same length. Of course, if the same number of toothpicks have been used, no matter what the shapes of the paths, they will be the same length.

Resources for Active Learning

Conservation of length tasks:

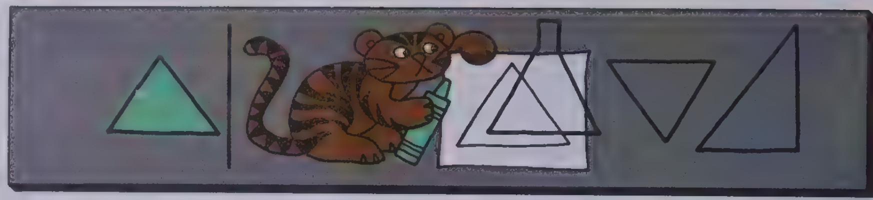
FREEDOM TO LEARN, pp. 123–124, Addison-Wesley

MATHEX: Measurement and Estimation No. 5, pp. 15–20, Encyclopaedia Britannica Publications Ltd.

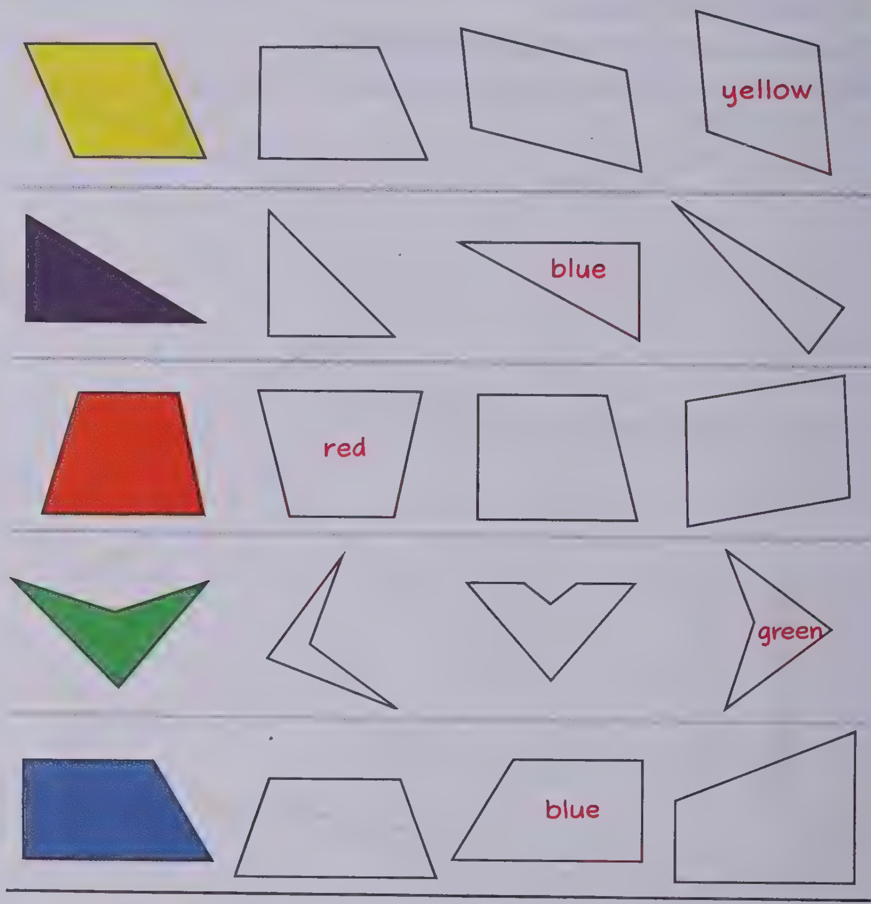
INQUIRY IN MATHEMATICS VIA THE GEOBOARD, Geo-cards 4/1–2, Walker

Teaching
Page g-29

Explain to the children that the first figure in each row matches one of the three figures in the row. When they decide which of the three this matching figure is, they should color it. In order to help them find out which it is, distribute tracing paper. Suggest that after first guessing, they check their choice by tracing the first figure. Then they can rotate, slide, and flip the tracing paper over the other figures to see which figure the first one matches. Such an activity can provide a background for transformations in later years of school. When the children have completed the page, discuss their choices. Point out differences between the first figures and the two figures in each row which do not match them. For example, in the second row, discuss that even though all of the shapes shown are triangles, only one matches the first figure in size and shape. Also point out that the matching figure in the last row, has been flipped, that is, it appears as the mirror image of the first figure.



Color the figure that is the same size and shape as the first figure.



Objective

Given a set of geometric figures, the child will be able to match the first figure with another which is the same size and shape.

Pre-Book Activity

Materials

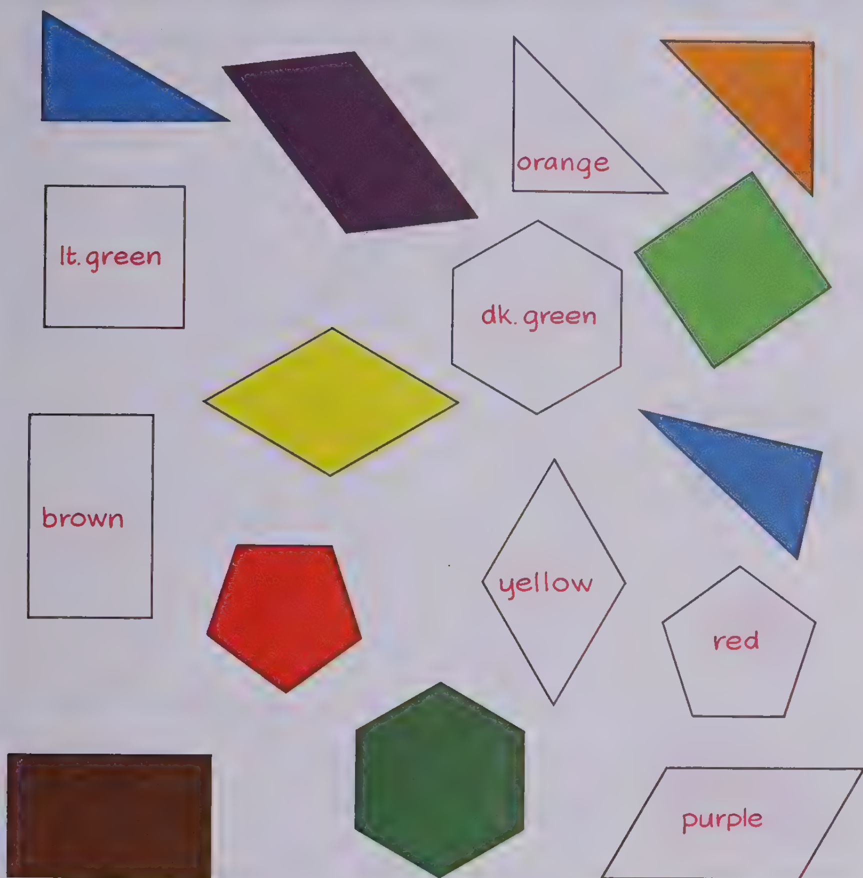
tracing paper

Display for the children a variety of shapes which have three or four sides. Compare pairs of these shapes by holding them on top of one another. Ask children questions such as: "Do they fit one upon the other?" "Do you think they're the same size?" Show children how they might flip, rotate, and slide the pieces over and around each other to see if they fit. Point out that some shapes (for example their small red and blue attribute pieces) fit exactly over each other no matter how they are turned. Point out that other shapes never match at all corners, no matter how they are turned or flipped.

Follow-up

Children often enjoy preparing a bulletin board to show various shapes. You might do this in the form of a graph. For example, show at the bottom of the graph the figure and word for triangle, rectangle, square, and circle. Then have the children identify various objects in the room according to the shapes which they suggest and put the name of each object on a slip of paper. The slips of paper can then be placed over the appropriate terms to form a bargraph. For example, a reading group

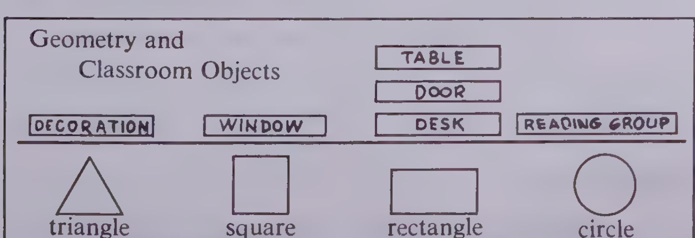
Color the inside of the figures so figures that have the same size and shape have the same color.



TEACHING
Page g-30

Explain to the children that some of the figures on this page have the same size and shape. They are to color the inside of each of these matching figures so that figures that have the same size and shape have the same color. If any children have difficulty finding the congruent figures, suggest that they trace the colored shape and then move the traced copy around on top of the others to find the one it matches.

may make them think of a circle; or a table, door, or desk could make them think of a rectangle, and so on. You can use a graph such as this to have the children consider questions such as: "Which figures seem to be suggested most by the ordinary things around us?" "Which figures did we have a hard time finding suggestions for?" "Which figure seems to be the easiest to find suggestions for?" and so on.



It would also be possible to extend an activity such as this by having children include on the graph not just labels which name the objects, but pictures of objects which they have found in magazines and books which make them think of the specific geometric shapes.

RESOURCES FOR ACTIVE LEARNING

Transformations:

MATHEX: Geometry No. 4, Activity 3, p. 20, Encyclopaedia Britannica Publications Ltd.

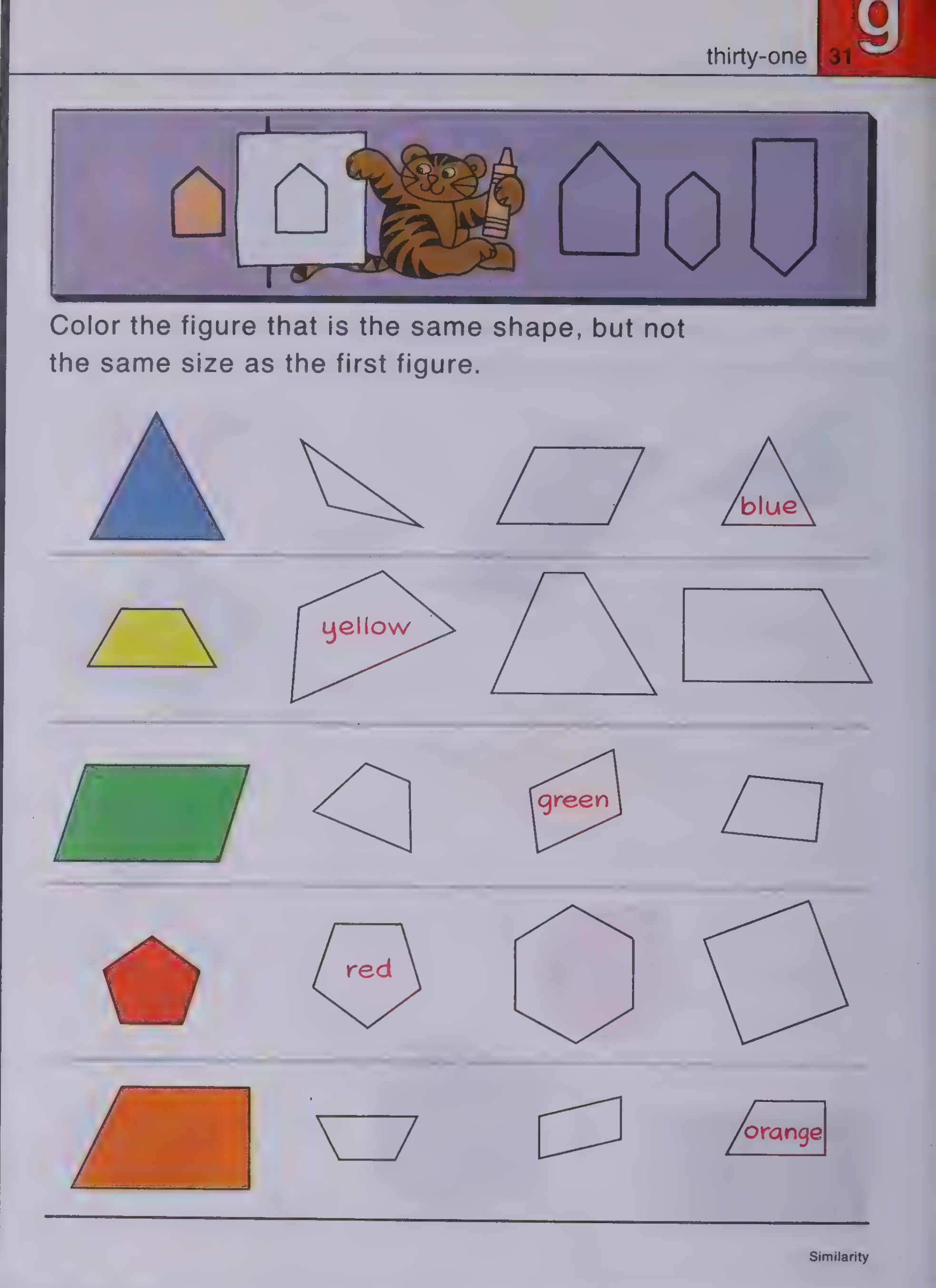
NOTES ON MATHEMATICS IN PRIMARY SCHOOLS, pp. 178–181, Cambridge University Press

Nuffield Project: SHAPE AND SIZE 2, pp. 56–61, Wiley

THINK AND COLOR, pp. 6–23, 93–95, Educational Science Consultants

Teaching
Page g-31

Explain to the children that each row has one figure which is the same shape but not the same size as the first figure in the row. They might want to trace the first figure in the row and place it on top of each of the other figures in that row to find the one with the matching shape. They should then color this same shape figure. As the children work, discuss with them the ways in which the shapes differ.


thirty-one 31 g

Color the figure that is the same shape, but not the same size as the first figure.

Similarity

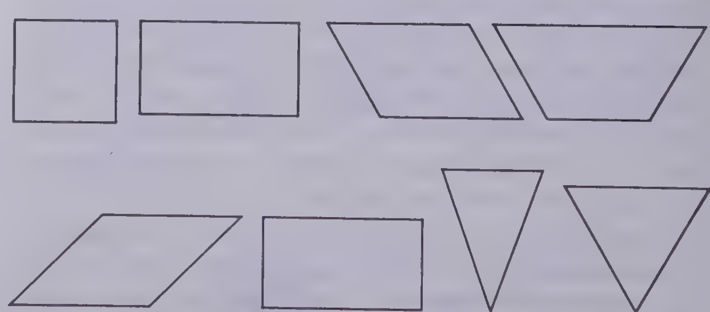
Objective

Give a set of figures of varying sizes, the child will be able to find a figure that is the same shape but not the same size as the given figure.

Pre-Book Activity

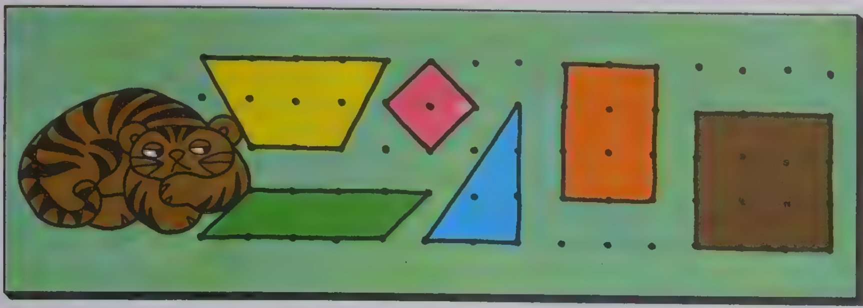
Display for the children various sets of triangles, squares, and other polygonal shapes, which are not similar. Point out various differences in shape, for example compare an equilateral triangle to an isoceles triangle. It is not intended that the terms equilateral and isosceles be used with the children. Simply help them to develop their ability to perceive differences between shapes which have some similarities.

Sample Comparisons:

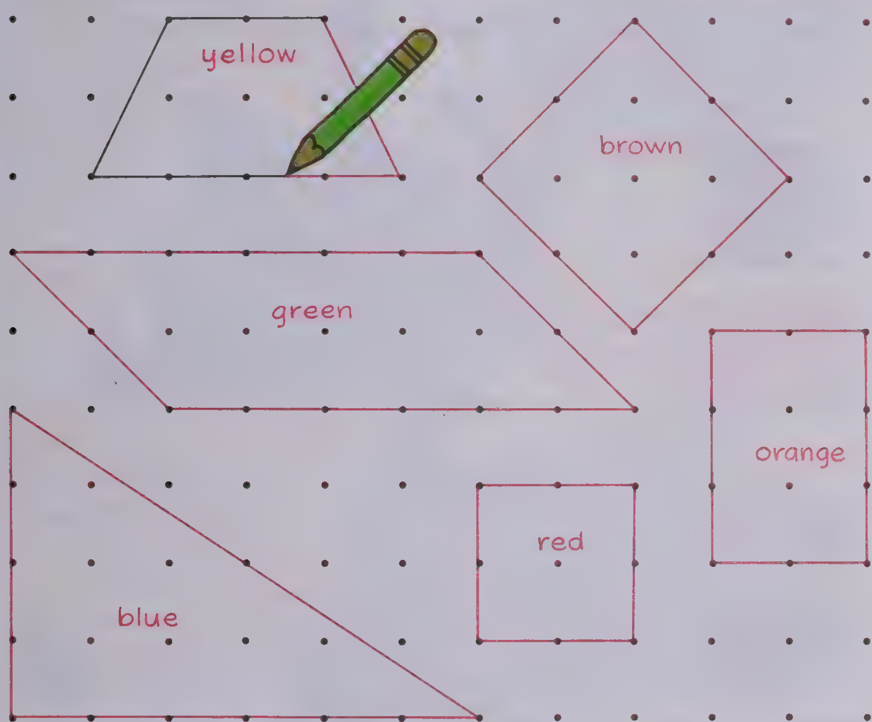


Follow-up

If geoboards are available, you might challenge children with tasks such as the following:



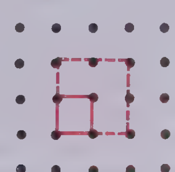
Draw figures that are the same shape as the ones above, but larger. Color figures that are the same shape the same color. Answers will vary. Examples are given.



Teaching
Page g-32

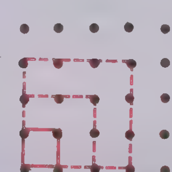
Read the directions with the children. Help them realize that they may use the dots as guides in drawing the shapes. Since the dots at the top of the page are separated by smaller distances they may simply connect the corresponding dots in the main section of the page to form a larger shape. It would be helpful for some children if you would show a transparency of this page on the overhead projector, asking volunteers to draw the larger shapes for all to see.

1) Build two squares (rectangles) so that each side of one is twice as long as each side of the other.



2) Build two triangles so that one side of one triangle is twice as long as a side of the second.

3) Build three rectangles so that the lengths of the top sides could be measured as 1, 2 and 3.



Directions such as these should be given orally to the children.

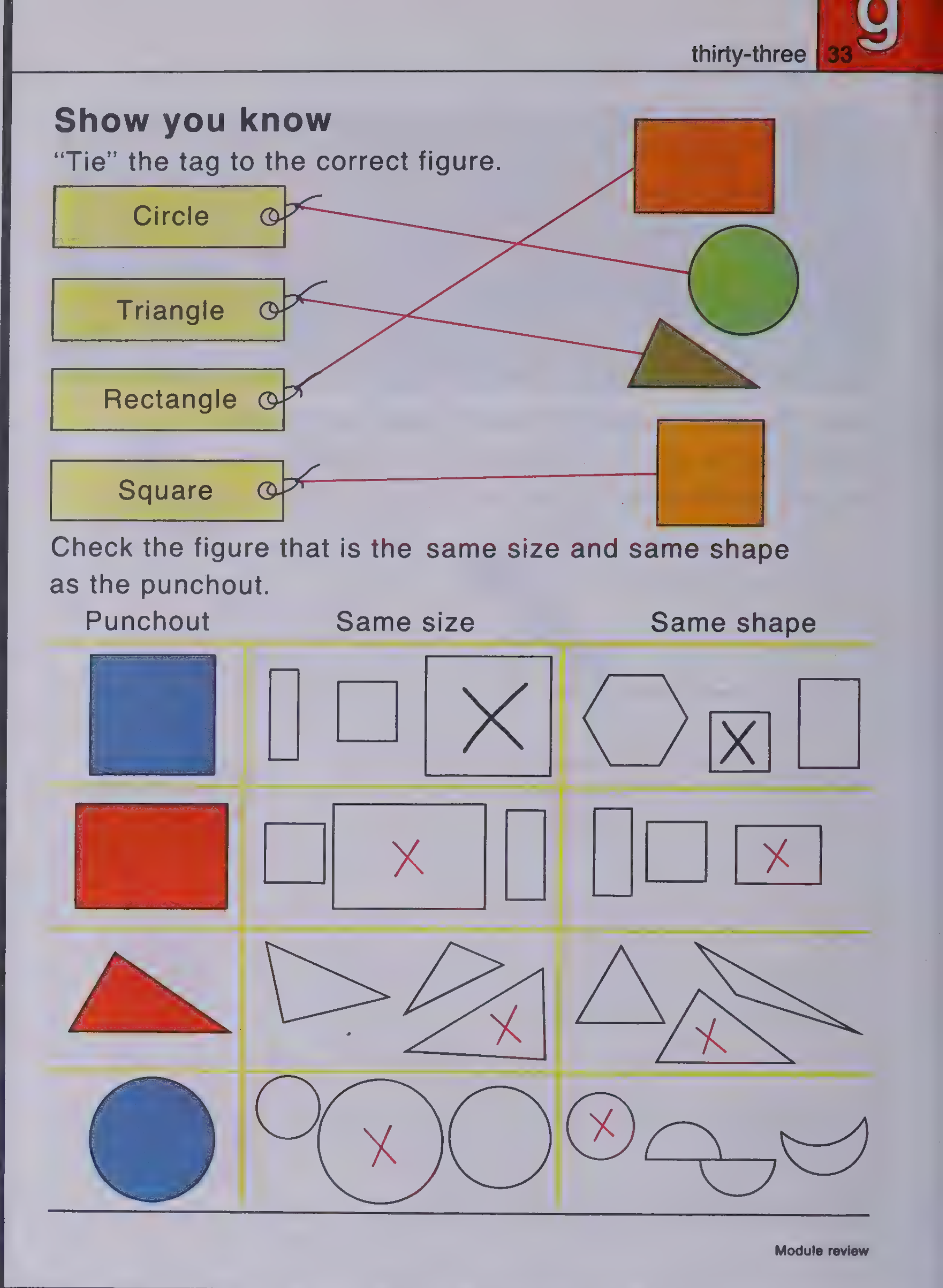
Resources for Active Learning

ACTIVITIES IN GEOMETRY, pp. 62 – 63, Addison-Wesley

NOTES ON MATHEMATICS IN PRIMARY SCHOOLS, "Similarity," pp. 149–156, Cambridge University Press

Teaching
Page g-33

Explain to the children that this page has two sections. In the top they must simply match the figure to the correct term. In the bottom section of the page, the children have two things to do. First, they must compare the attribute piece that is on the left-hand side with the figures in the centre under the column marked "Same size." In this column they should mark that figure which is the same size as their attribute piece. In the column on the right, the figure may not be the same size. They should only mark the figure that has the same shape. It is not necessary that children use the actual attribute pieces to do this page, but many may find it easier if they do so.



Objective

The child will demonstrate his ability to work with the concepts presented in this module.

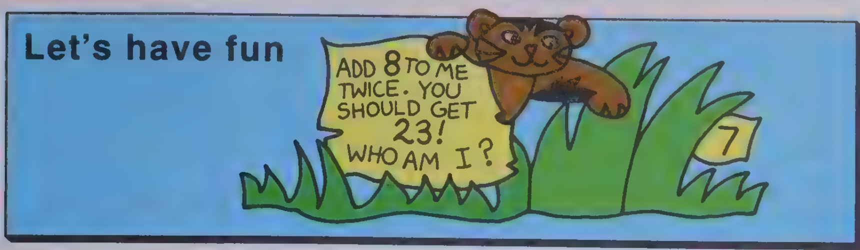
Pre-Book Activity

If necessary review the various shapes with the children or ask the children to take their attribute pieces and separate all of the squares or all of the triangles. That is, use an activity similar to the first investigation in this module as a review for the concepts studied throughout these lessons.

Follow-up

An appropriate follow-up for this module would be to encourage the children to work with other attribute games. For example, suggest that they try to make a train so that every piece is like the piece next to it in one way. Thus, they might put down the red triangle and next to it put a blue triangle. Next to the blue triangle, they might put a blue square because the color is the same. Next to the blue square they might put the large red square because the shape is the same. After children have worked with trains of this type, encourage them to build trains in which the figures next to each other are alike in two ways. For example, the small red triangle might be placed next to the large, red triangle because they are the same in color and shape or the small blue triangle might be placed next to the small red triangle

Let's have fun



Can you find the mystery number?

1. If you add 7 to me, you'll get 15. **Who am I?** 8

2. Add 5 to me. Then take away 2. You'll have 6. **Who am I?** 3

3. Add 4 to me. Then add 6 more. You should get 12. **Who am I?** 2

4. First take away 7. Then add 11. You'll get 20. **Who am I?** 16

5. Add me to myself. Then take away 5. You should get 7. **Who am I?** 6

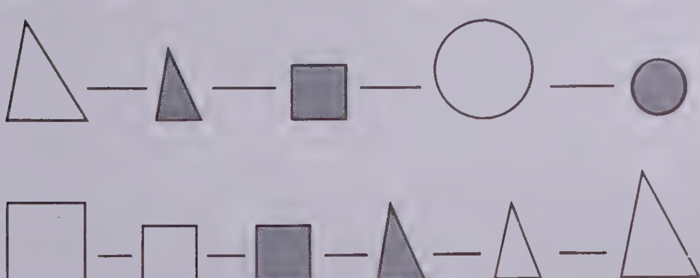
6. Add me to myself. Then add me on again. You better get 18. **Who am I?** 6

Puzzle problems

TEACHING
Page g-34

This change of pace page is a variation for the children as it gives puzzles to review basic facts. Explain to them that they can pretend that each sign is covering a hidden number. By working out each "Who am I?" puzzle they can find the hidden number. Give children an opportunity to share their answers when they have completed the page. Note that these puzzles use the missing addend concept.

because they are alike in size and shape. Then the small blue square might be placed next to the small blue triangle because they are alike in color and size. Children might work in groups of two or three to see if they can build trains of this type.



RESOURCES FOR ACTIVE LEARNING

INQUIRY IN MATHEMATICS VIA THE GEO-BOARD, "How Many Routes?" Geo-cards 6/1–3, Walker

MATHEMATICS IN MODULES: SPATIAL KNOWLEDGE • SK1 • Faces and Plane Shapes, Addison-Wesley

LIGHT GREEN MODULE, UNIT G

Addition and Subtraction Skills to 18 — Work Toward Speed

Pages g-35 to g-48

General Objectives

To provide an opportunity for the child to develop mastery of the combination facts for 18 or less

To review finding differences by thinking of missing addends

To further develop the understanding of related addition and subtraction equations

To motivate the child toward the development of the speed skills for the combinations of 18 or less

This module might be thought of as a bridge between development of power skills and speed skills for the combinations of 18 or less. It provides the child with an opportunity to reinforce his understanding. It also motivates the child to try to master these facts. Keep in mind that an ongoing objective throughout this book should be that children recall quickly the addition and subtraction facts of 18 or less. Some children will have already reached this goal. Others will do so during this module. Still others will need further study as presented in Unit H. The amount of time and emphasis given to this module will greatly depend on the needs of your children; however, most children will benefit from a thorough development of the module and from the practice of the facts provided in it.

Mathematics

The basic mathematics concepts of this module are identical to concepts previously developed for addition and subtraction. The chief difference in this module is that the underlying addition and subtraction concepts are used less in this module than they are in previous modules since the emphasis here is upon developing speed skills. That is, the child is not expected to use the basic mathematical concepts to arrive at answers to addition and subtraction equations. Rather, he is expected to begin mastery of these skills through memorization and through recognition of the fact that differences can be found by thinking of missing addends.

Teaching Light Green Module, Unit G

Approximate Time: 7 to 10 days

Materials

duplicated addition tables
duplicated forms for addition tables
flash cards for the combinations to 18
objects for set demonstrations
set of strips for each child
paper or tagboard squares, 2-by-2-centimetres

Your use of materials throughout this module will depend on the needs of the children. Some children may still depend on the concrete materials. They should be allowed to use these materials, but slowly be encouraged to try to wean themselves away from them. By this time most children will be able to work directly from the printed page which will help to motivate a desire to learn the combinations.

Evaluation of Progress

Since modules will be presented in Unit D to help children reach the ongoing goal of memorizing the combinations of 18 or less, you might help the children assess their ability in this area. Help them to find out which facts they already know and to recognize which facts still need more time and effort. A child's performance on a timed test might help you do this, although some children might not yet be ready for such an activity. Individual interviews with the children or oral quizzes can help you evaluate their mastery of these facts.

Resources for Active Learning

General Activities:

For games to practice basic facts, refer to the Introduction to the orange module in Unit G.

Manipulative Devices:

Cubical counting blocks (Milton Bradley, school supplier)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

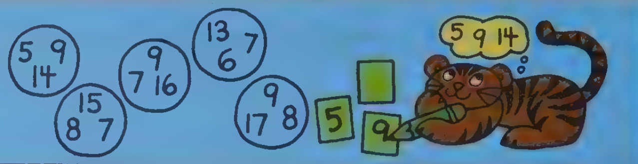
Commercial Games:

Cube puzzles (Creative Publications; Cuisenaire Co.; Edmund Scientific; Selective Educational Equipment)

Investigation
Page g-35

Guide the children through the top part of the page as suggested in the preparation. Explain that they should place their squares on the yellow spaces on page g-35 to make an equation. They should record this equation on the page and then make another equation. Note that the emphasis in this module will be on the development of skill with both addition and subtraction facts to 18; therefore, the attempt here is to memorize one of the harder fact families now. When the children have finished their equations, it would be helpful to have them explain their results to each other. Encourage them to explain which set they chose and to show the four equations which they were able to find using it.

Let's do



Memorize one of these sets of numbers.

8 7 15 | 14 5 9 | 9 7 16 | 13 7 6 | 9 8 17

Put your numbers on papers this size.

Can you place your papers on the yellow spaces below to make equations? Record the equations you find. Answers will vary. Example uses first set.

8 + 7 = 15

7 + 8 = 15

15 − 8 = 7

15 − 7 = 8

Skills—sums and differences less than 18

Purposes

To introduce the addition and subtraction facts to 18

To motivate children toward the ongoing objective of memorizing the facts to 18

Preparation

Materials

three 2-by-2-centimetre squares for each child

The top part of the text page g-35 may be used as the preparation for this lesson. Point out the sets of numbers at the top of the page and explain to the children that they should memorize one of these sets of numbers in order to play a memory game. After the children have memorized one set of numbers and have written each of the numbers in the set on a 2-by-2-centimetre square, ask them to put the squares back on their desk. Call out two numbers and ask someone whose set contains those numbers to name the third number. Call out another two numbers such as 7 and 16 and ask a child to name the third number in his set. After the children catch on, you might simply call out one number in a set and have children respond with two numbers. For example, if you call out the number 8, a child might respond 9 and 17, or 7 and 15. It would not be necessary to mention addition or subtraction at this time, although with some groups of children you may wish to.

Let's talk

Can you solve Sue's equations?

$15 - 7 = \boxed{8}$

$15 - 8 = \boxed{7}$

Can you find each of these differences?

$9 + 7 = 16$

$16 - 9 = \boxed{7}$ $\quad$ $16 - 7 = \boxed{9}$

$4 + 8 = 12$

$12 - 4 = \boxed{8}$ $\quad$ $12 - 8 = \boxed{4}$

$8 + 6 = 14$

$14 - 8 = \boxed{6}$ $\quad$ $14 - 6 = \boxed{8}$

$4 + 9 = 13$

$13 - 4 = \boxed{9}$ $\quad$ $13 - 9 = \boxed{4}$

Discussion
Page g-36

The material on this page should be used to emphasize the relation between addition and subtraction. One of the most important points developed throughout the module is that children can think of finding differences by thinking about addition. As you discuss this page, you might stress the idea of a family of facts. Just as the children were able to find four equations for one set of numbers on the investigation page, here each set of numbers may be used in two addition and two related subtraction equations. Children who still have difficulty finding the sums and differences to 18 should be allowed the use of counters, number line, and strips; but, in general, children should be discouraged from using such aids and strive to work toward mastery of the facts. Even though children might talk about the power skills which might be used to find the differences, stress that because they know an addition fact, they will be able to find a subtraction fact without depending on concrete materials. Thus, their mastery of facts need only include the addition facts. Since it would be time consuming to develop each sum using a power skill, stress that it is important that they *begin* to memorize the addition facts and learn the method of using the missing addend as Sue is doing at the top of page g-36.

Follow-up

It would be helpful at this time to encourage children to make flash cards for each of the sums to 18. If such flash cards have already been developed for the yellow and red modules of Unit F, encourage the children to work in pairs and use the flash cards to help them master these facts.

To help children understand the equation families for a given set, duplicate an exercise similar to the one in following column.

Write 2 addition and 2 subtraction equations.

× × × × × × × ×

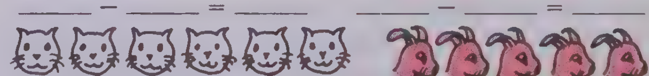
8 + 6 = 14
14 − 6 = 8

× × × × × ×

6 + 8 = 14
14 − 8 = 6

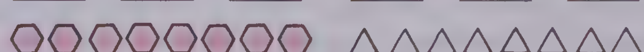
____ + ____ = ____
____ − ____ = ____

____ + ____ = ____
____ − ____ = ____



____ + ____ = ____
____ − ____ = ____

____ + ____ = ____
____ − ____ = ____



____ + ____ = ____
____ − ____ = ____

____ + ____ = ____
____ − ____ = ____

Teaching
Page g-37

Discuss the illustration at the top of the page and have children explain how knowing the fact $7 + 7 = 14$ has helped the tiger to find the fact $7 + 8 = 15$. Point out that since 1 has been added to one of the addends, 1 must be added to the sum. Thus, the new sum must be 1 greater than the sum of the double, 14. Note that in the last three examples, two has been added to one of the addends so the new sum has been increased by two. Although this type of logical reasoning to find sums from facts that we already know is a power skill, such a page is intended as a lead-in to speed skill development. Keep in mind that actual memorization of the facts is an ongoing objective. If some children still need counters, number lines or the use of the strips to find their sums, allow them to use them, but encourage them to try to master the addition facts.

Solve the equations.

Since $5 + 5 = 10$, I know $5 + 6 = \boxed{11}$.

Since $7 + 7 = 14$, I know $7 + 6 = \boxed{13}$.

Since $8 + 8 = 16$, I know $8 + 9 = \boxed{17}$.

Since $6 + 6 = 12$, I know $6 + 7 = \boxed{13}$.

Since $5 + 5 = 10$, I know $5 + 6 = \boxed{11}$.

Since $9 + 9 = 18$, I know $9 + 8 = \boxed{17}$.

Since $8 + 8 = 16$, I know $8 + 7 = \boxed{15}$.

Since $6 + 6 = 12$, I know $6 + 8 = \boxed{14}$.

Since $5 + 5 = 10$, I know $5 + 7 = \boxed{12}$.

Since $7 + 7 = 14$, I know $7 + 9 = \boxed{16}$.

Objective

Given an addition fact which is the double of some number such as $7 + 7 = 14$, the child will be able to use it to find sums such as $7 + 6$ and $7 + 8$.

Pre-Book Activity

As a pre-book activity for this lesson it would be helpful to review the doubles for sums to 18. If some children still choose to use concrete materials, have them find two strips which match a train. For example, ask them to build the number 14 with the 10-strip and the 4 strip, and then to find two strips of the same color which match this train. Most children should be able to review the doubles with an oral review, such as "What's My Number?" Say: "I'm thinking of a number which when added to itself gives me ten. What's my number?" or "I'm thinking of a number which if I add it to itself gives me 12. What's my number?" Or you might simply review what we mean when we speak of doubles, such as $7 + 7 = 14$. You might ask questions such as, "What is the double of 8?" or "What number when doubled is 16?" Throughout this review, you might also stress that numbers such as 17, 15, 13 are not the doubles of any number. That is, only the even numbers 10, 12, 14, 16, and 18 are doubles of other numbers.

Solve the equations.

7 + 5 = 12	4 + 8 = 12
7 + 7 = 14	9 + 5 = 14
6 + 4 = 10	3 + 8 = 11
6 + 7 = 13	8 + 8 = 16
9 + 6 = 15	4 + 9 = 13
5 + 6 = 11	7 + 4 = 11
6 + 8 = 14	9 + 3 = 12
8 + 5 = 13	7 + 8 = 15

Complete the tables.

Add 5	
6	11
4	9
7	12
9	14

Add 3	
9	12
7	10
8	11
6	9

Add 6	
7	13
8	14
6	12
9	15

Practice finding sums

TEACHING
Page g-38

This might be a good chance for the children to test their skills with the addition facts to 18. Encourage them to work the top of the page independently. Then they can check their answers using any method they prefer: either the doubles method or any of the power skill methods studied previously. You might also challenge the children to work these exercises as fast as they can, but be sure they understand that the main purpose of this activity is to see whether they have to use a power skill or if they already know some of the facts from memory. As the children see how quickly they are able to find the sums, help them realize that it is just as important to get the correct answer as it is to be able to find the sums quickly. However, they should also understand that the sooner they are able to memorize the facts, the sooner they will be able to compute with improved speed. A later module in Unit H will put stronger emphasis on speed so do not overemphasize speed at this time. Give directions as needed so children know how to complete the tables at the bottom of the page.

FOLLOW-UP

Some children might be aided by the use of an addition table. Duplicate blank addition tables. Give children opportunity to complete them using any power skill they choose. Then encourage them to use the table while they work through the module.

For more practice with the "doubles," direct the children to copy exercises like those in the following column from the chalkboard or overhead projector.

Find the sums.

2	3	4	5	6	7	8	9
+2	+3	+4	+5	+6	+7	+8	+9
2	3	4	5	6	7	8	9
+3	+4	+5	+6	+7	+8	+9	+8

Teaching
Page g-39

Explain to the children that at the top of the page they should find the sums for each of the addition equations. At the bottom of the page, the sum is given but one of the addends is missing. Point out the dashed numeral on the first missing addend leaf. The missing addend should be thought of as a covered number in these equations. Remind them of the relation between addition and subtraction facts as shown in the demonstration art. You might point out to the children that the equations at the bottom of the page are related to equations that they worked at the top of the page; however, they should be able to find the missing addend without specifically finding the addition equation at the top of the page. Note that the goal here is to move toward the speed skill; however, if a child is not ready for work without materials, use the suggestion mentioned in the follow-up to help him use concrete materials.

Find these sums.

8 + 4 = 12	9 + 3 = 12
3 + 8 = 11	8 + 6 = 14
7 + 8 = 15	7 + 4 = 11
9 + 2 = 11	6 + 7 = 13
8 + 5 = 13	9 + 7 = 16

Now find the "hidden" addend.

8 + 5 = 13	3 + 8 = 11
7 + 8 = 15	6 + 7 = 13
7 + 4 = 11	8 + 4 = 12
9 + 3 = 12	9 + 2 = 11
8 + 6 = 14	9 + 7 = 16

Sums and missing addends

Objective

Given an addition equation with one missing addend, the child will be able to find the missing addend.

Pre-Book Activity

Materials

18 counters
a small bag

Display a bag in which you have put an unknown number of counters. For example, put eight counters in the bag without letting the children know how many you have put in it. Ask a child to come up and place five counters into the bag. Then ask another child to count the total number of counters in the bag. Write an equation to show the situation on the chalkboard. For example, $\square + 5 = 13$. Then have the children talk about what number of counters must have been in the bag before they added the 5. Work through several examples of this type. Point out to the children that to aid them in finding the number of counters that were in the bag in the first place, they need simply think of the addition fact needed to arrive at the sum or the total number of counters. Thus, in order to find the solution or missing addend for $\square + 5 = 13$, if they know from memory that $8 + 5 = 13$, then they can simply identify the missing addend from their known fact. From the demonstration you might proceed to writing equations that have missing addends on the chalkboard without actually using the counters in the bags. For example, have a child write an equation on the chalkboard, but have him cover one of the addends that he has written. Then have another child try to guess

Find the sums.

$4 + 8 = \boxed{12}$	$8 + 5 = \boxed{13}$
$7 + 6 = \boxed{13}$	$6 + 8 = \boxed{14}$
$6 + 5 = \boxed{11}$	$3 + 8 = \boxed{11}$
$8 + 8 = \boxed{16}$	$1 + 9 = \boxed{10}$
$5 + 9 = \boxed{14}$	$9 + 3 = \boxed{12}$
$8 + 9 = \boxed{17}$	$8 + 7 = \boxed{15}$

Solve the equations.

$\boxed{8} + 5 = 13$	$\boxed{9} + 3 = 12$
$\boxed{8} + 7 = 15$	$\boxed{5} + 9 = 14$
$\boxed{3} + 8 = 11$	$\boxed{8} + 9 = 17$
$\boxed{6} + 8 = 14$	$\boxed{6} + 5 = 11$
$\boxed{1} + 9 = 10$	$\boxed{7} + 6 = 13$
$\boxed{4} + 8 = 12$	$\boxed{8} + 8 = 16$

Teaching
Page g-40

The equations at the top of the page are related to the equations at the bottom of the page in which children should find the missing addends. However, it is not necessary to point this out for most children. They should be able to work the equations with the missing addends simply by recalling the addition fact related to each equation. Be sure the children understand that in the top section they are to find the sums and in the bottom section, they are not to find the sum of the numbers, but to fill in the missing addend. Check the answers with the children and, if necessary, work through some equations using the bags and counters suggested in the pre-book activity.

the number which is covered. Again stress that if the addition fact is known, they can simply identify the missing addend.

Follow-up

Strips may be used to reinforce the concepts of missing addend.

A follow-up for the more capable children would be to give them time to have a flash-card day. For example, ask the children in adjoining rows to be partners. One member of each pair holds the cards and allows the other to answer. To score the game, count the number of correct answers given without hesitation. These scores might be recorded on a chart under the date and various flash-card days may be scheduled periodically to encourage the children to improve their skills and observe their own progress.

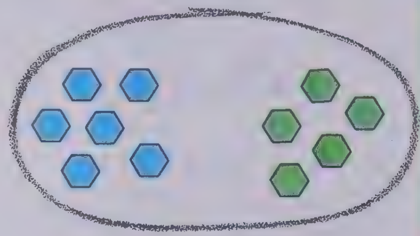
Teaching
Page g-41

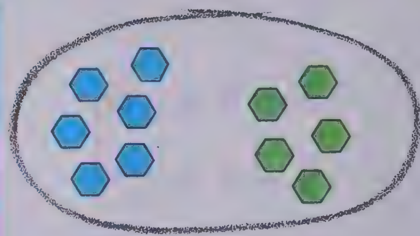
Use the illustration at the top of the page to introduce related addition and subtraction equations. You might also use the box of donuts to explain that a dozen is a group of 12. Then call attention to the first set at the top of the page. Ask the children to give the number of each of the colored subsets, and the number of the total set. Point out the relationship between the equations and the set.

Move to the second set, and again ask the children to give the total number in the set, and in the subsets. Remind them that they can think of separating the total set into two subsets. Ask how many would be left in the ring if the green set of 5 were taken away. Relate the equation to the set picture. Ask for the difference $11 - 5$, and have the children write 6 in the answer box provided. Emphasize that the set pictures are the same and that thinking about the addition combination $6 + 5$ will help them find the difference $11 - 5$.

Then direct the children to complete the page. You might suggest that they draw a set as an aid in solving each subtraction equation, but encourage children to use the related addition facts as their aid.

Solve the equations.





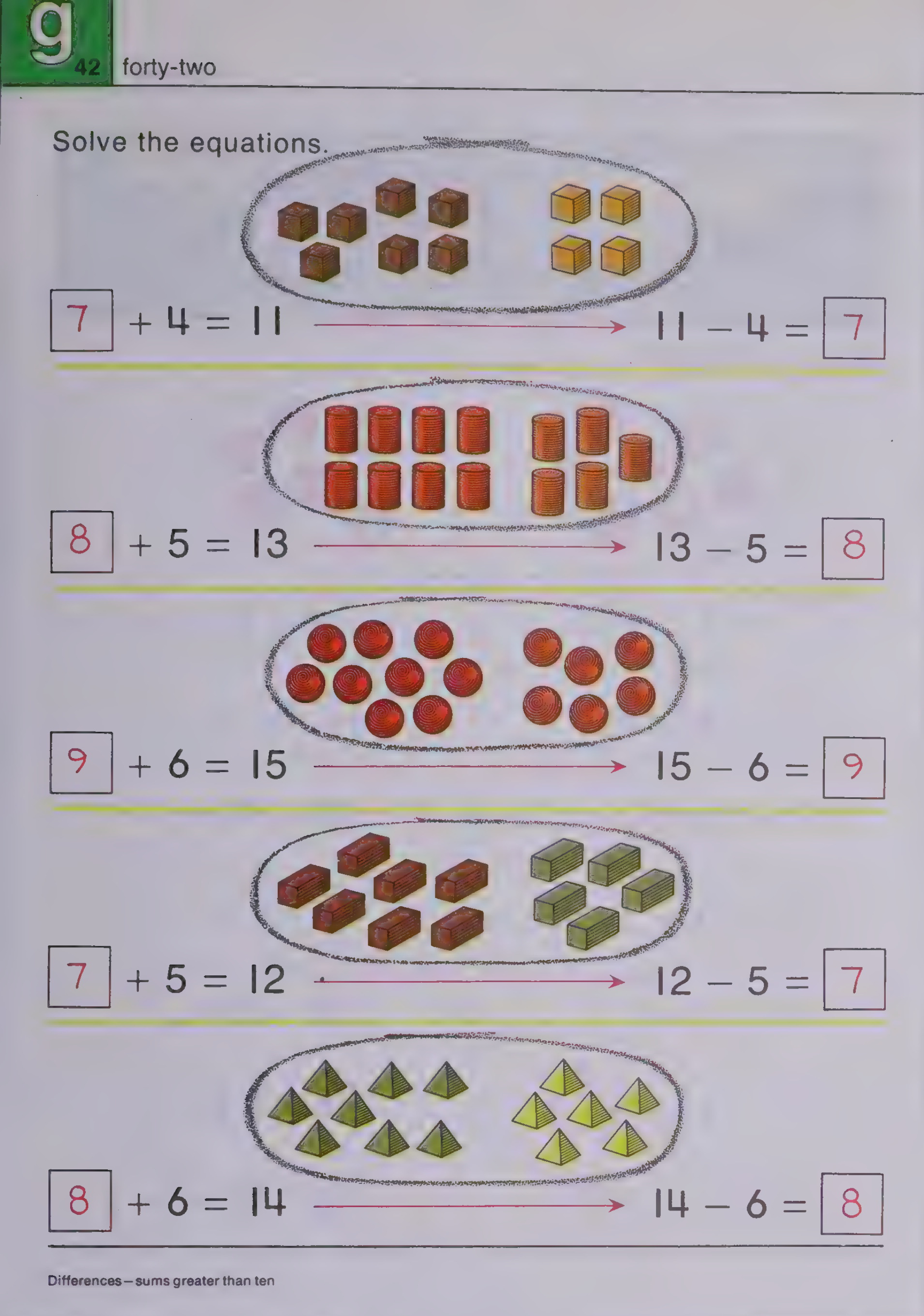
$6 + 5 = 11$	$11 - 5 = \boxed{6}$
$7 + 6 = 13$	$8 + 7 = 15$
$13 - 6 = \boxed{7}$	$15 - 7 = \boxed{8}$
$6 + 8 = 14$	$9 + 3 = 12$
$14 - 8 = \boxed{6}$	$12 - 3 = \boxed{9}$
$8 + 8 = 16$	$8 + 9 = 17$
$16 - 8 = \boxed{8}$	$17 - 9 = \boxed{8}$

Objective

Given an addition equation with a missing addend, a child will be able to find the missing addend and use it as the difference in a related subtraction equation.

Pre-Book Activity

According to the needs of the children, you might wish to prepare for pages g-41 and g-42 with two separate pre-book activities. For example, give a brief demonstration to illustrate the inverse relation between addition and subtraction: Exhibit a set of 9 objects. Ask the children to identify the number of this set. Then show a set of 5 objects and place these with the set of 9. Now ask the children to identify the total number. Write the equation $9 + 5 = 14$ on the chalkboard. Remove 5 objects from the set and write the equation $14 - 5 = 9$. Continue giving related equation pairs. Then, to extend this demonstration to include an addition equation with a missing addend, display a set of 13, grouped as 7 and 6, in the centre of the flannelboard. To the left place the equation $\square + 6 = 13$, and ask: "If one subset has 6 objects, how many are in the other set?" Complete the equation by placing a 7 in the placeholder box and saying that $7 + 6 = 13$. Next display the equation $13 - 6 = \square$ to the right of the set. Say, "We have 13 in the set. If I remove 6, how many are left?" Remove 6 objects as you are talking. After a child correctly answers that 7 remain, put the numeral in the placeholder box and repeat the equation $13 - 6 = 7$. Emphasize the process by saying: "If I have 7 and add 6, I get 13. If I then subtract 6, I will have the 7 I started with." Work through several such examples. Remember to begin with the


g 42 forty-two

Solve the equations.

$\boxed{7} + 4 = 11 \longrightarrow 11 - 4 = \boxed{7}$

$\boxed{8} + 5 = 13 \longrightarrow 13 - 5 = \boxed{8}$

$\boxed{9} + 6 = 15 \longrightarrow 15 - 6 = \boxed{9}$

$\boxed{7} + 5 = 12 \longrightarrow 12 - 5 = \boxed{7}$

$\boxed{8} + 6 = 14 \longrightarrow 14 - 6 = \boxed{8}$

Differences—sums greater than ten

Teaching
Page g-42

Call attention to the set in the first frame. Emphasize that the subsets help show that a difference can be found by finding a missing addend. Ask the children to identify both subsets and to determine the number in the entire set. Then direct the children to count the number of objects in the first subset and to complete the first equation by writing 7 in the placeholder box. Relate the subtraction equation to the picture of the set by explaining that there are 11 in all. Then ask what the difference of 11 − 4 is. Observe that it is 7 and that in finding the difference, they are really finding a missing part of a total set.

If the children are capable, direct them to complete the page by themselves. If they require more help, continue working through the rest of the page with them. Handling sets on a flannelboard may help some children understand these inverse relations.

placeholder in the addition equation and help the children to realize that because they know the addition fact they can easily find the difference in the related subtraction equation.

Follow-up

Adapt the matching game suggested on page 45 to treatment of related facts. Use small index cards to make 15 addition combinations and 15 related subtraction combinations. Use these for the matching game.

To determine the children's understanding of addition and its inverse, prepare a work sheet similar to the one in the following column.

Write + or − in each ○ to make a true equation.	
15 = 9 ○ 6	8 ○ 7 = 15
15 ○ 9 = 6	8 = 7 ○ 1
12 = 7 ○ 5	4 ○ 9 = 13
12 ○ 7 = 5	7 = 13 ○ 6
13 = 9 ○ 4	6 ○ 7 = 13
13 ○ 9 = 4	13 = 7 ○ 6
14 = 8 ○ 6	13 ○ 6 = 7
14 ○ 8 = 6	9 ○ 6 = 15

Resources for Active Learning

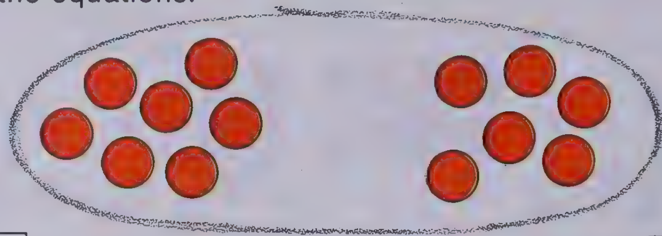
MATHEMATICS IN MODULES: ALGEBRAIC RELATIONS • AR1 • Open Sentences and Truth Sets, Addison-Wesley

Teaching
Page g-43

Again use the demonstration art to introduce the page. Then call attention to the set at the top of the page and to the pair of equations just below it. Ask for a volunteer to give the number of objects in the entire set and in each of the two subsets. Point out the relationship between finding the missing addend and finding the difference. Ask a child to give the number of orange circles on the left and then elicit from them that this is the solution to each equation.

Encourage the children to complete the page independently, if possible. If some children need more help, work through several examples with them until they feel that they can finish the page by themselves.

Solve the equations.



$\boxed{7} + 6 = 13$ $\quad$ $13 - 6 = \boxed{7}$

$\boxed{6} + 6 = 12$ $\quad$ $\boxed{6} + 4 = 10$

$12 - 6 = \boxed{6}$ $\quad$ $10 - 4 = \boxed{6}$

$\boxed{7} + 4 = 11$ $\quad$ $\boxed{7} + 5 = 12$

$11 - 4 = \boxed{7}$ $\quad$ $12 - 5 = \boxed{7}$

$\boxed{7} + 7 = 14$ $\quad$ $\boxed{9} + 6 = 15$

$14 - 7 = \boxed{7}$ $\quad$ $15 - 6 = \boxed{9}$

Objective

Given an addition equation with a missing addend and a related subtraction equation, the child will be able to find the difference for the subtraction equation by finding the missing addend of the addition equation.

Pre-Book Activity

It would be helpful to play an oral warm-up game with the children. For example, start with the doubles $4 + 4 = 8$, and the child should respond $8 - 4 = 4$. Or give, $5 + 5 = 10$, and the child should respond $10 - 5 = 5$. Once the children have caught on to the response, you might divide the class into two teams so that one child on the first team must give a correct addition equation and then a child on another team should respond with the related subtraction equation. You might want to have addition equations with missing addends printed on cards so that a child from the first team would pick up a card showing a missing addend equation. He should complete the equation. Then the second child should respond with the related subtraction equation and give the difference.

Solve the equations.

$\boxed{8} + 6 = 14$ $14 - 6 = \boxed{8}$	$\boxed{5} + 6 = 11$ $11 - 6 = \boxed{5}$
$\boxed{9} + 9 = 18$ $18 - 9 = \boxed{9}$	$\boxed{9} + 3 = 12$ $12 - 3 = \boxed{9}$
$\boxed{8} + 7 = 15$ $15 - 7 = \boxed{8}$	$\boxed{8} + 8 = 16$ $16 - 8 = \boxed{8}$
$\boxed{8} + 5 = 13$ $13 - 5 = \boxed{8}$	$\boxed{9} + 8 = 17$ $17 - 8 = \boxed{9}$
$\boxed{4} + 8 = 12$ $12 - 8 = \boxed{4}$	$\boxed{7} + 7 = 14$ $14 - 7 = \boxed{7}$

Teaching
Page g-44

Since page g-44 is simply an extension of the exercises on page g-43, the children should not have difficulty in understanding what to do. Encourage them to work these exercises independently as they did those on page g-43; however, if necessary, work examples with children who have difficulty. Any child who still is dependent on concrete materials might benefit from using the strips or individual counters.

Follow-up

Use a review sheet similar to this one to check the children's progress in learning combinations greater than 13.

Match the sums.

8 + 9	
5 + 10	
7 + 7	14
9 + 7	15
7 + 8	16
5 + 9	17
10 + 6	18

As a free time or enrichment activity, encourage the children to create number riddles. For example:

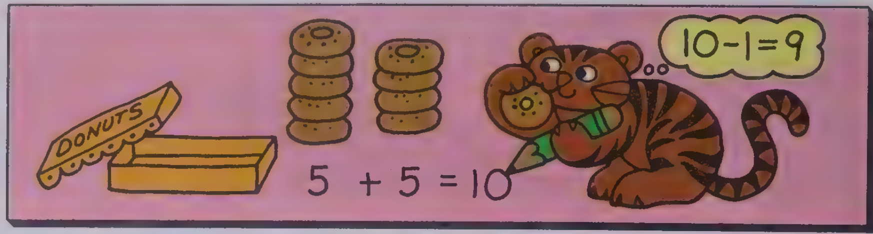
> If you add me to 10, the sum is 16. If you subtract me from 10, the difference is 4. What number am I?

More capable children may give more difficult clues such as:

> I am less than 20. I am greater than 10. I am an even number. If you subtract 4, the difference is 10. What number am I?

TEACHING
Page g-45

Point out that the equations in the illustration are simply examples of finding sums and differences. Read the directions at the top of the page with the children and instruct them to find the sums and differences. Caution them to look at the operation signs carefully. Mention that the addition exercises are in the left frame and the subtraction exercises are on the right. Encourage the children to work as many of these as possible by relying on their memory of the basic addition facts. Then carefully give directions for the lower half of the page. Explain that in the centre section they should complete each table by following the rule at the top of the table and either adding or subtracting the number indicated. Finally, at the bottom of the page, they should write a fact family for the set that is shown. Two equations which they should trace over and complete have been begun for them.



Find the sums and differences.

$6 + 5 = 11$	$7 + 7 = 14$	$8 + 5 = 13$	$12 - 5 = 7$	$13 - 7 = 6$	$11 - 6 = 5$
$4 + 8 = 12$	$8 + 7 = 15$	$9 + 8 = 17$	$12 - 6 = 6$	$11 - 4 = 7$	$14 - 7 = 7$

Complete each table.

Add 5	
6	11
7	12
8	13

Subtract 5	
11	6
12	7
13	8

Subtract 3	
10	7
11	8
12	9

Write 4 different equations.

7 + 5 = 12

12 − 5 = 7

5 + 7 = 12

12 − 7 = 5

OBJECTIVE

Given addition or subtraction exercises related to the sums through 18, the child will be able to find the sum or the difference.

PRE-BOOK ACTIVITY

Provide an oral warm-up using addition and subtraction combinations. For example, you might play "What's My Rule?" Ask a child to give you two numbers less than 10. Respond by naming their sum. Continue asking other children for two numbers less than 10 and responding with their sum. Remind the children to fold their arms as soon as they know your rule. When you call on a child whose arms are folded, he should ask you to give two numbers less than 10, and respond with their sum. If he does this correctly, say: "You're right. Don't tell anyone what my rule is." After most of the children have their arms folded, say "new game," and change the rule. If you change to subtraction, ask for two numbers less than 18, and subtract the smaller from the larger.

Harry the Hippopotamus

Harry the hippopotamus is the biggest, fattest, slowest hippopotamus in the river. But Harry is nice and everyone likes him. He has more friends than just about anyone else. You can find out more about Harry and his friends in these problems.

1. Harry weighs 6 metric tons. His best friend weighs only 5. How much do they weigh together? 11 tons
2. One day Harry moved only 4 metres. The next day he moved 8 metres. How far both days? 12 metres
3. Harry ate 6 trees for lunch and 7 for dinner. How many trees did he eat? 13
4. Harry has 15 crocodile friends. He has 8 monkey friends. How many more crocodiles? 7
5. When Harry falls into the river he splashes out enough water to fill 7 swimming pools. How much does he splash out in two falls? 14 pools of water
6. Harry sunbathed for 7 hours. This made him so tired he slept for 8 hours. How long in all? 15 hours
7. Harry is 13 years old. His favorite lion friend is 9. How much older is Harry? 4 years

Story problems

Teaching
Page g-46

Since this page involves a good deal of reading, you may wish to read the problems with the children, or to assign the children into groups according to their reading ability so that some groups will be able to work the problems independently. Even if you read the problems with the children, encourage them to think through the arithmetic procedures by themselves. It would be helpful to show the equation for each problem as the children discuss it. For those who do the problems independently, be sure that they are given an opportunity to share their solutions for the problems. You might encourage more capable children to make up other problems which deal with Harry the Hippopotamus, or encourage them to invent another animal character and make up a set of problems for him.

Follow-up

Test the 100 basic addition facts on duplicated sets of addition tables similar to these:

Write in the sums.

+	1	9	2	7	5
4					
6					
0					
8					
3					

+	4	6	0	8	3
4					
6					
0					
8					
3					

+	4	6	0	8	3
1					
9					
2					
7					
5					

+	1	9	2	7	5
1					
9					
2					
7					
5					

To help children having a difficult time learning higher combinations, prepare a review sheet showing patterns, like this:

Find the differences.

$12 - 2$	$12 - 3$	$12 - 4$	$12 - 5$	$12 - 6$	$12 - 7$	$12 - 8$	$12 - 9$
$13 - 3$	$13 - 4$	$13 - 5$	$13 - 6$	$13 - 7$	$13 - 8$	$13 - 9$	$13 - 10$
$14 - 4$	$14 - 5$	$14 - 6$	$14 - 7$	$14 - 8$	$14 - 9$		
$15 - 5$	$15 - 6$	$15 - 7$	$15 - 8$	$15 - 9$			
$16 - 6$	$16 - 7$	$16 - 8$	$16 - 9$				

Teaching
Page g-47

Since this is a review page, you might allow the children to work the page independently and use it as an evaluative instrument. Explain to the children that in the first frames they should simply find the sums of the equations. In the second and third sections, they should find the missing addend and then by thinking of the missing addend, find the difference in the related subtraction equation. In the fourth section, they should find the sums or the differences according to the signs indicated. Finally in the last section of the page, they should read and solve the problems. Help any child who has difficulty reading the problems, but encourage him to think through the solution independently. Be sure to check the page with the children. Work through any difficult spots and explain them carefully.

Show you know

Solve.

$6 + 6 = \boxed{12}$ $7 + 7 = \boxed{14}$

$7 + 5 = \boxed{12}$ $9 + 6 = \boxed{15}$

$4 + 9 = \boxed{13}$ $5 + 6 = \boxed{11}$

$\boxed{5} + 7 = 12$ $\boxed{7} + 6 = 13$

$12 - 7 = \boxed{5}$ $13 - 6 = \boxed{7}$

$\boxed{7} + 4 = 11$ $\boxed{7} + 7 = 14$

$11 - 4 = \boxed{7}$ $14 - 7 = \boxed{7}$

$\begin{array}{r} 5 \\ +8 \\ \hline 13 \end{array}$ $\begin{array}{r} 8 \\ +3 \\ \hline 11 \end{array}$ $\begin{array}{r} 5 \\ +9 \\ \hline 14 \end{array}$ $\begin{array}{r} 12 \\ -4 \\ \hline 8 \end{array}$ $\begin{array}{r} 13 \\ -4 \\ \hline 9 \end{array}$ $\begin{array}{r} 11 \\ -7 \\ \hline 4 \end{array}$

$\begin{array}{r} 9 \\ +6 \\ \hline 15 \end{array}$ $\begin{array}{r} 7 \\ +5 \\ \hline 12 \end{array}$ $\begin{array}{r} 8 \\ +8 \\ \hline 16 \end{array}$ $\begin{array}{r} 14 \\ -6 \\ \hline 8 \end{array}$ $\begin{array}{r} 12 \\ -9 \\ \hline 3 \end{array}$ $\begin{array}{r} 15 \\ -6 \\ \hline 9 \end{array}$

Tom's dog had 13 fleas.
Ann's dog had 9.
How many more fleas did Tom's dog have. 4

Tom's dog scratched 8 times an hour.
Ann's scratched 6.
How many scratches? 14

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

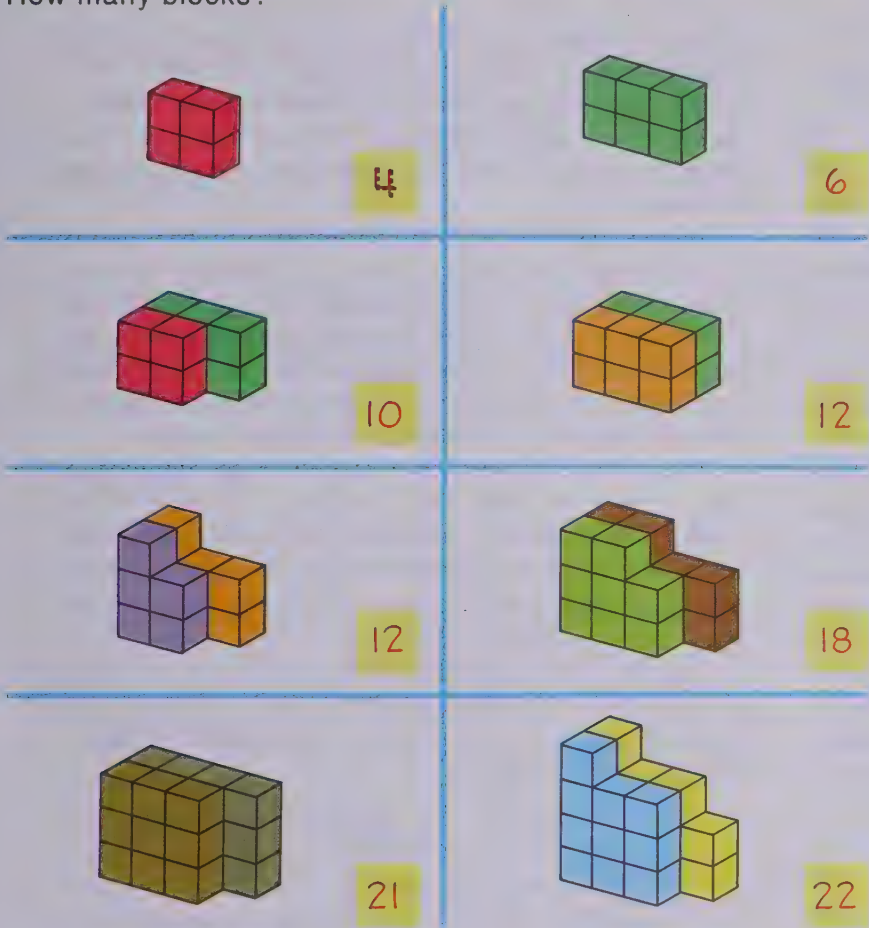
To prepare for this lesson, give the children an oral review of addition and subtraction facts through 18, with particular stress on finding differences by thinking of missing addends. For example, you might use a chain reaction review. Ask a child to give the sum 7 + 9. If he answers correctly, he may ask another child to give a sum or a difference and so on around the room. Alternatively, you might use a chain reaction in which the children work a problem as you call it out step by step. For example, you might say: "Begin with 5, add 7, subtract 3. What's your answer?" (9) Or, "Begin with 15, subtract 6, subtract 6 more. What's your answer?" (3)

Follow-up

To provide children with further practice with sums, duplicate a worksheet such as the one in the next column.

Let's have fun

How many blocks?



Teaching
Page g-48

Since this is a change of pace page, it should be treated with a light touch. Explain to the children that they are to count and figure out how many blocks there are in each frame and write the number in the yellow box provided. To help the children count the unseen blocks, you might explain that in each stack the partially hidden layer has at least as many as the full view layer. It would also be helpful to have the children build the actual stacks after they have tried to count the number of blocks from the pictorial illustration. If children have difficulty, you might want them to build the stacks with the blocks *before* they count how many there are.

Find the missing numbers.

5	+7	12	−3	9	−4	5	+6	11
15	−6	9	−6	3	+3	6	−6	0

Put + or − to make each equation true.

6 ____ 5 = 11   10 ____ 2 = 12

6 ____ 5 = 1   10 ____ 2 = 8

Children might also enjoy the game of "Roll-a-pair." A pair of wooden or cardboard cubes, with faces two to three centimetres square, which have the numerals 4 through 9 painted on each cube, can be used to practice higher combinations. "Roll-a-pair" is played by a group of two, three, or four children. One child in each group rolls the cubes. He then quietly calls out the combination and chooses someone in his group to give the correct sum.

To practice subtraction a third cube having numerals for 12 to 18 should be made to be used with one cube of the original pair. The child throwing the cubes calls the subtraction phrase and chooses another child to give the difference.

Resources for Active Learning

Counting blocks and volume:

DEVELOPMENTAL MATH CARDS, C²18, Addison-Wesley

MATHEX: Measurement and Estimation No. 5, Test 2, pp. 7–8, 11, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: SHAPE AND SIZE (2), pp. 25–26, 33, Wiley

DARK GREEN MODULE, UNIT D

Multiplication

Pages g-49 to g-62

General Objectives

To introduce the concept of multiplication
To associate products with groups of equivalent sets
To associate multiplication with repeated addition
To introduce the symbol for multiplication

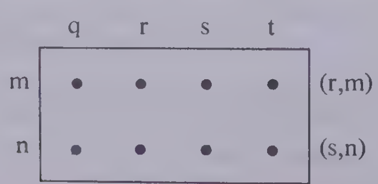
The chief emphasis in this module is upon understanding the basic concept of multiplication. Therefore, all of the work with finding products is kept at the power skill level; that is, the children are encouraged to use a variety of methods for determining the product of any two numbers. While the investigation and discussion which introduce the module give the children an opportunity to look at a variety of ways to think about products, there is a very clearcut sequence in the way multiplication is introduced lesson by lesson. The first lesson focusses upon the product of two numbers, through the use of equivalent sets. This lesson is followed by material involving the use of strips and number line jumps to find products. Finally, the children are given an opportunity to relate the concept of sets of equivalent sets to that of repeated addition in finding products. Therefore, the sequence of ideas moves from the concrete materials, sets and strips, to the semi-concrete number line concept, to the more abstract concept of repeated addition.

Mathematics

Just as we began the study of addition by defining an operation on sets, we begin the study of multiplication with such a definition. Consider two sets, $C = \{q, r, s, t\}$ and $D = \{m, n\}$. Determine from these sets the set of all pairs of elements such that the first member of the of the pair is from C and the second is from D. Thus each of the four elements of C is paired with one of the two elements of D. This set has eight pairs, and is known as the Cartesian product of C and D (written $C \times D$). The set is sometimes referred to as a product set. The elements of $C \times D$ are

(q, m)	(r, m)	(s, m)	(t, m)
(q, n)	(r, n)	(s, n)	(t, n)

The figure indicates these pairs in a rectangular array.



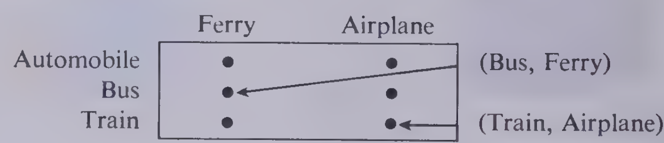
Careful examination of this array will reveal how it is related to the multiplication concept.

The Cartesian product $C \times D$ has 8 elements (pairs), and 8 is the product of 4 and 2. The numbers 4 and 2 are called factors of 8, and 8 is a multiple of each of these numbers.

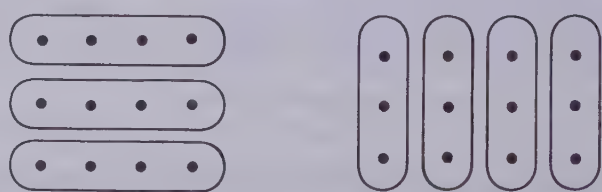
By changing the order of the members in each pair of $C \times D$, $D \times C$ is obtained. Thus $C \times D$ and $D \times C$ have exactly the same number of pairs. This illustrates the justification of the order principle (commutativity) for multiplication. We demonstrate this principle for children by having them consider rectangular arrays, first by rows, then by columns. Consider the following example:

If you can go from City X to City Y by automobile, bus, or train and from City Y to City Z by ferry or airplane, then you can go from City X to City Y to City Z in 6 ways. These six ways are: (automobile, ferry); (automobile, airplane); (bus, ferry); (bus, airplane); (train, ferry); (train, airplane).

This set of six pairs illustrates the *Cartesian product* of two sets. The cardinal number of the Cartesian product can be represented in a convenient figure called an *array,* as shown below:



Each dot in the preceding array represents one way to go from X to Y to Z. You should present arrays to children in their early exposure to multiplication. Consider the following:



3 sets of 4 — 4 sets of 3

These arrays illustrate the fact that if a and b are whole numbers, then $a \times b = b \times a$. This is the commutative (order) principle of multiplication.

Rectangular arrays also can be used for the repeated addition interpretation of multiplication. We can consider $5 + 5 + 5 = 3 \times 5$ as an extension of the distributive principle. If $a, b, c, d, \ldots$ are whole numbers, then $a \times (b + c + d + \ldots) = (a \times b) + (a \times c) + (a \times d) + \ldots$

This development applied to $5 + 5 + 5 = 3 \times 5$ is

$$5 + 5 + 5 = (5 \times 1) + (5 \times 1) + (5 \times 1)$$
$$= 5 \times (1 + 1 + 1) = 5 \times 3 = 3 \times 5$$

Other principles are involved. Of course, you should not expect to present these ideas to the children when considering $5 + 5 + 5$. Instead, merely use the set interpretation of multiplication, that is, 3 sets of 5 are associated with the product, 3×5.

Teaching Dark Green Module, Unit G

Approximate Time: 7 to 10 days (optional)

Materials

a centimetre ruler for each child, if possible.
counters
demonstration number line
felt objects and flannelboard
objects to be used for set demonstrations
a set of strips for each child

Vocabulary

equivalent sets	pairs	same-color train
multiplication	product	times

The centimetre strips provide excellent individual material for use in finding products. If centimetre rulers are not available, then the illustrations in the book might be used, or you might duplicate a copy of a ruler 25 centimetres long. Counters also provide excellent material for the children to use to build groups of equivalent sets. If you wish to extend the activities in the text to a study of Cartesian product sets, you might also include materials such as cups and saucers of different colors or pictures of complementary objects such as: sweaters and skirts, shoes and socks, hats and coats, scarves and gloves. These might then be used to demonstrate how many combinations or pairings might be obtained by matching the objects in the two sets. For example, if a child has two cups and three saucers, by pairing each cup with each of the three saucers the product 2×3 is illustrated.

Evaluation of Progress

Since new ideas are introduced, evaluating the children's progress might be difficult. We do not expect the children to master many multiplication combinations at this level, although some of the children will master a few. The chief emphasis should be on understanding the idea of multiplication rather than on mastering combinations. As you evaluate the children's daily work, you will need to be alert to those children who find products because they memorize certain combinations without understanding the ideas.

Resources for Active Learning

General Activities:

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN4 • Multiplication (A), Addison-Wesley

MATHEX: Operations No. 3, "Early Multiplication . . . Activities," pp. 29–30, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: MATHEMATICS BEGINS 1, "Multiplication," pp. 55–57; COMPUTATION AND STRUCTURE 3, "Multiplication," pp. 24–25, Wiley

Manipulative Devices:

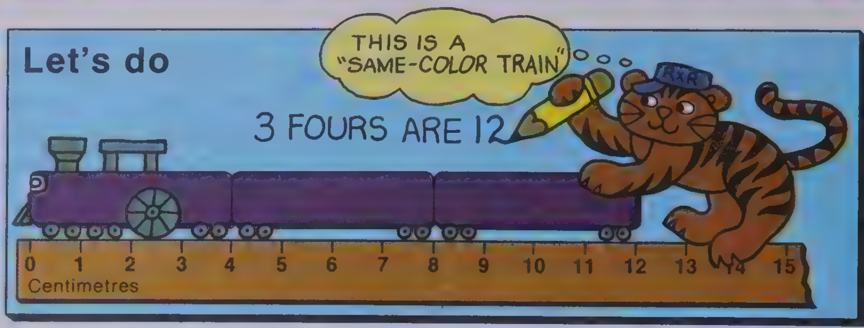
Cuisenaire Cubes, Squares and Rods (Cuisenaire Co.)

Commercial Games:

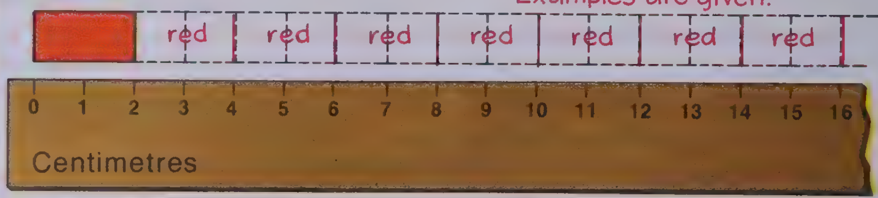
Pegboards (school supplier)

INVESTIGATION
Page g-49

Use the demonstration art at the top of the page to show how a "same-color train" can be built along the edge of a centimetre ruler and how the length of the train may be found by reading the number at the end of the train from the ruler. Discuss how this illustration shows that three fours are twelve; that is, a train made of three four-strips has the same length as the distance from 0 to 12 on the centimetre ruler. Explain to the children that in this investigation they are to make "same-color trains" and record the number of the strips which they used and the final length of the train. You might work through the first example with the children to remind them that they should cover the first strip even though it is pictured in the book, otherwise they might not count it among the strips used in their train. Note that they can make a train any length; they should simply record how many red strips, or twos, they used and the number where this train ended. After children have worked with the two and the three strips, encourage them to make other trains either with other colored strips or to make different length trains with these same strips. Have the children show the trains which they made on the overhead projector. Be sure that they describe each train with an expression such as "four twos are eight" or "three threes are nine."

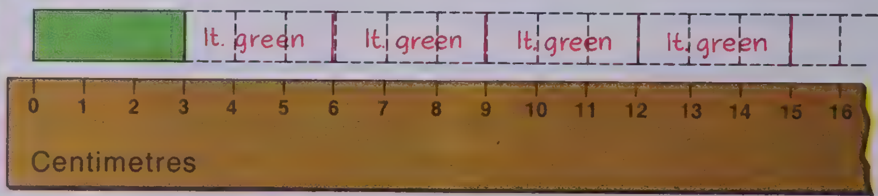


Can you make a red "same-color train."
Use as many strips as you want.
Tell about your train below the ruler. Answers will vary. Examples are given.



8 twos are 16 .

Now try a light green train.



5 threes are 15 .

Can you make a "same-color train" of your own and tell about it?

PURPOSES

To introduce the basic concepts of multiplication

To prepare the child to read and write multiplication equations

To provide readiness for working with simple multiplication facts

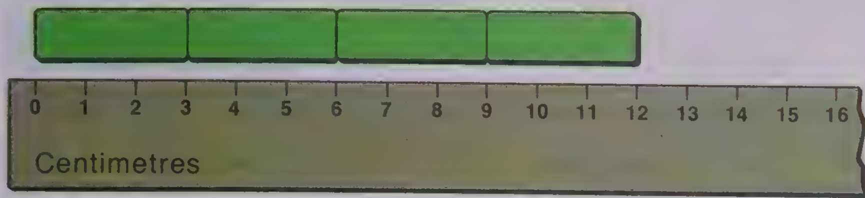
PREPARATION

Materials

sets of strips for each child

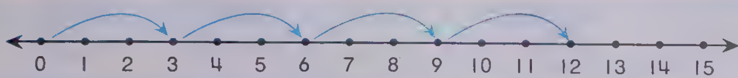
As a preparation for this investigation, it would be helpful to explain the meaning of the phrase "same-color train" to the children. Ask the children to take their six-strip (the dark green strip) and make a train that is the same length as this strip using only strips that are the same color. The children may then make a train using six one-strips or three two-strips or two three-strips. All of these are correct. Remind the children that when they build a train, their strips should be placed end to end. If necessary, give children another example to work; however, move into the investigation as quickly as possible.

Let's talk



4 strips of 3 → How long? 12 centimetres

4 sets of 3 → How many? 12



4 jumps of 3 → End where? 12

4 threes are 12.

We write: $4 \times 3 = \boxed{12}$

We say: Four times three is 12.

Find the missing numbers.

3 fives are 15.

We write: $3 \times 5 = 15$

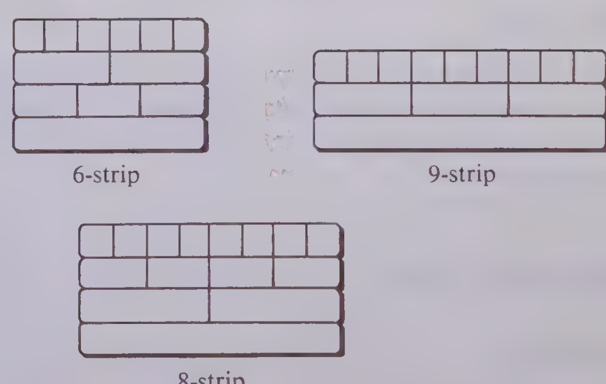
We say: Three times five is 15.

Discussion
Page g-50

This discussion page simply introduces the children to various ways in which the multiplication concepts will be developed throughout the module. Since the multiplication symbol, or times symbol, and the way to read and write a multiplication equation are new, they should be developed carefully. It would also be helpful to work through examples with each of the materials shown. For example, discuss the first illustration of the strips and ruler and relate it to the activity which the children performed in the investigation. As you discuss the second illustration which shows discrete objects in equivalent sets, use demonstration materials to show other groups of equivalent sets. For example, show five sets of three flannelboard objects. In each case, have the children count the total number of objects. Also, it would be helpful to use a demonstration number line, particularly to show the children that the jumps on the number line are not jumps of one space as they have used in the past, but rather they are jumps of a certain multiple, two, three, four, and so on. Since all of these methods of thinking of multiplication will be developed throughout the module, do not expect mastery of these power skills now. Finally work through the examples at the bottom and help children complete the multiplication phrases and equations.

Follow-up

Ask the children to take a strip, such as the eight-strip, and find how many "same-color trains" they can make which match this eight-strip. Or, have them take the ten-strip and find how many "same-color trains" they can make. Although every child should not be expected to make trains for every strip, it would be helpful to have the children share their results. Note with them that, except with their white strips, it is not possible to make a same-color train for the three-, five-, and seven-strips. If children enjoy an activity of this type, a group of them might be encouraged to record the "same-color trains" which they made for the strips and post them on a bulletin board as illustrated in the next column.



Teaching
Page g-51

Use the art to discuss the equation $2 \times 2 = 4$. Then call attention to the set of keys. Work through the first exercise with the children. Ask them to read the questions and encourage them to answer the questions. Emphasize that we write $2 \times 3 = 6$, and we say: "Two times three is six."

Help the children read and answer each question in the second exercise with the boxes of balls. Use demonstration materials as needed to clarify these examples.

WE SAY:
TWO TIMES TWO IS FOUR

HOW MANY PAIRS OF SHOES? 2.
HOW MANY IN EACH PAIR? 2.
HOW MANY IN ALL? 4.

WE WRITE:
$2 \times 2 =$

Fill the blanks. Solve the equation.

How many sets of keys? 2

How many in each set? 3

How many in all? 6

2 threes are 6.

We write:	We say:
$2 \times 3 = \boxed{6}$	Two times three is 6.

How many boxes of balls? 3

How many in each box? 4

How many in all? 12

3 fours are 12.

We write:	We say:
$3 \times 4 = \boxed{12}$	Three times four is 12.

Objective

Given an illustration of equivalent sets of objects, the child will be able to find the total number of objects in the sets and to record this fact by writing a multiplication equation.

Pre-Book Activity

Materials

felt objects for a flannelboard
counters and cups for demonstrating equivalent sets
sets of counting blocks

Ask eight children to come to the front and form two sets of four and then to form four sets of two, or to form one set of eight. As the children form these groups, use phrases such as two fours are eight, four twos are eight, one eight is eight. Then ask six children to come to the front of the room. Ask them to form three same-size groups and point out that three twos are six, two threes are six and so on. A flannelboard and felt object materials might then be used to demonstrate and exhibit multiplication equations. For example, display two sets of five triangles on the flannelboard. Ask the children how many sets there are. Ask them the number of triangles in each set. Finally, ask them the number of triangles in all. Stress the number of sets, the number of objects in each set, then the number in all. Write on the chalkboard, 2 fives are ten. Underneath that, introduce the equation $2 \times 5 = 10$. Review with the children how the times sign is read. You might also explain that the answer for a multiplication sentence is called the product.

Find the missing numbers.

3 twos are 6.

$3 \times 2 = \boxed{6}$

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3 threes are 9.

$3 \times 3 = \boxed{9}$

2 fives are 10.

$2 \times 5 = \boxed{10}$

4 fours are 16.

$4 \times 4 = \boxed{16}$

5 threes are 15.

$5 \times 3 = \boxed{15}$

2 fours are 8.

$2 \times 4 = \boxed{8}$

Teaching
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Call attention to the illustration in the first frame. Ask the children to point out how many objects there are in each set, how many sets there are, and how many objects there are altogether. Help them relate the illustration to the phrase "3 twos are ____" and to complete the multiplication equation $3 \times 2 = \square$. Be sure that in the second frame they realize that the equation refers to the letters and stress that there are three letters in each word. Encourage them to complete the remaining frames independently.

Follow-up

Provide children with a duplicated sheet showing sets in each of eight sections. Ask the children to ring the equivalent sets, as directed, and then to show a multiplication equation for the result.

Ring the sets. Write the equation.			
Ring sets of 3. 3 × 3 = 9	**** **** **** Ring sets of 4. ___ × ___ = ___	Ring pairs. ___ × ___ = ___	Ring sets of 5. ___ × ___ = ___
Ring sets of 5. ___ × ___ = ___	Ring sets of 3. ___ × ___ = ___	Ring sets of 4. ___ × ___ = ___	Ring pairs. ___ × ___ = ___

Mathematics

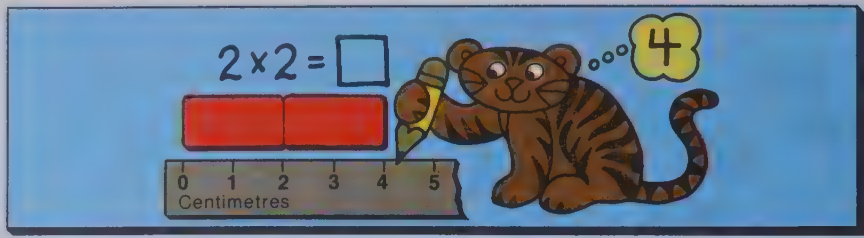
In this lesson we have used sets of disjoint equivalent sets. To determine the total number of objects in such a group of sets, the number of objects in the union of these sets must be found. This number can be found by counting or by adding. If skip counting or addition is used, it is necessary to know the number of each set and the total number of sets. Multiplication phrases and equations are developed to express these ideas.

Resources for Active Learning

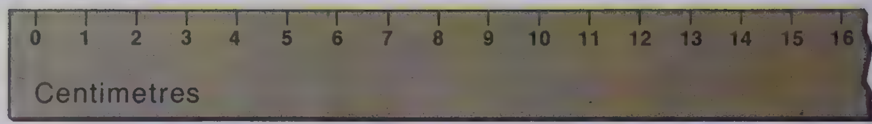
MATHEMATICS IN MODULES: WHOLE NUMBERS • WN4 • Multiplication (A), Addison-Wesley

Teaching
Page g-53

Children will need their strips to work this page. Explain how to interpret the illustrated problem in each frame. For example in the first frame, the 3 next to the red strip means that they should use three red strips. After they have built a train of three red strips on the ruler, they should determine where this train ends and then fill in the blank showing the product of the multiplication equation 3 × 2. Work through as many examples of this kind as necessary; however, if the children are capable, encourage them to work several exercises independently. When they are finished, have volunteers explain how they found the products and show the corresponding trains on the overhead projector or with demonstration strips on the flannelboard.



Use this ruler and your strips to help you solve each equation.



3
3 × 2 = 6

2
2 × 5 = 10

5
5 × 3 = 15

3
3 × 4 = 12

3
3 × 5 = 15

4
4 × 3 = 12

4
4 × 4 = 16

4
4 × 2 = 8

Multiplication – strips

Objectives

Given a multiplication equation, the child will be able to find the product by building a chain of strips along the edge of a centimetre ruler.

Given a multiplication equation, the child will be able to find the product by repeated jumps on the number line.

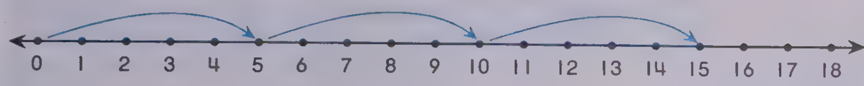
Pre-Book Activity

Materials

a centimetre ruler for each child, if possible
a demonstration number line
set of centimetre strips for each child

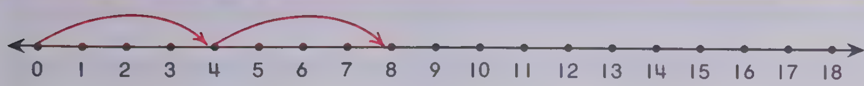
Use demonstration strips or strips on the overhead projector and a transparent centimetre ruler to build a same-color train and then write a related multiplication equation. After the children have discussed this equation, ask them to think of the edge of the centimetre ruler as a number line and to think of the jumps as being the distance from the beginning of each strip to the end of each strip. Thus, if you show 3 five-strips on the centimetre ruler, tell the children to think of the distance of the first strip as being a jump from 0 to 5, the second strip from 5 to 10, and the third strip from 10 to 15. Then remove the strips and ask the children to picture these jumps on the demonstration number line. It is important to show them how to make jumps of the multiples that they are using, in this case, 5, since most of the children's previous work with the number line has used jumps of units or one spaces.

Solve the equations.

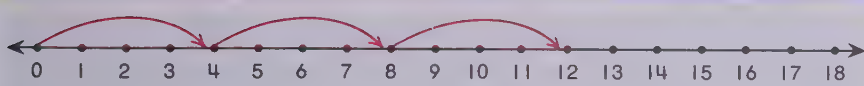


$3 \times 5 = \boxed{15}$

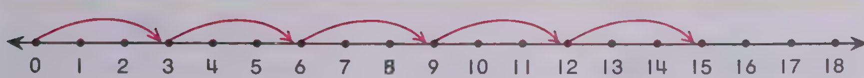
$4 \times 3 = \boxed{12}$



$2 \times 4 = \boxed{8}$



$3 \times 4 = \boxed{12}$



$5 \times 3 = \boxed{15}$

Teaching
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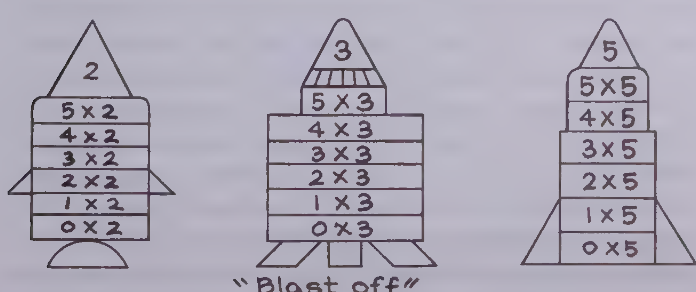
Explain to the children that they may think of the number line as the edge of the centimetre ruler. It would be helpful to work through the top two examples with the children to be sure they realize how the jumps are pictured on the number line. Explain that the number lines given with the next three equations should be used as an aid in solving the equations. You might remind the children that in reading an equation such as two times four, they may think two fours and this will help them to figure out what size jump they should show on the number line. You might present children with other equations to draw on their number lines.

Follow-up

For group practice on multiplication facts, draw rockets of various sizes and shapes on the chalkboard. Label them 2, 3, 4, or 5, and write the corresponding "family" of multiplication facts on each. For example:



Across a bulletin board or display area, show a long path (100 or more spaces) to the moon or a planet such as Venus or Mars. Include some spaces that are marked with directions such as

Meteor damage: *Go back 3 spaces.*
Space pirates sighted: *Zoom 10 spaces ahead.*
Space station: *Lose 1 turn to refuel.*
Refueled: *Take 2 turns* (and so on).

Divide the class into teams and let them choose a paper space vehicle. To begin the game, a child chooses a rocket such as Rocket 5. If he can begin at the bottom and give all the products, he gets a 6-space "rocket burn" for his team's space vehicle on the way to the moon. If he answers only two products correctly he only gets a 2-space "rocket burn."

Teaching
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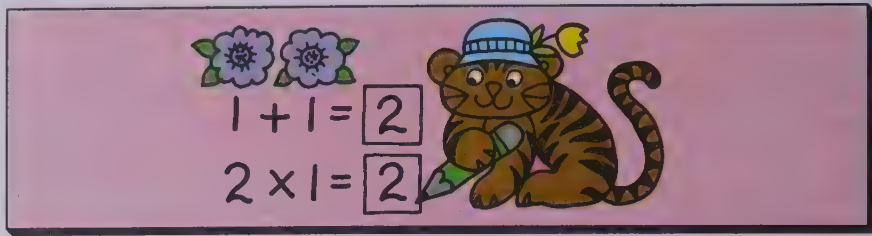
After reading the directions with the children, point out the correlation between the shaded sets and the numerals. Ask a child to give the sum of $4 + 4 + 4$. Observe that the children can also think about the problem as 3 sets of 4, and write the multiplication equation $3 \times 4 = \square$. Instruct the children to write 12 in the answer box as the product.

Work through the remaining exercises with the children, if necessary. Explain that they should first find the sum, and then the product, in each frame.



Find the sum. Then solve the equation.

$4 + 4 + 4 = 12$ $3 \times 4 = \boxed{12}$

$3 + 3 + 3 + 3 = \boxed{12}$ $4 \times 3 = \boxed{12}$

$3 + 3 = 6$ $2 \times 3 = \boxed{6}$

$2 + 2 + 2 = \boxed{6}$ $3 \times 2 = \boxed{6}$

$3 + 3 + 3 = 9$ $3 \times 3 = \boxed{9}$

$3 + 3 + 3 = \boxed{9}$ $3 \times 3 = \boxed{9}$

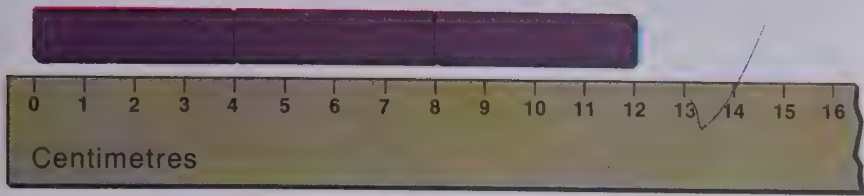
Objective

Given a repeated addition problem, the child will be able to find the sum and use the addition problem to find the product for a related multiplication problem.

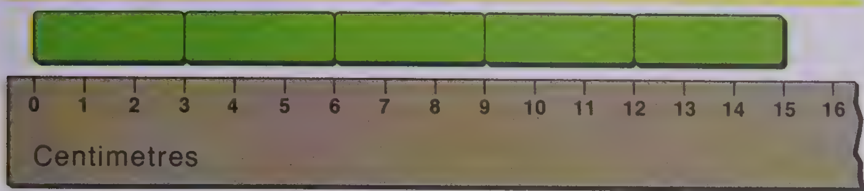
Pre-Book Activity

To encourage children to do some creative thinking on their own, divide them into five or six groups. Prepare five or six stations with same-size groups of counters. For example, you might have three rows of counters with four counters in each row, or two rows of counters with two counters in each row, or three rows of counters with two counters in each row. Label each station and then ask each group of children to study each station, record what they see, and then move to another station. Next to each station, it would be helpful to have written down a repeated addition equation related to that station. When the children go to each station, explain that they should try to figure out how they can describe this same situation or group of sets by using a multiplication equation rather than an addition equation. Encourage the children to discuss each station with their group. If some appear without many ideas, ask questions such as: "How many sets do you see?" "How many are in each set?" "Can you write a multiplication equation showing this?" Or, you might ask, "How many groups of three are there at this station? Can you express this in terms of a multiplication equation?" After the children have spent time studying each station, ask volunteers to explain the multiplication equation that they have written for each. If necessary, use a flannelboard demonstra-

Solve the equations.

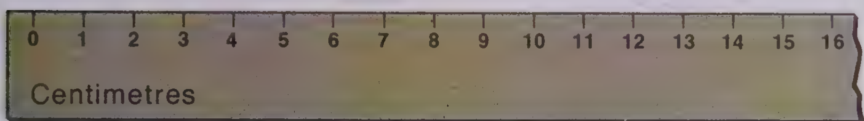


$4 + 4 + 4 = \boxed{12}$ $3 \times 4 = \boxed{12}$



$3 + 3 + 3 + 3 + 3 = \boxed{15}$ $5 \times 3 = \boxed{15}$

Use this ruler and your strips to help you solve the equations.



$2 + 2 + 2 + 2 + 2 = \boxed{10}$ $5 \times 2 = \boxed{10}$

$5 + 5 + 5 = \boxed{15}$ $3 \times 5 = \boxed{15}$

$3 + 3 + 3 + 3 = \boxed{12}$ $4 \times 3 = \boxed{12}$

Teaching
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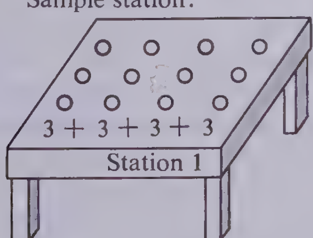
Explain to the children that if they wish they might use their strips to help them work the equations on this page, but that they might also work simply from the illustrations. In the top two examples, children first should find the sum of the repeated addition equation and then the product of the multiplication equation. Continue to use the terms, sum and product, so that the children develop correct usage of these words mainly by your example. Explain to them that in the bottom section of the page, they should work with their strips and use the centimetre ruler shown to build the related repeated addition and the multiplication equations. You might extend this page by putting other examples of repeated addition equations and related multiplication equations on the chalkboard and having the children make trains with their strips on the edge of the centimetre ruler.

tion to show repeated addition equations and multiplication equations for the same group of counters.

Sample station:



Follow-up

The matching game suggested on page 45 may be used with multiplication facts and also with repeated addition equations and multiplication equations. Make fifteen small cards showing the multiplication facts up to 5×5. On fifteen other cards show the same facts, but change the order. Put one set of fifteen in each half of the matching game board. You might also make fifteen small cards showing repeated addition equations and a matching set of fifteen cards showing related multiplication equations. Again put one set of fifteen in each half of the matching game board.

Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "Peg Counting," E-1, Addison-Wesley

Teaching
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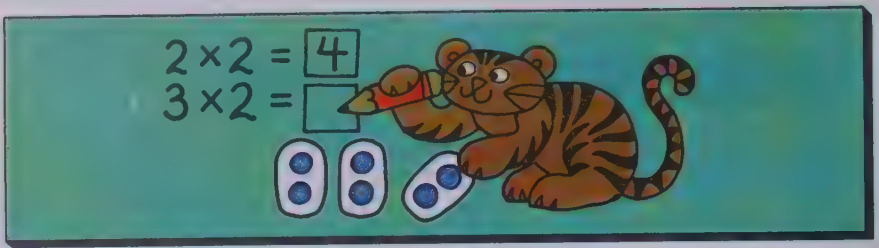
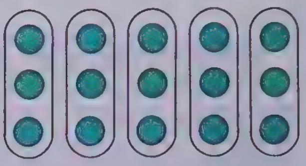
Encourage the children to talk about how they can use the sets of threes to solve the equations. Stress with them how important it is that they understand what each multiplication equation means; that is, the first equation, 2 × 3, means that they must think of two threes. They can use their illustrated sets of threes to help them find the product, six. Similarly, for the next equation 3 × 3, they may simply think three threes and again use their illustration to find the product. Point out that they may think of these patterns in terms of repeated addition or as so many sets of three. You might also point out that the products form a sequence which they have heard before during their skip counting activities. You might want to work through all of the equations on this page with the children. In any event, allow time for a discussion of the ideas when they have finished the exercises.



Use the sets of three to help you solve these equations.

2 × 3 = 6   4 × 3 = 12

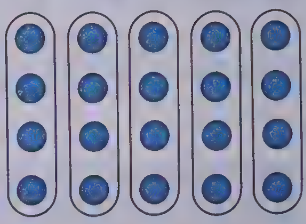
3 × 3 = 9   5 × 3 = 15



Use the sets of four to help you solve these equations.

2 × 4 = 8   4 × 4 = 16

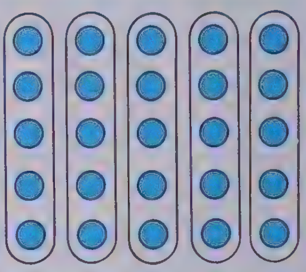
3 × 4 = 12   5 × 4 = 20



Use the sets of five to help you solve these equations.

2 × 5 = 10   4 × 5 = 20

3 × 5 = 15   5 × 5 = 25



Objective

Given multiplication problems which involve successive multiples of 3, 4 and 5, the child will be able to use the pattern to help him find the products.

Pre-Book Activity

You can help children see the patterns when multiplying by 2, 3, 4, or 5 by using a brief oral game of skip counting. For example, play the Buzz game in which children count, but instead of saying the number name for a specific multiple, such as, three, they say the word Buzz. Thus they would say: "One, two, Buzz, four, five, Buzz, seven, eight, Buzz," and so on. To relate this counting to the patterns, after you have worked up to numbers such as 15 or 18, you might simply have the children skip count through the numbers for which they had to say Buzz, thus you will skip count: "Three, six, nine, twelve, fifteen, . . ." and so on. You might also simply begin with the combination and work in a pattern. For example, begin with 3 × 1, then 3 × 2, 3 × 3 and so on. Continue this until most of the children understand the idea of building a pattern of combinations. If necessary you might combine this latter activity with demonstration materials on the flannelboard.

Use this number line to help you solve the equations about twos.

$2 \times 2 = \boxed{4}$ $\quad$ $6 \times 2 = \boxed{12}$

$3 \times 2 = \boxed{6}$ $\quad$ $7 \times 2 = \boxed{14}$

$4 \times 2 = \boxed{8}$ $\quad$ $8 \times 2 = \boxed{16}$

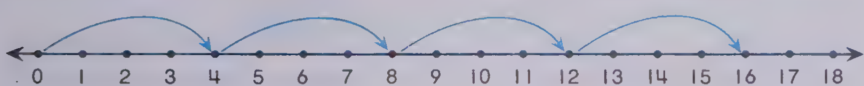
$5 \times 2 = \boxed{10}$ $\quad$ $9 \times 2 = \boxed{18}$

Solve the equations about threes.

$3 \times 3 = \boxed{9}$ $\quad$ $5 \times 3 = \boxed{15}$

$4 \times 3 = \boxed{12}$ $\quad$ $6 \times 3 = \boxed{18}$

Solve the equations about fours.



$4 \times 4 = \boxed{16}$ $\quad$ $2 \times 4 = \boxed{8}$

$3 \times 4 = \boxed{12}$ $\quad$ $1 \times 4 = \boxed{4}$

Teaching
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Explain to the children that the equations on this page form patterns similar to the patterns studied on the previous page. Here the patterns are developed using the number line. Again review with the children that the jumps on the number line are not single space jumps, but that they follow the multiplication family that the children are studying. Stress the pattern of the products as the children work through each family of facts and point out how the products form a sequence which they have heard before in their skip counting activities. For variety, you might ask those children who finish early to find the products of problems you write on the chalkboard in vertical form.

Follow-up

Many children will enjoy filling out a multiplication table up to the five times five facts; however, no mastery by the children should be expected at this time. Note that the multiplication table shown does not include a column for zero. Throughout this module, the multiplication involving zero has been omitted simply to avoid ambiguities which often develop. Children will have plenty of time to work with the zero facts in later grades. If examples with zero come up naturally, treat them lightly, simply in terms of a natural explanation such as five zeros is zero, or zero fives is zero and so on.

×	1	2	3	4	5
1					
2					
3					
4					
5					

Resources for Active Learning

MATHEX: Numeration No. 2, "Arrays," pp. 41–42, Encyclopaedia Britannica Publications Ltd.

Teaching
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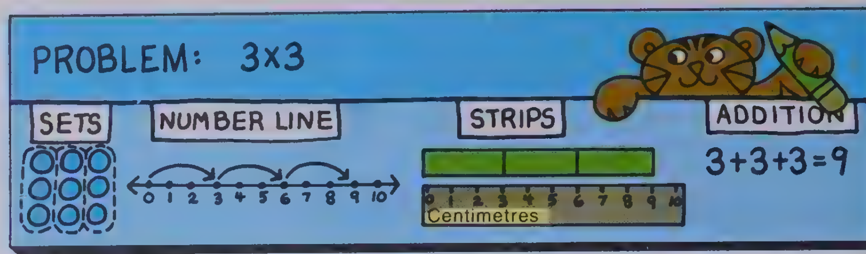
Use the illustrations at the top of the page as a basis for discussing the power skill methods that have been developed in the previous lessons. For example, you might take a sample equation and ask the children to explain how this equation might be solved using sets. Then have someone show how the demonstration number line might be used to solve the equation. Have a volunteer show how it might be solved using his colored strips. Finally show how the equation might be solved by thinking in terms of repeated addition. Work through as many examples as you think necessary to review the various methods developed in the module. Then encourage the children to solve the equations using whichever method they choose. You might work together with those children having difficulty and discuss methods which they can use to solve the equations.



Use the ways you like best to help you solve these equations.

$2 \times 3 = \boxed{6}$	$4 \times 3 = \boxed{12}$
$3 \times 4 = \boxed{12}$	$4 \times 4 = \boxed{16}$
$2 \times 2 = \boxed{4}$	$2 \times 5 = \boxed{10}$
$5 \times 2 = \boxed{10}$	$2 \times 4 = \boxed{8}$
$3 \times 5 = \boxed{15}$	$5 \times 3 = \boxed{15}$
$3 \times 3 = \boxed{9}$	$5 \times 5 = \boxed{25}$

Objective

Given a multiplication problem involving a fact below five times five, the child will be able to find the product by working with sets, strips, the number line, or by thinking of multiplication as repeated addition.

Pre-Book Activity

Materials

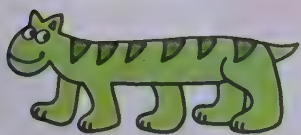
centimetre strips
counters or other objects for building sets
demonstration number lines

Assign the children to groups of three or four. Give each group one of the three kinds of materials listed above. Write three or four multiplication equations on the chalkboard and then ask each group to use their strips, number lines, or counters to solve the equations. After sufficient time, ask a volunteer from each group to explain to the whole class how they used their materials to help them find the products. If time permits, you might have students make up multiplication word problems such as the following:

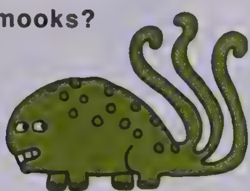
1. Karen got two long ropes of licorice. She cut each into three pieces. How many pieces of licorice did she have then?
2. One hot day Kris and Ken sold lemonade. In one hour they sold five glasses at three cents each. How much money did they collect that hour?
3. Three boys are playing. Each boy is playing with four cars. How many cars are there altogether?

Make-believe Animals

1 A **fleek** has 2 heads. How many heads on 4 **fleeks?** 8



2 A **mook** has 5 legs. How many legs on 3 **mooks?** 15



3 A **muk** has 3 tails. How many tails on 4 **muks?** 12

4 Each **burt** has 4 wings. How many wings on 2 **burts?** 8

5 Each **boog** has 2 eyes to look forward, 2 eyes to look sideways, and 2 eyes to look backward. How many eyes? 6

6 Each **flook** has 3 horns on his head. How many horns on 3 **flooks?** 9

7 A **munck** has 4 arms with 3 fingers on each arm. How many fingers? 12

8 Each **chack** has 4 legs. How many legs on 4 **chacks?** 16

Make up an animal story of your own. Can you draw a picture of your animal?

Story problems

TEACHING
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It is important to work through this page together with the children, both to help them read the strange names for the animals and to see how multiplication applies in each problem. However, as you work through the problems, encourage the children to do as much independent thinking as possible. One way to do this is to encourage the children to draw a picture to correspond to the problem. For example, in problem number 1 they might try to draw four fleeks. Since each fleek has two heads, they should find out that they must draw a total of eight heads. It would be helpful to show a multiplication equation for each problem worked out in this manner. If the children are capable, you might encourage them to work in small groups, but most children will benefit from your guidance. The directions at the bottom should be treated with a light touch. Encourage those who wish to do so to create an animal and then write a multiplication story for it. Sample problems the children write might be used on the bulletin board with the pictures of the animals the children create.

FOLLOW-UP

Less verbally oriented children might benefit from more simplified problems or short stories such as the following.

1) Box of crayons. 5 rows. 5 crayons in each row. How many crayons?
2) 4 bike racks. 4 bikes per rack. How many bikes in all?
3) Basketball game. 2 teams. 5 boys on a team. How many in the game?
4) 4 packs of gum. 5 sticks in each pack. How many sticks of gum?
5) 3 letters. Stamps cost 5 cents each. How much for stamps?

As material for more capable children, duplicate an exercise similar to the one following.

Write +, −, or × in each ◯.	
$3 \bigcirc 2 = 1$	$3 \bigcirc 3 < 6$
$3 \bigcirc 2 = 5$	$3 \bigcirc 3 > 6$
$3 \bigcirc 2 = 6$	$4 \bigcirc 2 > 6$
$5 \bigcirc 3 = 8$	$4 \bigcirc 2 < 6$
$5 \bigcirc 3 = 15$	$1 \bigcirc 1 < 1$
$5 \bigcirc 3 = 2$	$1 \bigcirc 1 > 1$

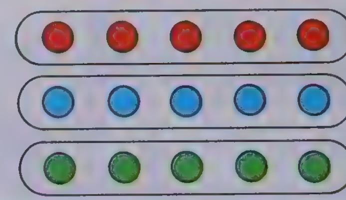
Teaching
Page g-61

Explain to the children that the illustrations review the various ways they have used to solve multiplication equations. Encourage the children to find the products by referring to the illustration in each section. Explain that the equations at the bottom may be solved in any way they choose. As the children work, if you wish to evaluate the understanding certain children have of the concepts presented in this module, you might ask them to explain their favorite method and show how they use it to solve a multiplication equation.

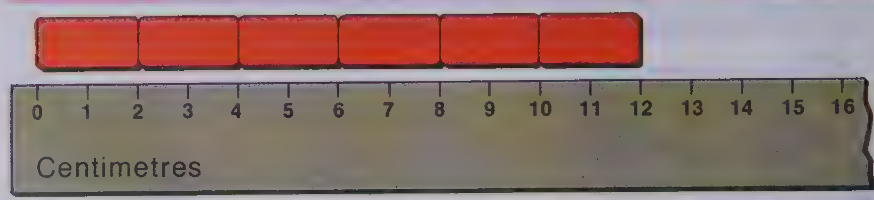
Encourage independent work for this page. When the children have finished, allow time for them to discuss any ideas which you think need further clarification.

Show you know

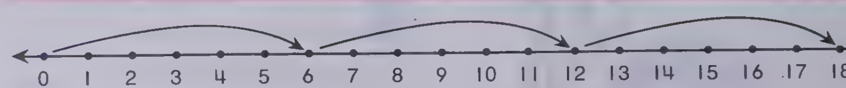
Solve the equations.



$3 \times 5 =$ 15



$6 \times 2 =$ 12



$3 \times 6 =$ 18

$4 + 4 + 4 =$ 12 $\quad$ $3 \times 4 =$ 12

$2 \times 5 =$ 10 $\quad$ $3 \times 3 =$ 9

$3 \times 2 =$ 6 $\quad$ $5 \times 3 =$ 15

$4 \times 3 =$ 12 $\quad$ $4 \times 4 =$ 16

$2 \times 2 =$ 4 $\quad$ $4 \times 5 =$ 20

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Note: Since this entire module is an extension module, it should be considered optional material for all the children. Thus, no child should be required to show mastery of this material as a basis for further work at this level.

Pre-Book Activity

Children might enjoy a review game such as "Product Detective." Write the products for factors of five or less on cards. Place these at random on the chalk tray. Say: "There is a product missing. The missing product tells how many in three sets of five. Can anyone find the missing product?" A child who knows the product should then find that product card along the chalk tray. Keep the factors under five and add some props like a paper detective badge for each child. If you wish to further review the power skills developed in this module, you might write sample equations on the chalkboard and ask volunteers to explain how they might solve each one.

Let's have fun

ACROSS	DOWN
① 2×3	② 2×6
③ 8+4	③ 6+4
④ 2×5	

1 6		2
	3 1	2
4 1	0	

Work the puzzle.

1 9	2 9		3 2	3	4 4
5 2	0				6
	6 1	7 6	8 9		7
9 7		10 5	0	11 1	
5				12 1	13 1
14 1	0	0		15 1	2

Across

1. One less than 100
3. 132 + 102
5. 4 fives
6. 100 + 60 + 9
10. 723 − 222
12. 7 + 4
14. ten tens
15. 3 × 4

Down

1. 9 tens and 2
2. 897, 898, 899, 900, __?__
4. 143 + 324
7. 20 + 40 + 3 + 2
8. 10 less than 100.
9. 700 + 50 + 1
11. 1 hundred, 1 ten and 1
13. 7 + 5

Cross numeral puzzle

Teaching
Page g-62

This change of pace page presents a number puzzle for the children. It will be necessary to explain the *across* and *down* directions carefully. Be sure they understand that the numbers under the across column should be written in the normal manner from left to right across the rows and that the numbers under the down column should be written vertically. For example, the first answer for the down column 9 tens and 2 should ordinarily be written as 92, side by side, but for this *down* answer, they must write their answer in vertical form. If children do the across problems first, they should not have difficulty seeing how to write the answers for the down columns.

Follow-up

Children might enjoy working with a different way of writing products. For example, present three or four examples with the products shown as indicated below. Then ask children to study these to see if they can figure out a rule to help them find the missing numbers.

(2, 4) — 8
(3, 2) — 6
(1, 4) — 4
(2, 3) — ?
(5, 1) — ?
(2, 2) — ?
(1, 2) — ?

You might also present charts for the children to try to complete.

Boys	1	2	3	4	5	6
Shoes	2	4	?	?	?	?

Cars	1	2	3	4	5
Wheels	4	8	?	?	?

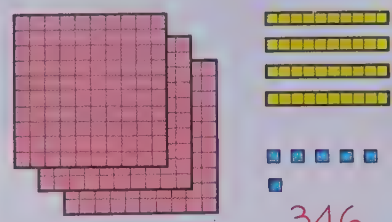
Teaching
Page g-63

Explain to the children that this page reviews some of the topics which they have studied in this Unit. Then read the directions with them. Explain that in each top frame they should write the numeral which will tell how many small squares are pictured. Note that the second frame reviews the place-value concept; child must use zero here to show that there are zero tens shown.

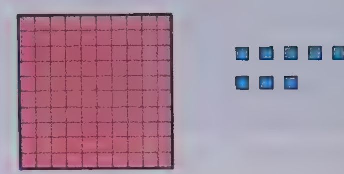
In the second section they should simply find the sums and differences. Finally at the bottom they should try to solve each word problem. Help any child who has difficulty reading these exercises, but encourage him to do his solutions independently.

Looking back

How many?



346



108

Find the sums and differences.

641	136	493	838	796	457
+ 233	+ 540	+ 406	− 205	− 125	− 230
874	676	899	633	671	227
333	224	600	669	978	588
+ 256	+ 775	+ 287	− 516	− 423	− 123
589	999	887	153	555	465
364	703	470	778	895	369
+ 115	+ 192	+ 500	− 601	− 195	− 118
479	895	970	177	700	251

126 children ride buses to school. 233 children walk to school. How many children in all? 359

A table is 265 centimetres long. It is 124 centimetres wide. How much longer than wide is it? 141

Cumulative review

Objective

The child will demonstrate his ability to work with the concepts presented in Unit G.

Pre-Book Activity

Review skip counting by tens. Ask the children to count orally, while you write the first 20 numerals on the board (10, 20, 30 . . ., 200). Then, practice on particular trouble spots such as the sequence from 190 to 210 or from 790 to 810. Next, have the children count by hundreds.

More children can participate in this review if you make an oral game out of it. For example, give two consecutive multiples of 100, such as 500 and 600. Then call on someone to give the next two multiples in that sequence. Ask others whether the child responded correctly, and if not, instruct them to correct what he said.

You might also write a few sums and differences *which do not require regrouping* on the chalkboard for children to review.

Solve the equations.

$7 + 5 = \boxed{12}$	$8 + 5 = \boxed{13}$
$8 + 3 = \boxed{11}$	$9 + 3 = \boxed{12}$
$9 + 5 = \boxed{14}$	$6 + 4 = \boxed{10}$
$6 + 7 = \boxed{13}$	$9 + 8 = \boxed{17}$
$\boxed{8} + 3 = 11$	$\boxed{8} + 6 = 14$
$11 - 3 = \boxed{8}$	$14 - 6 = \boxed{8}$
$\boxed{5} + 9 = 14$	$\boxed{9} + 8 = 17$
$14 - 9 = \boxed{5}$	$17 - 8 = \boxed{9}$
$2 \times 2 = \boxed{4}$	$2 \times 5 = \boxed{10}$
$3 \times 2 = \boxed{6}$	$3 \times 4 = \boxed{12}$
$2 \times 4 = \boxed{8}$	$2 \times 6 = \boxed{12}$
$4 \times 4 = \boxed{16}$	$5 \times 4 = \boxed{20}$

Cumulative review

Teaching
Page g-64

The top sections of this page may be used by all the children who have completed Unit G, but the bottom section should only be attempted by those who studied the optional module on multiplication. Remind children that they have studied many power skills to help them find answers to basic facts. However, encourage them to try to solve the equations at the top from memory. The equations in the middle review missing addends and inverse relation between addition and subtraction. It would be helpful to review with the children any topics which need further clarification.

Follow-up

Children will benefit from continued work with short story problems. Write problems similar to the following on the chalkboard or encourage children to make up problems of their own which they may then exchange.

1. Allowance was 25 cents. Bought a glider for 12 cents. How much left?
2. Model car costs 77 cents. Piggy bank has 65 cents. Need how much?
3. Birthday money 75 cents. Necklace costs 55 cents. How much change?

Sums to 18 – Speed Skills

Pages h-1 to h-8

General Objectives

To develop speed skills for the addition combinations to 18

To maintain as an ongoing objective the memorization with quick recall of the addition combinations to 18

This module opens with a memory game which should serve to stimulate the child to memorize a few of the more difficult addition combinations. The addition table is presented as an instrument which contains interesting patterns and may be used throughout the module. Since the next module develops the skill of finding differences by thinking of them as missing addends, this module stresses the addition combinations to 18. If a child knows the sums with quick recall, he will have a much easier time finding differences to the related subtraction facts. This module will provide practice pages for the children including several games as aids in improving and retaining mastery of the basic facts.

Mathematics

Many of the ideas in this module are extensions of mathematical concepts already developed. See the Mathematics section of dark green module for Unit G.

Teaching Yellow Module, Unit H

Approximate Time: 4 to 6 days

Materials

sets of flashcards for each child
scissors

Since the emphasis of this module is on speed skills for the addition combinations, few concrete materials are required; however, if some children still need sets, or a number line, or strips to help them find the sums, allow them to use them. Encourage them to start abandoning these aids and memorizing the facts as soon as they are able. The games suggested in the teacher comments should help the children develop speedy recall for these facts.

Evaluation of Progress

Page h-7 may be used to evaluate the children's knowledge of the addition facts; however, if you wish to determine the speed with which children are able to recall these facts, you might help them time themselves on this page. Children should be made to feel that they are not competing with each other but with themselves and that the purpose of such a test is simply to help them assess their ability and determine which facts need more study.

Resources For Active Learning

General Activities:
Basic facts games:

DEVELOPMENTAL MATH CARDS, C#4, Addison-Wesley

MATH ACTIVITIES, Games 3/31–79, pp. 97–119, Allyn and Bacon

MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Ltd.

Commercial Games:

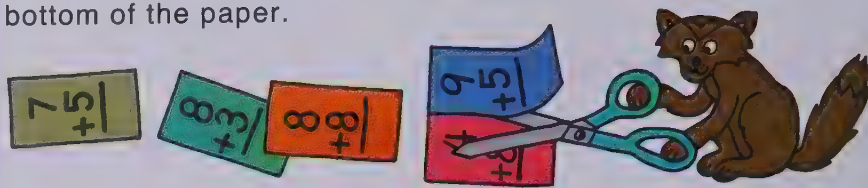
For games to develop competence with the basic facts, refer to the Introduction of the blue module, Unit E.

INVESTIGATION
Page h-1

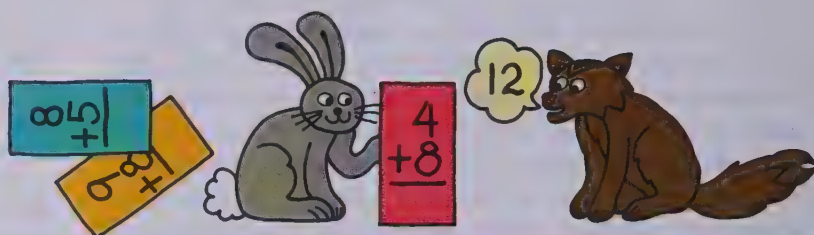
Explain to the children that they should choose three of their fact cards and memorize these sums. When they think that they can recall the sums from memory, they should work with a partner to test themselves. If the children have sets of flashcards, you might prefer to have them use other combinations; however, the combinations chosen for the investigation emphasize the harder addition facts. Not all of the children will be able to master the facts now, but recall that an ongoing objective is for the children to learn these facts from memory and eventually to eliminate the use of concrete materials. Remind the children that at this time they need memorize only three of these facts in order to do the investigation.

Let's do

Cut out the **fact cards** at the bottom of the paper.



Choose 3 of them to memorize.
Have a classmate test you.



7 + 5 12	6 + 8 14	7 + 6 13	8 + 3 11	7 + 8 15
8 + 5 13	8 + 8	9 + 8 17	4 + 8	9 + 5 14

12

PURPOSE

To motivate children to develop the speed skill for the addition facts with emphasis on the harder facts

To give the children an opportunity to assess their present skill in recalling some of the harder addition facts from memory

PREPARATION

Due to the nature of the investigation, it would be helpful to use the preparation time to help the children prepare the cards which they will use for the investigation page. Distribute the text pages and explain to the children that they should cut out the fact cards at the bottom of the paper. Have them be particularly careful so that the top part of the page will be saved for the discussion on page h-2.

Let's talk

Complete the addition table. Can you find some patterns in the table?

+	0	1	2	3	4	5	6	7	8
0	0	1	2	3	4	5	6	7	8
1	1	2	3	4	5	6	7	8	9
2	2	3	4	5	6	7	8	9	10
3	3	4	5	6	7	8	9	10	11
4	4	5	6	7	8	9	4 + 6 10	11	12
5	5	6	7	8	9	10	11	12	13
6	6	7	8	9	6 + 4 10	11	12	13	14
7	7	8	9	10	11	12	13	14	15
8	8	9	10	11	12	13	14	15	16

Discussion
Page h-2

The addition table is provided to stimulate discussion of the addition facts and their orderly arrangement. Some children may benefit from using the addition table throughout the module as an aid. It would be helpful to duplicate copies of an addition table that includes the sums of nine. It is important that the children complete the table accurately and learn how to use it. First, explain the meaning of the sums 6 + 4 and 4 + 6 written above the dashed numeral 10. Help them see how a number from the column on the left is added to a number from the row on the top and the sum is put in the box where the row and column for those numbers meet on the table. After you have helped the children complete the addition table, use it as a basis for discussing various patterns. For example, you might first point out the consecutive order of the numbers both horizontally and vertically. Then you might also point out that if children imagine a line from the top left corner to the bottom right corner, they could think of one side of the table matching the other side of the table. Also point out the diagonals which extend from the top right-hand corner to the bottom left-hand corner. Point out that the first row and the first column match the blue headings because of the identity property of zero.

Follow-up

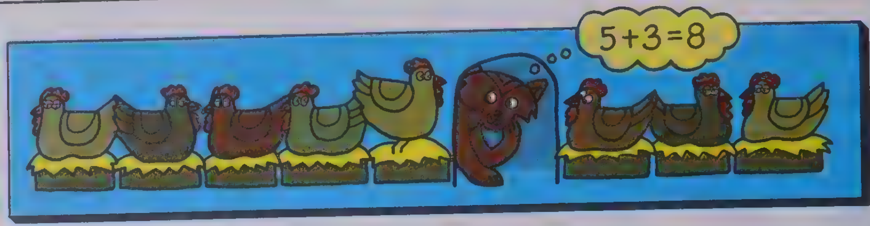
Suggest to the children that they work with a partner as they did in the investigation and use the fact cards to test each other. It would also be helpful if you duplicate a worksheet such as the one shown in the next column on which children can make a record of the facts that they do not know and of their progress in learning these facts.

Sample record:

Date	3/13	3/17	3/20	3/24	3/27
Sums to Learn	7+8 5+9 6+9 8+9 6+8 5+8 7+7	5+9 8+9 6+8 7+7 6+7	7+8 6+9 6+8 7+9 5+8 8+8 9+8	8+9 7+8 9+8 9+7	6+8 9+7

Teaching
Page h-3

You might explain to the children that since one of the main purposes of the module that they are studying is to develop their speed and accuracy in finding the sums, they might try to work through the equations at the top as quickly as possible and then go back and carefully check them against their addition table or do them more slowly and see if they would still give the same answer. Be sure they understand how to complete the tables that are shown at the bottom of the page.



Solve the equations.

$4 + 8 = \boxed{12}$

$7 + 4 = \boxed{11}$

$8 + 9 = \boxed{17}$

$2 + 6 = \boxed{8}$

$7 + 3 = \boxed{10}$

$9 + 5 = \boxed{14}$

$4 + 5 = \boxed{9}$

$7 + 8 = \boxed{15}$

$6 + 7 = \boxed{13}$

$8 + 3 = \boxed{11}$

$6 + 8 = \boxed{14}$

$8 + 8 = \boxed{16}$

Complete the tables.

Add 4	
7	11
6	10
4	8
8	12

Add 7	
8	15
7	14
5	12
6	13

Add 5	
8	13
5	10
6	11
9	14

Practice—sums to 18

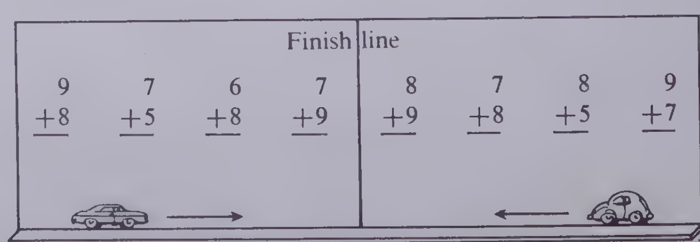
Objective

Given addition problems written horizontally or vertically, the child will be able to find the sums.

Pre-Book Activity

Most children will benefit from a game which motivates them to respond quickly to the addition sums. The Auto-Race Game suggested on page 164 is such a game.

Divide the class into two groups. Give each group an auto and have the children place them at opposite ends of the chalk tray. Have the more difficult sums written on the chalkboard as shown below. One member of each team begins by solving the problem nearest his team's car. When he finishes, the next member continues with the next problem, moving the car along the tray. The first team whose car gets to the finish line wins. If you wish, you can establish the rule that you will correct each team's work as the relay is in progress. Thus, if you notice an incorrect answer, you might erase that answer and require another member of the team to write the correct sum before allowing their car to move another place. In this way, children will have to strive for both speed and accuracy.



Chalkboard Auto Race

Find the sums.

7	5	3	5	2	8
+ 5	+ 6	+ 6	+ 9	+ 5	+ 2
12	11	9	14	7	10
4	3	8	5	6	8
+ 4	+ 7	+ 6	+ 4	+ 7	+ 7
8	10	14	9	13	15
8	4	7	8	4	6
+ 5	+ 8	+ 4	+ 8	+ 2	+ 9
13	12	11	16	6	15

Ring the correct sum.

7 + 8	14	15	16
9 + 5	14	15	16
8 + 8	14	15	16
9 + 6	14	15	16
9 + 7	14	15	16
6 + 8	14	15	16

Teaching
Page h-4

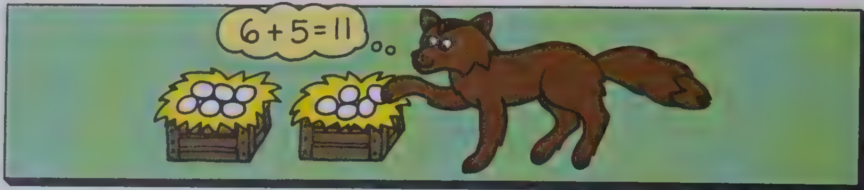
Read the directions at the top of the page with the children. Remind the children that the addition problems in vertical notation present problems really no different from those given in equation form. Again you might suggest that they work through these problems as quickly as possible and then go back and check their work more slowly. Point out the bottom section of the page. Here they should choose the correct sum, from the three that are given and ring that sum.

Follow-up

Try this arithmetic review game. Seat the children cross-legged on the floor and establish the rhythm, pat, pat, snap, snap. (The pats indicate hands patted on the thighs, and the snaps indicate snapping fingers.) Then while you snap your fingers, give a combination. Continue the patting, but warn the child on your right that he must give the answer during the next snap, snap. He then continues the game by giving a new combination during the snap-snap phase of the rhythm, and the child to his right must answer during the next snap-snap phase and so on. Play several trial games and then challenge the children to see if they can play and keep the rhythm unbroken until each child has had a turn.

Teaching
Page h-5

Ask the children to read the directions at the top of the page. Point out that they should first find the sums for each equation and then find the sums for the addends written in vertical notation. You might again remind them that they should strive to find all of the sums as quickly as possible and then go back and check their answers to see that they have done them correctly. You might suggest that they use their addition table to help them check their answers.



Find the sums.

3 + 4 = 7	5 + 7 = 12
9 + 8 = 17	4 + 6 = 10
4 + 7 = 11	6 + 8 = 14
7 + 7 = 14	9 + 6 = 15
7 + 2 = 9	6 + 5 = 11
8 + 4 = 12	4 + 9 = 13
6 + 9 = 15	9 + 7 = 16

8 +8 16	2 +6 8	9 +3 12	5 +9 14	6 +7 13	3 +7 10
8 +3 11	5 +8 13	9 +9 18	9 +0 9	8 +7 15	8 +6 14

Objective

Given addition equations or addends written in vertical notation, the child will be able to find the sums. He will also be able to recognize incorrect sums.

Pre-Book Activity

Provide an oral review of combinations as a warm-up. For example, say: "I'm thinking of the sum of 8 and 7." "What's my number?" Or, say: "If you add 5 to my number you get 14." "What's my number?" Include some combinations that have sums less than 10 to review these also. Remember, that since one of the most important points of this module is to develop speed skill for addition facts, you should encourage the children to respond as quickly as possible.

Grade Jeanie's paper.

Name *Jeanie*

Find the sums.

1. 7 + 4 = 11
2. 3 + 9 = 12
3. 6 + 6 = 12
~~4.~~ 8 + 2 = 11
5. 5 + 9 = 14
6. 6 + 3 = 9
~~7.~~ 8 + 7 = 14
~~8.~~ 7 + 9 = 15
9. 0 + 9 = 9
~~10.~~ 7 + 6 = 15
11. 8 + 8 = 16
12. 6 + 4 = 10
13. 9 + 7 = 16
~~14.~~ 5 + 8 = 14

Give the sum for each

10	13	9	12
11	6	5	11
11	7	4	11
12	13	9	10

12	8	17	13
11	3	8	11
14	5	9	14
13	8	17	12

Teaching
Page h-6

Explain to the children that Jeanie worked out a set of equations just as they did on page h-6; however, Jeanie made a few mistakes. Therefore, they should go through the equations and see if they can find the mistakes that Jeanie made. With some children you might want to point out that they should write the correct answer beside those equations which Jeanie did incorrectly. At the bottom of the page, point out that they should add the numbers in the green boxes both down, across, and diagonally. You might work through a few examples with them and have them trace over the dashed numerals. Be sure to give children an opportunity to check their work and recognize any errors.

Follow-up

Single-column addition tables like the following may help children who are having difficulty learning a family of addition facts accurately.

Add 8	Add 8	Add 9	Add 9
3	7	2	5
4	5	3	2
5	3	4	7
6	9	5	6
7	8	6	4
8	4	7	8
9	6	8	9
		9	3

You might also use an arithmetic review game similar to a spell-down to provide both fun and practice for children. For a "Drill-down," choose two evenly matched teams. Give the first player on one team a combination. If he answers by giving the correct sum within twenty seconds, continue by giving a combination to the next player on the same team. If he fails to answer or is incorrect, he must sit down, and the opposing side gets a turn to compete. The team with the greatest number of players left standing after all have had a turn is the winner.

Teaching
Page h-7

Since the purpose of this module is to develop in the children the ability to find the correct sums and to do so with speed, you might encourage them to do this page as quickly as possible. If your classroom has a clock, you might suggest to the children that they time themselves as they do the page and then, if possible, jot down the time so that they can remember how long it took them to complete the exercises. Or, you yourself might like to make a note of how fast the children are working. It is important that children have a chance to assess their ability to work with speed and correctness. For example, if a child works through the page very quickly but gets many incorrect answers, he should slow down his work and strive for more accuracy; however, another child who works very slowly and gets mostly correct answers should be encouraged to work more quickly.

Show you know

Find the sums.

$8 + 3 = \boxed{11}$

$3 + 2 = \boxed{5}$

$7 + 6 = \boxed{13}$

$2 + 5 = \boxed{7}$

$6 + 9 = \boxed{15}$

$5 + 7 = \boxed{12}$

$8 + 2 = \boxed{10}$

$4 + 5 = \boxed{9}$

$3 + 7 = \boxed{10}$

$8 + 6 = \boxed{14}$

$8 + 7 = \boxed{15}$

$4 + 8 = \boxed{12}$

$7 + 2 = \boxed{9}$

$4 + 9 = \boxed{13}$

$2 + 4 = \boxed{6}$

$4 + 7 = \boxed{11}$

9 + 5 = 14	3 + 6 = 9	9 + 6 = 15	5 + 6 = 11	4 + 4 = 8	7 + 7 = 14
6 + 6 = 12	9 + 8 = 17	8 + 5 = 13	8 + 8 = 16	5 + 5 = 10	6 + 7 = 13

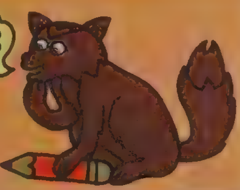
Objective

The child will demonstrate his ability to work with the skills developed in this module.

Pre-Book Activity

Various relay games might provide the children with a stimulating review of the facts which they have been studying. For example, draw an addition table on the chalkboard. You might write the numerals in order from 0 to 9 for the first row and the first column or, if the children are capable, you might put the numbers down in random order. Then divide the class into two evenly matched teams. Give each team a different colored piece of chalk. The first child on one team should go to the chalkboard and write a sum in any box that he chooses. Then he should pass the chalk to the second child on his team. Meanwhile, the first child in the other team should go to the chalkboard and write a sum. Thus, members of the two teams are alternately writing a sum. When all of the spaces have been completed, the sums can be checked and the number of correct answers for each team recorded. A variation of a relay with addition tables would be to write addition tables similar to those shown in the follow-up on page 249 on the chalkboard. In this way you might divide the class into a greater number of evenly matched teams.

Let's have fun



2, ~~3~~, 4, 5, ~~6~~, 7, ~~8~~, ~~9~~, 10

Can you put these numbers in the squares so the sum in any row →, column ↓ or diagonal ↖ ↙ is 18?

2, ~~3~~, 4, 5, ~~6~~, 7, ~~8~~, ~~9~~, 10

The puzzle is started for you.

5	10	3
4	6	8
9	2	7

Magic square—practice with sums

TEACHING
Page h-8

This change of pace page provides the children with a magic square number puzzle. Explain to them that they are to use only the numbers written at the top, and they must put them in the squares so that every diagonal, every row and every column gives the sum 18. You might point out that 3 plus 6 plus 9 gives the sum of 18 and that that is what you mean by the numbers in the diagonal adding up to equal 18. When children have completed this puzzle, you might provide them with others as suggested in the follow-up.

FOLLOW-UP

The following are other magic squares children might enjoy after doing page h-8.

1	4	4
6	3	0
2	2	5

8	1	6
3	5	7
4	9	2

3	5	4
5	4	3
4	3	5

6	7	2
1	5	9
8	3	4

5	6	1
0	4	8
7	2	3

3	10	5
8	6	4
7	2	9

(Numerals in color should be left for the children to fill-in.)

RESOURCES FOR ACTIVE LEARNING

For resources on Magic Squares, refer to the yellow module of Unit F.

Sums and Differences to 18

Pages h-9 to h-20

General Objectives

To help children master the addition and subtraction combinations through 18

To review and extend the idea of finding the differences by thinking of missing addends

To stress the importance of word problems in arithmetic

To provide practice with the combinations in word problem situations

This module culminates the formal development for addition and subtraction skills through 18. It also serves as a preparation for use of the facts in regrouping as studied in the modules on the addition and subtraction of two-digit numbers. Children should be encouraged to memorize these facts; however, emphasis should depend on the ability of the children. Memorization of the facts should only follow a clear understanding of how to solve the addition and subtraction exercises for combinations through 18. Thus, if a child still has difficulty finding sums and differences, he should not be expected to memorize the facts. However, most children at this level should be encouraged to learn the facts for speedy recall. Again, it is emphasized that a child need only memorize addition facts, that differences of subtraction exercises might be found by thinking of a missing addend. Thus, if a child knows only the addition combinations through 18, he should be able to find both sums and differences for combinations through 18.

Teaching Orange Module, Unit H

Approximate Time: 6 to 8 days

Materials

flashcards for combinations through 18
scissors

Notice that the list of materials is very brief. Since this module emphasizes the memorization of the facts, the use of concrete materials is not emphasized. If a child still needs to use concrete materials, he should be allowed to do so, but hopefully most children will have progressed beyond this stage and will be able to concentrate on memorizing the combinations.

Evaluation of Progress

To evaluate the children's mastery of both addition and subtraction combinations through 18, examine carefully a large number of exercises that the children have done. Keep in mind that the time element is important. To evaluate the children's skill in handling combinations, you must present enough material to show whether or not the children can respond quickly and accurately or whether they must still rely on counting or some concrete device.

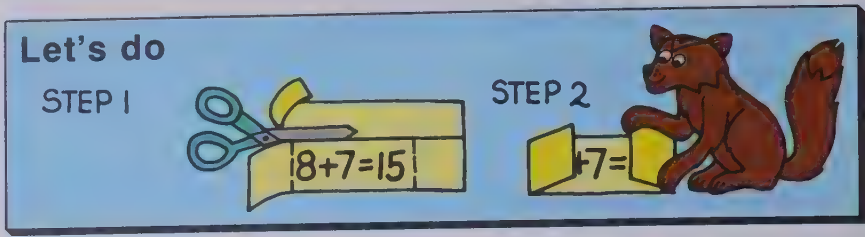
Resources For Active Learning

General Activities:

For games to practice basic facts, refer to the Introduction to the yellow module in Unit H.

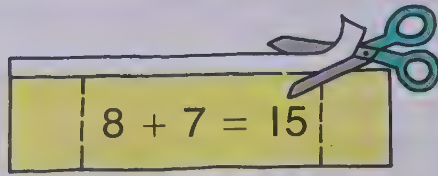
Investigation
Page h-9

You will probably want to suggest to the children specific guidelines for a game using the equation slips. For example, first suggest that they work with a partner. Show the children how either end of their slip of paper might be folded. If the left end is folded on the dotted line, the first addend of the equation will become a missing addend. If the right side of the slip of paper is folded in, the sum will be hidden. Suggest that one child show a slip of paper with the sum hidden, and his partner must tell him the hidden number. Then the first child should show either that same equation or another equation with an addend hidden. Thus, each turn consists of answering a sum and answering a missing addend equation. Suggest that the children keep a record of their responses. They might score points, giving one point for responding correctly with the sum and two points for correctly supplying the missing addend. After the points have been recorded, the partners should exchange roles. As children work, you might suggest other rules for them to follow, or they might like to make up some of their own. In any case, notice that the intent here is not only to review with the children the sums which they just studied in the previous module, but to introduce the topic of the missing addend.

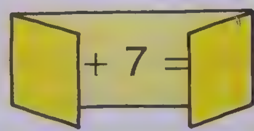


Cut and fold.

Step 1



Step 2



8 + 7 = 15	8 + 5 = 13
7 + 6 = 13	4 + 8 = 12
5 + 6 = 11	9 + 5 = 14
7 + 5 = 12	3 + 8 = 11
6 + 8 = 14	9 + 3 = 12

Purposes

To motivate the children to master the addition and subtraction facts to 18

To reintroduce the idea of finding a difference by thinking of a missing addend

Preparation

Since children must prepare their own materials for this investigation, it would be beneficial to use this preparation time for cutting and folding their slips of paper. First, explain that they should very carefully cut off the bottom section of the investigation page and save the top section for discussion. You might even choose to collect the top sections of the pages. Guide the children to cut on the solid, dark lines. Be sure that they understand that the dashed lines indicate a line for folding not for cutting. You might also distribute envelopes to the children so that each child can store these equations in an envelope.

Let's talk

Explain how to find the differences easily.

$9 + 8 = 17$ → $17 - 8 = \boxed{9}$

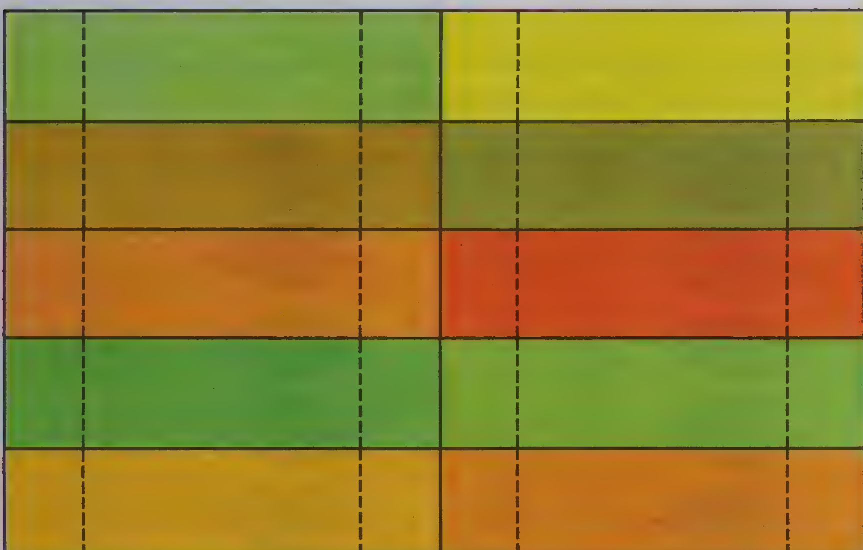
$9 + 8 = 17$ → $17 - 9 = \boxed{8}$

$6 + 7 = 13$ → $13 - 7 = \boxed{6}$

$6 + 7 = 13$ → $13 - 6 = \boxed{7}$

$78 + 49 = 127$ → $127 - 49 = \boxed{78}$

$78 + 49 = 127$ → $127 - 78 = \boxed{49}$



Discussion
Page h-10

If necessary, write the equations shown on the top of page h-10 on the chalkboard so you can work through the illustration together with the children. Point out how the addition fact is related to both of the subtraction equations shown at the ends of the arrows. Help the children see that the difference may be thought of as one of the addends, and when they are looking for the difference, they can really be thinking about finding a missing addend. Notice that the third example illustrates this relationship well since the children have not yet studied regrouping which would be necessary to solve these subtraction equations if they did not have the addition fact given. Since the addition fact is given they can think of the equation $127 - 49 = \square$ and relate it to the addition fact, finding that the missing addend is 78. Similarly, they can think of the other missing addend in the equation $127 - 78 = \square$ and find the answer to be 49. You might also explain to the children that one of the main objectives of this module is to help them be able to recall the subtraction and addition facts relatively quickly; however, stress with them that if they know the addition facts, the subtraction fact may be derived simply by thinking of the difference as a missing addend.

Follow-up

As a follow-up for this lesson, you might distribute a worksheet showing equations similar to those at the top of page h-10. Give addition facts which the children would not ordinarily know how to solve as shown below.

Find the differences by studying the addition equations.

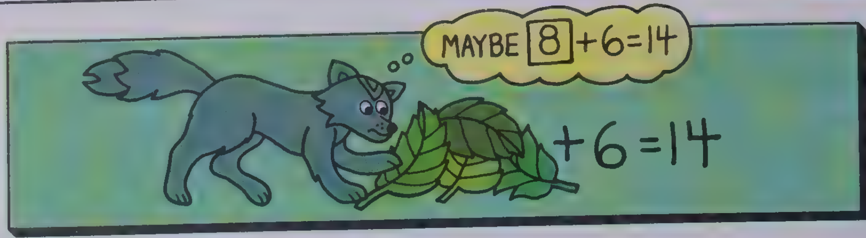
	$16 - 7 = \square$
$9 + 7 = 16$	$16 - 9 = \square$
	$104 - 79 = \square$
$25 + 79 = 104$	$104 - 25 = \square$
	$124 - 88 = \square$
$36 + 88 = 124$	$124 - 36 = \square$

Resources For Active Learning

MATHEX: Operations No. 3, "Writing Equations," pp. 4–8, Encyclopaedia Britannica Publications Ltd.

Teaching
Page h-11

Point out the two sets of directions on this page to the children. Note that in the top section they simply find the sums in the addition equations. In the second section, they are to find the missing addends. You might point out to the children that the missing addend equations at the bottom are related to the equations at the top. Thus, for each equation at the bottom, there is an addition fact which they have found in the top section. You might suggest to the children that they first find the sums from memory and then check their answers with their addition table. Then they should continue through the second part of the paper and try to find the missing addends.



First find these sums.

$6 + 5 = \boxed{11}$	$8 + 7 = \boxed{15}$
$4 + 8 = \boxed{12}$	$8 + 6 = \boxed{14}$
$7 + 6 = \boxed{13}$	$6 + 6 = \boxed{12}$
$5 + 9 = \boxed{14}$	$4 + 9 = \boxed{13}$
$7 + 5 = \boxed{12}$	$8 + 3 = \boxed{11}$

Now find these "hidden" addends.

$7 + 5 = 12$	$8 + 7 = 15$
$8 + 6 = 14$	$7 + 6 = 13$
$6 + 5 = 11$	$8 + 3 = 11$
$4 + 9 = 13$	$4 + 8 = 12$
$6 + 6 = 12$	$5 + 9 = 14$

Objective

Given an addition equation in which one addend is missing, the child will be able to find the missing addend with relatively quick recall.

Pre-Book Activity

Use an oral warm-up activity to review addition sums to 18. It would be particularly helpful to work with the addition equations that are shown on page h-11 and h-12. For example, you might take a page yourself before the children have a copy and use the addition equations shown to review the facts with the children. For example, you might say: "I'm thinking of the number that is the sum of 6 and 5." And then, to relate that sum to a missing addend, you can say to another child: "I'm thinking of a number which when added to 5 gives me 11." Since the emphasis in this module is on the development of speed skills, it should not be necessary to demonstrate these sums with display objects or concrete materials; however, with some children a review which uses concrete materials might be helpful.

Find the sums.

4 + 9 = 13	8 + 3 = 11
5 + 4 = 9	5 + 5 = 10
7 + 3 = 10	5 + 8 = 13
6 + 9 = 15	8 + 4 = 12
7 + 7 = 14	9 + 7 = 16
3 + 9 = 12	4 + 7 = 11

Solve the equations.

8 + 4 = 12	9 + 7 = 16
5 + 5 = 10	4 + 9 = 13
5 + 8 = 13	4 + 7 = 11
6 + 9 = 15	5 + 4 = 9
8 + 3 = 11	3 + 9 = 12
7 + 3 = 10	7 + 7 = 14

TEACHING
Page h-12

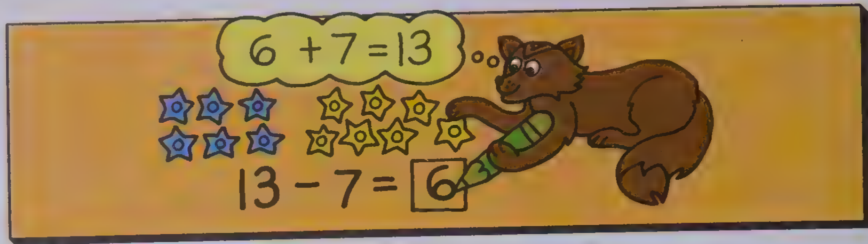
Point out to the children the top and bottom sections on this page. Here, too, the addition facts at the top are related to the equations at the bottom although they are not written in a particular order. Be sure to point out to the children that at the bottom a placeholder stands for the missing addend which they should supply. Again suggest that they first find the sums for the equations at the top and then check their answers with their addition table. When they are sure their answers at the top are correct, then they should continue to find the missing addends. Help them see that when they come to a missing addend equation which they are not sure of they can find an equation at the top to check their answer.

FOLLOW-UP

To provide children with further review of addition facts you might play the game "Guess the Addends." Use a pile of flash cards for addition facts. You might begin the game by picking up a card. Read off the sum on the card and the children should try to guess which addends are on the card. For example, if you pick up a card whose fact is 8 plus 8 equals 16, tell the children "My sum is 16; can you guess my addends?" Then they can simply take turns guessing any pair of numbers which equals that sum. If you wish to adapt this game so that children can play in twos or threes and score points, instruct the children to take a group of flash cards which show addition facts. Each player should take a turn picking up a card. When he picks up his card. he should call out the sum that is on the card. Then each player should write down a pair of numbers which they think would be addends for that sum. If a child gives a correct pair of addends for a sum, even though it is not the pair on the card, he might be given one point. If a child happens to guess the exact pair of numbers which are the addends for that sum, he might be given two points. The other players can act as judges for each other to decide if a player deserves the one point for having a correct pair of numbers which when added give the sum on the card.

Teaching
Page h-13

Point out to the children that each frame on this page has a pair of related equations in it. One, the addition equation which shows a missing addend, and the other, a subtraction equation. You might ask the children: "What is the relationship between the numbers for placeholders in each frame?" Stress that the same number will be the correct solution for each equation. Note that exercises such as this automatically encourage the children to find differences by thinking of missing addends. Encourage the children to work through the page independently striving to do the problems as quickly as possible. When they finish, suggest that they use their addition table to check their answers.



Solve the equations.

$\boxed{7} + 5 = 12$	$\boxed{9} + 4 = 13$
$12 - 5 = \boxed{7}$	$13 - 4 = \boxed{9}$
$\boxed{5} + 6 = 11$	$\boxed{3} + 7 = 10$
$11 - 6 = \boxed{5}$	$10 - 7 = \boxed{3}$
$\boxed{8} + 6 = 14$	$\boxed{7} + 8 = 15$
$14 - 6 = \boxed{8}$	$15 - 8 = \boxed{7}$
$\boxed{8} + 5 = 13$	$\boxed{9} + 3 = 12$
$13 - 5 = \boxed{8}$	$12 - 3 = \boxed{9}$
$\boxed{4} + 7 = 11$	$\boxed{8} + 6 = 14$
$11 - 7 = \boxed{4}$	$14 - 6 = \boxed{8}$

Missing addend and differences—inverse relation

Objective

Given a subtraction equation, the child will be able to find the difference quickly by using the missing addend method.

The purpose of studying missing addends is for children to be able to eventually find differences. On page h-13 the children are given related addition and subtraction equations. By thinking of the missing addend in the addition equation, the child will be able to find the difference in the subtraction equation. The purpose of studying this relation is to enable the children to find the difference in subtraction equations. Eventually, many children will respond to the subtraction facts strictly from memory as they do the addition facts, but meanwhile it is only necessary for the children to know the addition facts to master the subtraction facts.

Pre-Book Activity

Chain games such as the following will provide a review of the addition facts for the children.

Sample chain games:
Start with 7 . . . add 5 . . . subtract 6 . . . What's your answer? (6)
Start with 9 . . . add 8 . . . subtract 1 . . . subtract 9 . . . What's your answer? (7)
Start with 15 . . . subtract 9 . . . add 7 . . . subtract 8 . . . What's your answer? (5)
(The speed with which you give each step should be adjusted to the ability of the children.)

Find the differences.

13 − 7 = 6	10 − 4 = 6
10 − 6 = 4	13 − 5 = 8
8 − 5 = 3	12 − 6 = 6
11 − 2 = 9	15 − 8 = 7
18 − 9 = 9	8 − 8 = 0
12 − 7 = 5	14 − 5 = 9
14 − 8 = 6	13 − 8 = 5
9 − 5 = 4	11 − 4 = 7

13	12	15	11	7	10
−6	−8	−6	−3	−4	−5
7	4	9	8	3	5

11	16	9	14	12	13
−5	−9	−3	−6	−9	−4
6	7	6	8	3	9

Practice—differences to 18

Teaching
Page h-14

Ask the children to read the directions at the top of the page. Stress that these are subtraction exercises and that here they must find the differences. Remind them that in order to find the difference they need simply think of the related addition fact and then think of the addend that is missing. For example, for the first equation, they may simply think, "What number must I add to 7 to get 13?" and for the second equation, "What number must I add to 6 to get 10?" Point out that the vertical notation problems at the bottom of the page may be solved in a manner similar to the equations. When the children have finished, it would be helpful to check their responses together. As you check the answers you might wish to stress the use of the missing addend by giving the answers in the following manner. Since 6 + 7 is 13, 13 minus 7 is 6. Since 4 + 6 is 10, 10 minus 6 is 4, and so on.

Follow-up

As children progress in their ability to respond quickly to both addition and subtraction facts, you might again play the arithmetic game suggested for page h-4. This time, the game should include subtraction facts as well as addition facts. The game of "Roll-a-Pair," suggested for page g-48 might also be used at this time.

For additional practice on subtraction facts, duplicate, or write exercises similar to the following on the chalkboard.

Subtract 3

10		8	
6		4	
5		7	
3		9	

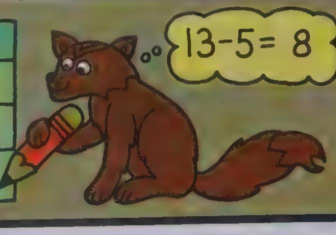
Subtract 4

10		9	
8		7	
6		5	
4		10	

Teaching
Page h-15

Direct the children to complete the tables at the top of the page. Point out to them that they should be careful to follow the rule given at the top of each table. Then explain to the children that they should try to figure out how many mistakes Mary Jo made. Tell them to mark the equations using the same marks you use in grading their papers, such as, a plus (+) mark for a correct answer and a check (✓) mark for an incorrect answer. You might also suggest that the children write the correct answer next to any equation that Mary Jo did incorrectly.

SUBTRACT 5	
11	6
12	7
13	



Complete each table.

Subtract 6	
10	4
11	5
12	6

Add 4	
8	12
9	13
10	14

Subtract 4	
12	8
13	9
14	10

Grade Mary Jo's paper.

Solve the equations. Name *Mary Jo*

7 + 6 = 13	12 − 5 = 7
✓ 8 + 2 = 11	✓ 4 + 6 = 11
✓ 9 − 4 = 13	✓ 7 − 6 = 13
10 + 6 = 16	18 − 8 = 10
12 − 7 = 5	✓ 10 + 7 = 3
✓ 15 − 5 = 9	16 − 9 = 7
9 + 2 = 11	7 + 9 = 16
✓ 14 − 7 = 6	5 + 5 = 10

Objective

Given problems involving the basic addition and subtraction facts, the child will be able to find the sums and differences.

Pre-Book Activity

Lead a short oral drill on addition and subtraction combinations through 18. For example, say: "I'm thinking of the sum 9 + 6. What's my number?" or "I'm thinking of the difference 15 − 8. What's my number?" After this, introduce word problems about the children themselves or situations familiar to them. Here are some suggestions.

1. Gloria and Debbie have birthdays on the same day. Gloria was nine and Debbie was seven. Which girl was older? How much older?
2. Debbie invited ten girls to her birthday party in the morning. Then four of the same girls went to Gloria's party that afternoon. How many girls went to just Debbie's party that day?
3. Gloria asked six boys and six girls to her party. Did she have more children at her party than Debbie? If so, how many more?

Carry the discussion of a single problem as far as possible with your class; that is, bring out comparisons among details in the problem and relate these findings to facts presented in earlier problems. Some children may be more successful with short story problems which

Freddie the Frog

Freddie the frog is the happiest frog in the pond. He is always jumping around and having fun. You can find out more about Freddie and his friends if you work the problems.

1. Freddie jumps 7 metres.
   Then he jumps 6 metres.
   How far does he jump? 13 metres

2. Freddie is in a jumping contest.
   He jumps 17 metres. His friend can jump only 8 metres. How much farther can Freddie jump? 9 metres

3. One day Freddie caught 9 flies.
   The next day he caught 5.
   How many did he catch in all? 14

4. Freddie went to see Tim the Turtle. He made 13 jumps to get there. He made only 9 jumps coming back. How many more jumps to get there? 4

5. Freddie saw a snail.
   It crawled 6 centimetres in one hour and 9 centimetres in another hour.
   How far in all? 15 centimetres

6. Freddie saw 14 fish by a rock in the pool.
   8 fish swam away. How many fish were left? 6

7. 13 birds were in a tree.
   Freddie made a big splash.
   8 birds flew away.
   How many were left? 5

Teaching
Page h-16

Many children will need your help in reading the problems on this page; however, encourage them to think through the solution of each problem independently. After you have read the problem, you might ask them to write an addition or subtraction equation that could be used to solve the problem. You might wish to work through the entire page with the children having them just write the equations which they may complete later. Alternatively, you might wish to have the children solve each problem as you work through it together. Help the children see that each problem really has two parts: the information given and the question that they are asked. Their challenge is to figure out how to use the information given to find the answer to the question that is asked.

give only pertinent facts; such as: "16 guppies. 7 sold. How many left?" It would be helpful to ask the children to write equations for each problem.

Follow-up

To encourage creativity and to improve the children's attitude toward story problems, try this novel approach.

Ask the children to write their favorite number between 0 and 10 on a piece of paper. Then have them silently count the number of objects of some kind that they can see in the room and record this number on their paper. Then each child makes up a story problem using his two numbers. The children can take turns reading or telling their stories to the rest of the class. Each child should be sure to try to solve the problem he has written.

You may wish to make duplicated copies of some of the better stories or post some of the more interesting story problems on the bulletin board.

Resources for Active Learning

MATH ACTIVITIES, Bulletin board 3–28, p. 96, Allyn and Bacon

MATHEMATICS IN MODULES: WHOLE NUMBERS • WN10 • Number Stories and Sentences, Addison-Wesley

Teaching
Page h-17

Read the directions at the top of the page with the children. Explain to them that on this page both the sets of equations and the set of vertical notation problems contain addition and subtraction problems; therefore, they should note the operation signs carefully. If the children are able you might ask them to time themselves as they work through this page. For example, point to the clock and write on the chalkboard the time that it says. Suggest that the children work the page as quickly as they can, but always try to get the correct answer. When they are finished, they should look up at the clock and write the time on their paper. Then when you are checking the pages, you might observe which children are working only for speed and need to slow down to try to get the correct answer and which children get many correct answers but take a long time in doing so. This latter group of children should be encouraged to find their answers a little bit quicker.

Find the sums and differences.

$8 + 4 = \boxed{12}$	$3 + 7 = \boxed{10}$
$6 + 5 = \boxed{11}$	$14 - 7 = \boxed{7}$
$4 + 9 = \boxed{13}$	$10 - 6 = \boxed{4}$
$13 - 6 = \boxed{7}$	$7 + 8 = \boxed{15}$
$11 - 5 = \boxed{6}$	$15 - 9 = \boxed{6}$
$12 - 8 = \boxed{4}$	$5 + 7 = \boxed{12}$

$$\begin{array}{r}9\\+3\\\hline 12\end{array}\quad\begin{array}{r}8\\+6\\\hline 14\end{array}\quad\begin{array}{r}6\\+7\\\hline 13\end{array}\quad\begin{array}{r}14\\-8\\\hline 6\end{array}\quad\begin{array}{r}13\\-5\\\hline 8\end{array}\quad\begin{array}{r}11\\-7\\\hline 4\end{array}$$

$$\begin{array}{r}7\\+7\\\hline 14\end{array}\quad\begin{array}{r}12\\-9\\\hline 3\end{array}\quad\begin{array}{r}13\\-7\\\hline 6\end{array}\quad\begin{array}{r}4\\+7\\\hline 11\end{array}\quad\begin{array}{r}8\\+5\\\hline 13\end{array}\quad\begin{array}{r}11\\-6\\\hline 5\end{array}$$

Practice—sums and differences

Objectives

Given addition and subtraction equations, the child will find the sums and differences relatively quickly.

Given word problems which are based on one of the basic addition or subtraction facts, the child will be able to solve the problems.

Keep in mind that mastery of the facts should not be expected from all children at this stage. There will be a great variety in children's ability to respond to these facts quickly. Some children will master the facts with little difficulty; others who find working for speed a detriment should be made to realize that it would be more important for them, if necessary, to work with concrete materials in order to get the correct answers rather than to try to work simply for speed and not get the correct answers.

Pre-Book Activity

Provide the children with a relay race with which to review the addition and subtraction facts. For example, write three or four tables similar to these on the chalkboard.

Add 4	
8	
7	
9	
3	
4	
6	

Add 5	
9	
5	
8	
6	
3	
7	

Add 6	
6	
8	
4	
7	
9	
5	

Jerry the Jet

Jerry the jet is a very big airplane. Jerry carries people all around the country. He is so fast that he can fly a long way in a short time. The problems tell you more about Jerry.

1. It takes 3 people to fly Jerry and 6 to take care of the passengers. How many? 9

2. Jerry can fly all the way across the country in 5 hours. How long to go across and back? 10 hours

3. One morning 13 jets were at the airport. 7 took off. How many still there? 6

4. Jerry visited 7 cities one week and 5 the next. How many? 12

5. Jerry likes to fly boys and girls. One trip he had 6 boys and 8 girls. How many children? 14

6. Jerry can fly 6 hundred kilometres in 1 hour. How far can he fly in 2 hours? 12 hundred kilometres

7. Jerry has 13 empty seats. 6 more people get on board. How many empty seats now? 7

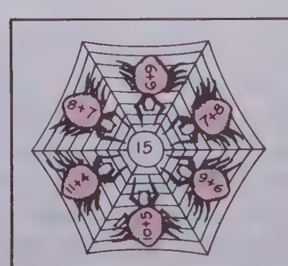
Teaching
Page h-18

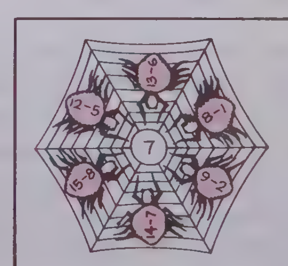
Read the paragraph at the top of the page with the children and use it and the illustrations provided to encourage children to briefly talk about their experiences with jet airplanes. With this motivation, you might want to guide the children through each of the problems; however, if their reading ability allows, encourage them to work the problems independently. It would be helpful to suggest that they write an equation for each problem. If you guide the children through the problems, you might have a child write an equation on the chalkboard for each problem after they have discussed what operation, addition or subtraction, should be used and what the equation should be.

Assign the children into groups and explain that the first child in each team should come up and fill in a sum on the table. When he finishes he should return and pass the chalk to the next member on his team who in turn should come up and write another sum. It is not necessary that the children add the sums in consecutive order, but explain that the goal of each team should be to finish the table as quickly as possible and that the first team to complete their table with correct sums wins. You might also use tables which review the subtraction facts.

Follow-up

To give children who seem to be having difficulty mastering the facts a change of pace activity, you might have them play the "Spider game." The basic material is a large web drawn on a piece of cardboard or posterboard. You also need circles the size of the centre of the web. On these circles, write the numerals from 1 to 18. Also make spiders on which you have written addition or subtraction phrases such as those shown in the illustration. Children should try to find as many spiders as possible whose sum or difference matches the centre number.





Teaching
Page h-19

In giving directions for this page, explain to the children that they are to find the sums and differences for the equations and for the vertical notation problems on the top parts of the page. Then explain that the two problems at the bottom are problems which they should try to read and solve independently. However, as they are working, if a child is having difficulty with the reading, he should be free to ask you to help him read the problem although he should try to do the arithmetic thinking independently.

Show you know

Find the sums and differences.

$8 + 4 = \boxed{12}$	$12 - 6 = \boxed{6}$
$5 + 6 = \boxed{11}$	$14 - 6 = \boxed{8}$
$7 + 7 = \boxed{14}$	$11 - 4 = \boxed{7}$
$7 + 8 = \boxed{15}$	$10 - 2 = \boxed{8}$
$2 + 8 = \boxed{10}$	$13 - 8 = \boxed{5}$
$5 + 7 = \boxed{12}$	$15 - 8 = \boxed{7}$

9	14	4	12	10	7
+5	−5	+7	−4	−1	+6
14	9	11	8	9	13
11	8	8	15	5	13
−9	+9	+8	−9	+8	−6
2	17	16	6	13	7

Betty had 15 cents. She spent 7 cents. How much does she have left? 8¢

Tom had 8 baseball cards. Len gave him 3 more. How many does he have now? 11

Module review

Objective

The child will demonstrate his ability to work with the addition and subtraction facts studied in this module.

Pre-Book Activity

An oral review game such as "What's My Rule?" might help the children prepare for the facts which they will work with on this evaluation page. Ask a child to give you a number less than ten. Respond with a number according to a rule such as, add 7. Remind the children to fold their arms when they know your rule. When you call on a child who knows the rule, he should ask you to give him a number less than ten, apply your rule and respond with the answer. If he is right, remind him to keep your rule a secret. Continue using the rule until almost everyone has his arms folded. Then continue the game by changing your rule.

Let's have fun

13	12	10
−8	−3	+5
5	9	15
F	O	X

Write the sum or difference on yellow. Use the code to put the letters on blue.

Code

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
A	C	D	E	F	I	H	N	O	R	S	T	U	W	X	Y

7	5	4	9	18	6	9	7	8	9
− 5	− 4	+ 4	+ 7	− 9	+ 7	− 6	+ 3	− 7	+ 5
2	1	8	16	9	13	3	10	1	14
C	A	N	Y	O	U	D	R	A	W

10	12	4	17	12	8	11	8	11	11
− 9	− 7	+ 9	− 9	− 4	+ 8	− 6	− 7	− 9	− 7
1	5	13	8	8	16	5	1	2	4
A	F	U	N	N	Y	F	A	C	E

Code — practice with sums and differences

Teaching
Page h-20

Read the directions with the children. Explain to them that they should first find the sums or differences and write them in the yellow boxes provided. Then they should follow the code and match the letters with the numbers to fill in all the blue boxes. When they have filled in all the blue boxes, explain that they should do what this sentence tells them to do. Be sure to provide them with paper to use for this purpose.

Follow-up

Suggest to more able children that they create reconstruction problems for sums of 18 or less or related differences. These children can trade papers with each other so that they may complete problems different from those they wrote. Sample problems are shown below.

$$\begin{array}{r} \boxed{10} \\ -\ 3 \\ \hline 7 \end{array} \qquad \begin{array}{r} \boxed{9} \\ +\ 5 \\ \hline \boxed{1}4 \end{array} \qquad \begin{array}{r} \boxed{7} \\ +\ 6 \\ \hline \boxed{1}3 \end{array} \qquad \begin{array}{r} 16 \\ -\boxed{8} \\ \hline 8 \end{array}$$

Provide children with sample problems which you write on the chalkboard or on a duplicated worksheet.

Resources for Active Learning

A CLOUDBURST, Vol. 1, Nos. 8711-2, Midwest Publications

Fractions

Page h-21 to h-30

General Objectives

To introduce the child to some number concepts other than concepts for whole numbers

To introduce the symbols and names for fractions, in particular, the fractions one half, one third, one fourth, two thirds, three fourths, two halves, three thirds, four fourths

To develop the idea of fractions with regard to both regions and sets

In this module children are introduced to the basic concepts of fractional numbers, such as halves, thirds, and fourths. The module begins with an experience in which the child physically partitions a region into a given number of same size regions, or fractional parts of the whole region. Other pages throughout the module stress not only fractional parts of regions, but also fractional parts of sets.

In teaching this module, you should keep in mind that the main emphasis should be upon an understanding of fraction concepts and the language associated with fractions, rather than upon the symbols that are used to denote fractions.

Mathematics

In this series, ideas about whole numbers were first introduced by working with concrete objects. Only after the children had much experience with physical sets did we bring out certain abstractions. The presentation of fraction concepts in this book parallels the development for whole numbers. Thus, the children are first exposed to many real physical situations involving ideas about fractions. Although the abstraction of these ideas does not come until later years, the important thing for you to recognize now is that this material provides a foundation which later will become a basis for building the abstract concepts of fractional numbers.

Teaching Red Module, Unit H

Approximate Time: 5 to 7 days

Materials

counters
crayons
demonstration felt objects for use on the flannelboard
flannelboard
materials which can be used as discrete objects for set demonstrations
overhead projector (optional)
paste or white glue
sheets of paper of various sizes

Vocabulary

fourths	one fourth	two fourths
fractions	one half	two thirds
half	one third	part
halves	three fourths	whole

In order to develop a good foundation for understanding fractional numbers, the child needs many physical experiences and activities involving fraction concepts. Activities such as sharing a candy bar with three friends or dividing a region into parts of equal size are valuable experiences in building fractional number concepts. You will find many such activities in the teaching suggestions accompanying the lessons. Adjust the material to the needs and ability of your children. Some children will need considerably more work with concrete objects before the concepts and symbols for fractions become meaningful to them.

Evaluation of Progress

Much of the child's understanding of fractions can be determined by his performance on the given pages. However, mastery of the concepts should not be stressed. Much of your evaluation might depend upon your observation of the children as they work through the activities suggested in the unit.

Resources for Active Learning

General Activities:
MATHEX: Numeration No. 2, "Fractions," pp. 48–52, Encyclopaedia Britannica Publications Ltd.

Manipulative Devices:
Discovery Blocks (Educational Teaching Aids)
Fraction Circles and Squares (Hammett; Lakeshore)
Mirror Cards (Webster, McGraw-Hill)
Mobiles (Webster, McGraw-Hill)
Symmetry patterns (Selective Educational Equipment)

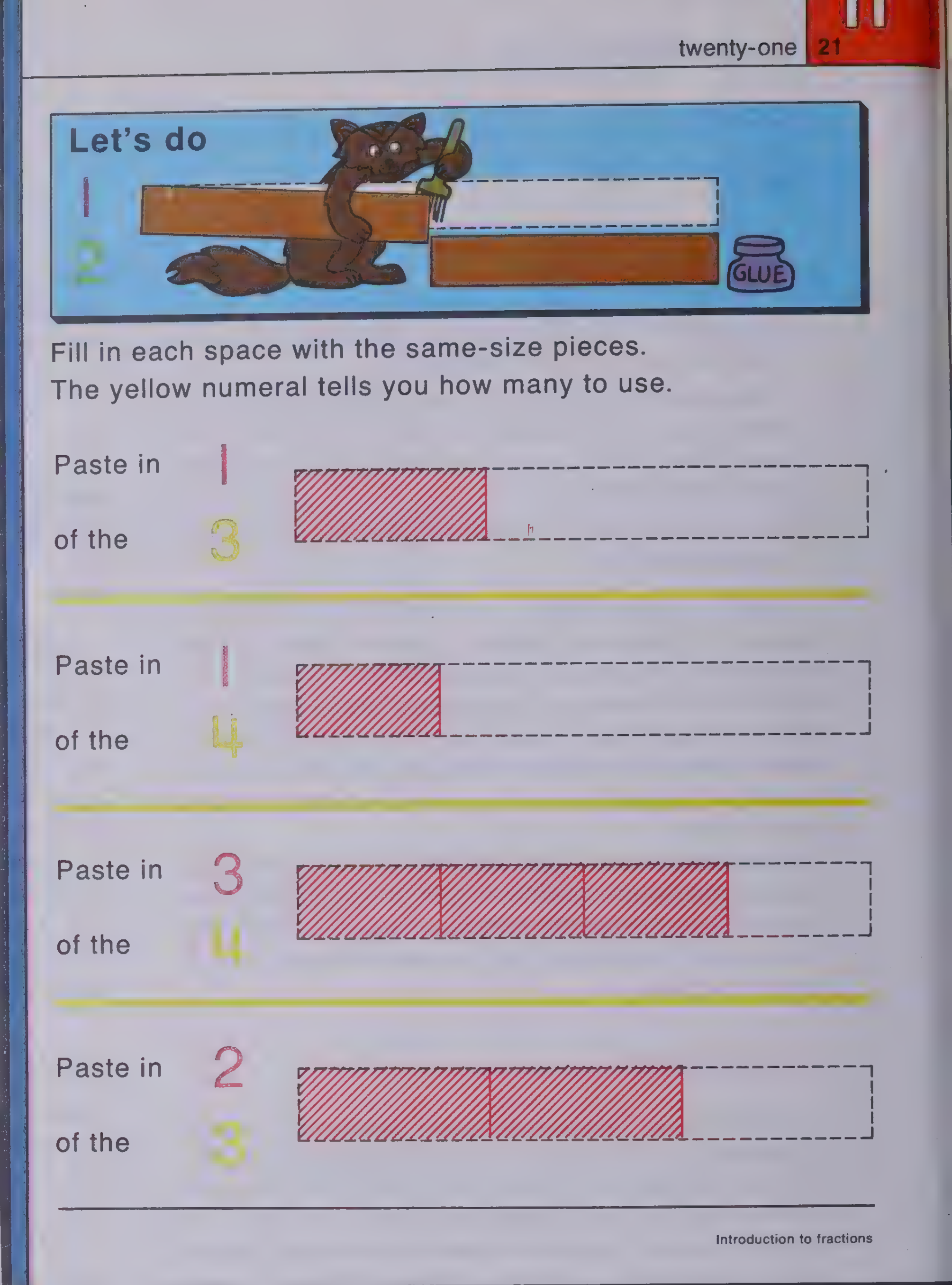
Commercial Games:
Competitive Fractions (Selective Educational Equipment)
Fraction Dominoes (Selective Educational Equipment)
Fractions Are As Easy As Pie (Milton Bradley, school supplier)

Investigation

Page h-21

Use the illustration at the top to explain this investigation. You might work through this art section as if it were an exercise even though the cutout pieces do not exactly fit the space the fox is using. Ask the children to find the longest pieces from among the cutout pieces. The longest pieces are the halves of the sections shown on the page. Explain to the children that they should try to fit these two longest pieces into the first rectangular region shown by the fox. Then explain how the fox is pasting one of these two parts onto the page. Use the phrase *one out of two pieces* to describe the piece which the fox is pasting. Point out that the yellow numeral, the 2, told them how many pieces they had to place in the rectangular region and that the pink numeral, 1, indicated how many of those same size pieces were *to be pasted* on the region. Work through the next region in a similar manner. As soon as possible encourage the children to complete the page with as little direction from you as possible. Summarize the instructions by explaining that they must first place the same size pieces on the rectangular region and then paste in as many of these pieces as the numeral indicates.

Note that the smallest cutout shows *fifths* of the rectangular region. It would be helpful to suggest that the children try to fit these into the regions according to your directions.



twenty-one 21

Let's do

1

2

GLUE

Fill in each space with the same-size pieces.
The yellow numeral tells you how many to use.

Paste in 1 of the 3

Paste in 1 of the 4

Paste in 3 of the 4

Paste in 2 of the 3

Introduction to fractions

Purposes

To introduce fractions by presenting interpretations involving both parts of regions and parts of sets

To provide the children with physical experiences related to fraction concepts developed in the module

Preparation

Materials

paste or white glue
cutout fraction pieces

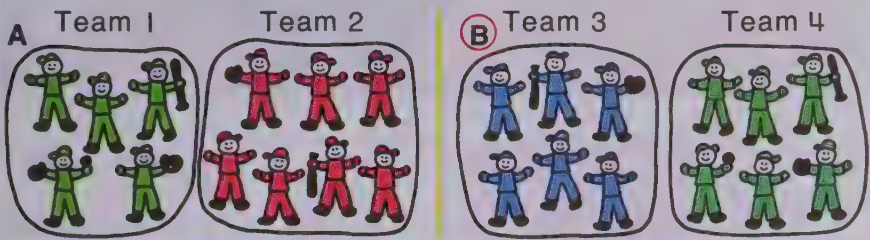
Guide the children in cutting out the 4 fraction pieces that you have duplicated. Since the children will paste these pieces on the booklet page, you need not be concerned about supplying envelopes for their storage.

(Make each bar 1.5 cm by 12 cm.)

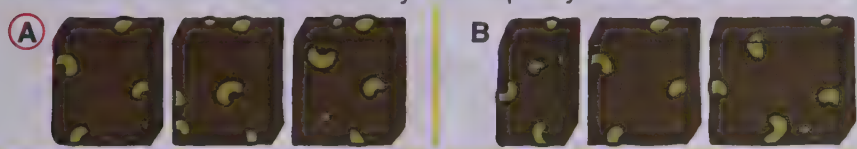
Let's talk

Which picture is correct, A or B?

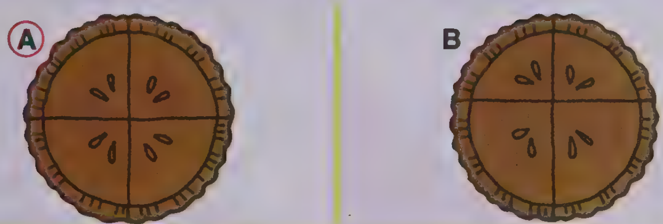
Half of the players are on each team.



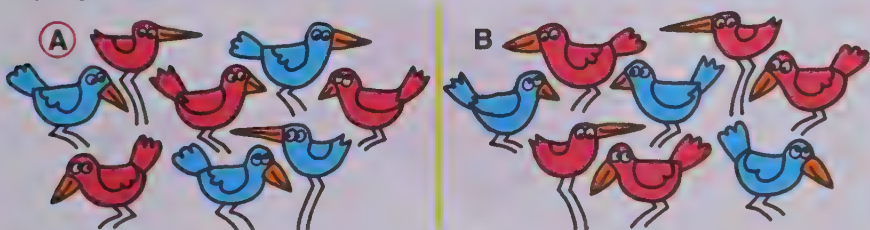
Three children share a candy bar equally.



Four pieces are the same size.



Half of the birds are red.



Discussion
Page h-22

The four sets of pictures provided on this page should be discussed with the children. Each of the four sections shows two pictures, one of which shows fractional parts, the other shows unequal parts. For each frame, read the description and give the children an opportunity to study the two pictures. Ask them to try to identify the picture which correctly shows the fractional parts described. Encourage the children to explain reasons for their choices. For example, in the first picture A, children might describe Team 2 as having one too many players, or one more than half of all the players. Use the terms *one half, one third, one fourth* as they arise naturally in the discussion of the frames. When possible, relate the discussion to the pasting activity of page h-21. For example, you might speak of each piece of candy bar in the second picture A as being one out of three same size parts of a candy bar. Observe with the children that fractions may be used with subsets of a set as well as with parts of a region. Keep in mind that discussion of these pictures should introduce, in a semiconcrete way, the common fractions which will be considered in other lessons of this module.

Follow-up

To develop the general meaning of fraction, provide children with experiences in sharing. Have various sets of objects in containers or envelopes labelled according to how many items are in each set. Then assign children into groups of three, four, five, and eight. Give each group a set arranged so that the set can be shared equally among all the members of the group. For example, give four children a set of 12, 16, or 20 items. Give three children a set of 9, 12, 15, or 18 items. Explain to the children that they should simply try to share the set equally and that in so doing each member of the group will have a fraction of the set. Suggest that they record what they do on cards. A sample recording for a group might be the following.

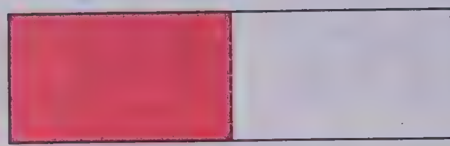
> There were 4 in our group.
> We had 12 counters.
> Each of us got 3.

The emphasis in an activity such as this is simply on allowing children in a group to have an experience of sharing items equally. It would not be necessary, for example, to discuss the fact that three twelfths is the same as one fourth or even to discuss the fraction $\frac{3}{12}$. The important thing to stress is simply that since each member in this group of 4 received an *equal number* of counters, each of them has one fourth of the whole set.

Teaching
Page h-23

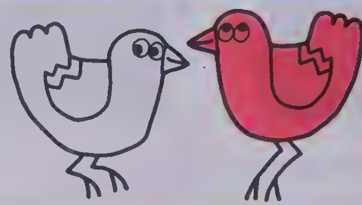
Use the illustration at the top of the page to initiate discussion of one half. Note both the set interpretations where one of two butterflies and two of four flowers have been colored and the region interpretation where one half of a circle has been shaded. Demonstrations using set collections and single objects are suitable. For example, get out a set of six checkers and divide these between two children. Point out that each child now has one half of the checkers. Write the fraction one half using a horizontal rather than a diagonal bar to separate the numerator from the denominator of the fraction $\frac{1}{2}$. Then refer to the exercises on page h-23. Point out that the directions for coloring the region or objects in the set are on the left side. The fractional part they will have colored has been printed on the right side. Thus, in the first rectangle, they should color one of the two parts. When they have done this, they will have colored one half, $\frac{1}{2}$, of the rectangle. Point out to the children that for the last exercise when they have colored two of the two parts of this figure, that is actually the same as coloring the whole figure. As children work through the page, encourage them to use both the phrase *one of two parts* and the term *one half*. It would also be helpful to point out the written words for the phrase *one half*.

Color 1 of the 2 parts.



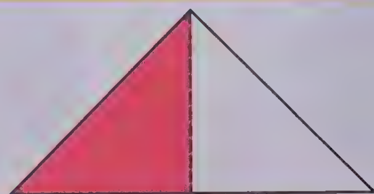
$\frac{1}{2}$ **(one half)** of the rectangle is colored.

Color 1 of the 2 birds.



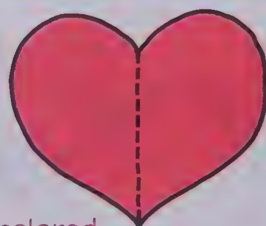
$\frac{1}{2}$ **(one half)** of the birds are colored.

Color 1 of the 2 parts.



$\frac{1}{2}$ **(one half)** of the triangle is colored.

Color 2 of the 2 parts.



$\frac{2}{2}$ **(two halves)** of the figure is colored.

Either half can be colored.

Objective

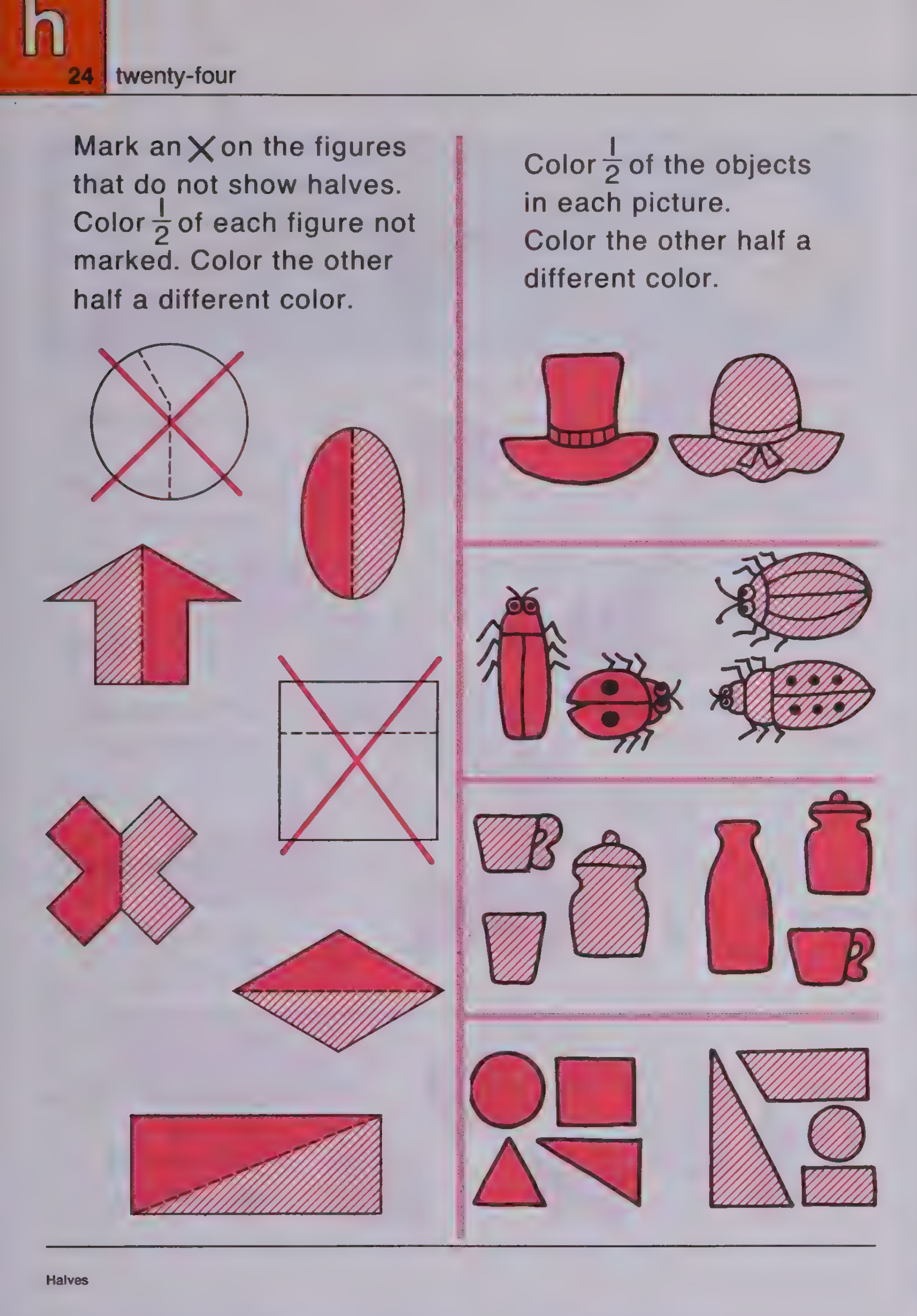
Given sets or regions divided (equally or not) into two parts, the students can select which of these are divided into halves.

Pre-Book Activity

Materials

different-sized sheets of paper

Assign the children into groups of two or three. Pass out sheets of paper of different sizes to the children so that each group has three or four sheets. Ask the children to show some different ways to fold the papers in half. Allow much freedom in this activity. Most children will probably fold the rectangles horizontally or vertically through the middle of the rectangle. A few students may fold a rectangle along one of its diagonals. To show that this kind of fold does divide the region into halves, cut along the diagonal and turn one piece over the other to show that they have the same size and shape. Thus, each piece is one half of the rectangular region.



h 24 twenty-four

Mark an X on the figures that do not show halves. Color $\frac{1}{2}$ of each figure not marked. Color the other half a different color.

Color $\frac{1}{2}$ of the objects in each picture. Color the other half a different color.

Halves

Teaching
Page h-24

Explain to the children that for the left section of the page, they should mark an "X" on the figures that do not show halves. Then they should go back and color one half of each figure that they did not mark; that is, one half of each figure that does show halves. After they have done this, they might then color the other half of the figures which show halves a different color. Some children will benefit from guidance in working through the right section of the page. Explain that each picture shows a set of objects. They should think about one-half of the objects shown and color the objects that would be in one half. Then they should take another color crayon and color the other half of the objects of that set a different color. If some children need to manipulate objects for this section of the page, allow them to do so.

Follow-up

Provide the children with crayons and large sheets of newsprint. Instruct the children to fold the sheets into eight sections and to follow the directions you write on the chalkboard. Sample directions might be:

1. Draw ten houses. Color one half of them red.
2. Draw six trees. Color one half of them green.
3. Draw eight lollipops. Make faces on one half of them.

Notice that even though the fraction referred to is one half in each case, the number of items in each set differs, giving children further experience in working with the set interpretation for fractions.

A follow-up for more capable children might be an extension of the right-hand section of page h-24. For example, refer to the first picture and ask them if they can tell you how many one half of two is. Continue down the column and ask: "What is one half of four?" or "How many is one half of six?" and "How many is one half of eight?"

Resources for Active Learning

DEVELOPMENTAL MATH CARDS, B^{1}6, D^{1}8, E^{1}10, Addison-Wesley

MATH ACTIVITIES, "Half a Rod," Game 6–6, pp. 216–217, Allyn and Bacon

Teaching
Page h-25

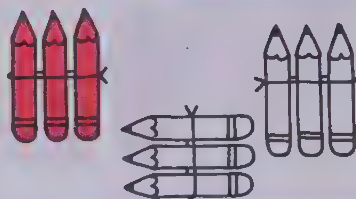
Use the illustration at the top of the page to relate this page to the pre-book activity involving thirds. Then explain to the children that on this page they are given regions or sets which have been divided into thirds and that they should color those three parts according to the directions on the left. Thus, in the first exercise they should color one of the three parts shown, then they should look on the right and read with you the explanation of what they have colored. Although some children would be able to work independently through the page, it will be helpful to guide them through it. Discuss in particular the section on the right which explains the coloring which they were instructed to do on the left.

Answers will vary. Examples are given.

Color 1 of the 3 parts shown

$\frac{1}{3}$ (one third) of the rectangle is colored.

Color the pencils in 1 of the 3 sets.

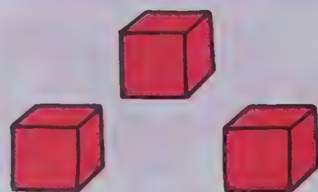


$\frac{1}{3}$ (one third) of the pencils are colored.

Color 2 of the 3 parts shown

$\frac{2}{3}$ (two thirds) of the rectangle is colored.

Color 3 of the 3 boxes shown



$\frac{3}{3}$ (three thirds) of the boxes are colored.

Objective

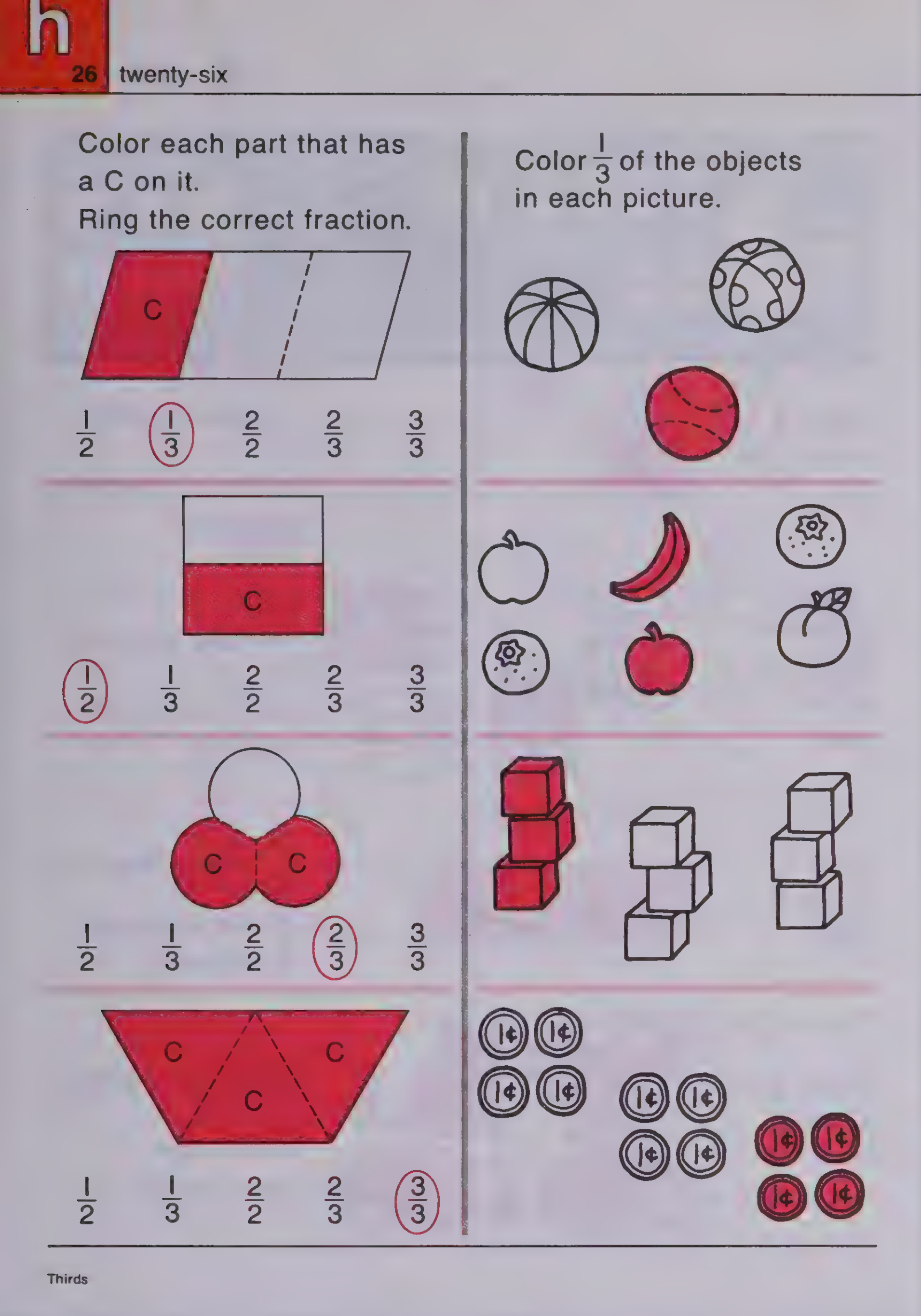
Given a set or region which has one half, one third, two thirds, or three thirds shaded or grouped, the child can select the correct fraction to describe the shaded or grouped part.

Pre-Book Activity

Materials

one strip of brown wrapping paper, approximately 10 cm by 30 cm, to each group of three children
12 counters for every group of three children

Assign the children into groups of three and give each group a strip of paper. Then say: "Pretend this is a candy bar." "Can you divide this strip so that each person in your group gets an equal amount?" Encourage children to talk about different ways of folding the strip of paper, but stress the most obvious which would be to divide the paper into sections of 10-by-10 squares. After the students have satisfactorily divided the strip so that each person has an equal amount, explain to them that since there are three equal parts and each of them has one of those parts, we say that each has one third of the whole strip and write the fraction $\frac{1}{3}$ on the chalkboard. Then pass out 12 counters to each group. Ask the children: "Can you divide these counters so that each person in your group has an equal number?" When they have done this, again write the fraction $\frac{1}{3}$ on the chalkboard and stress that each child in a group of three has one third


h 26 twenty-six

Color each part that has a C on it.
Ring the correct fraction.

C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{3}$

C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{3}$

C C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{3}$

C C C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{2}$ $\frac{2}{3}$ $\frac{3}{3}$

Color $\frac{1}{3}$ of the objects in each picture.

Thirds

Teaching
Page h-26

Although this lesson is concerned mainly with the study of thirds, the regions do include an exercise involving one-half. You might explain to the children that for the left side of the page they should color each part that has a "C" on it. Once they have colored this much of the region, they should study the fractions written underneath the region and decide which fraction describes the part of the region they have just colored. For the set interpretations on the right side of the page, explain that each picture shows a set of objects. They should think of one-third of the set and color the objects in that third. Some children might be helped by the opportunity to work with counters. Allow them to use counters if they choose. If necessary, put felt objects on the flannelboard and use set demonstrations related to the sets shown on this page.

of the total number 12. It would also be helpful to write the phrase one third to help children become familiar with the written words.

Follow-up

More able children may be ready to analyze story problems to develop a better understanding of fractions. For this purpose, you might put problems such as the following on task cards and suggest that these children work in pairs to answer the questions. Encourage them to use concrete objects as aids.

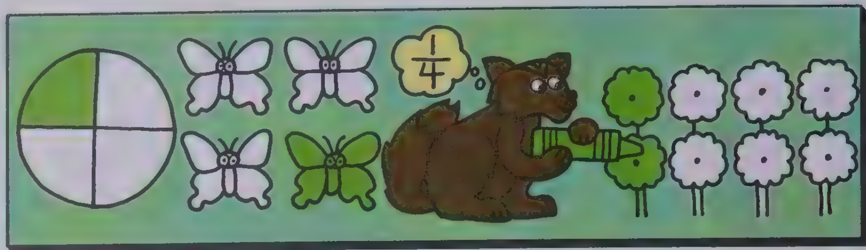
1. Philip had 12 trucks. Danny came to play. If Philip gave $\frac{1}{2}$ of his trucks to Danny, how many trucks did each boy have?
2. Kim, John, and Jack dug up some night crawlers to use fishing. They counted 15 worms in the can. They wanted to divide the 15 worms so each got the same number. Did they need to divide the set of worms into halves, thirds, or fourths? How many worms did each boy get?
3. Mildred raised guppies. Marge gave Mildred one half of her guppies. If Marge gave Mildred five guppies, how many guppies did Marge have to begin with?

Resources for Active Learning

MATH ACTIVITIES, "Rods and Thirds," Game 6–7, p. 217, Allyn and Bacon

Teaching
Page h-27

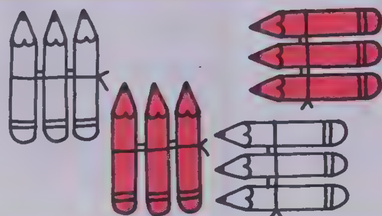
Use the illustration at the top to relate the pre-book activity to this study of fourths. As you work through the page with the children, focus on the idea that the regions or sets have been divided into four same size regions or subsets. Also take care to point out the notation which accompanies each exercise. Help the children to correctly read each of the fractions shown on the right side of the page. You might want to give special emphasis to the idea of two fourths, three fourths, and four fourths. It is not necessary at this time to discuss how two fourths may also be thought of as one half, but if a child comes up with this idea, explain that he is correct.



Color 1 of the 4 parts.

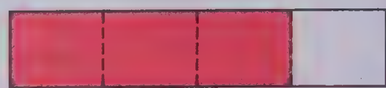
$\frac{1}{4}$ **(one fourth)** of the rectangle is colored.

Color the pencils in $\frac{2}{4}$ of the sets.



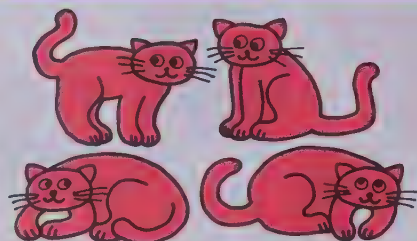
$\frac{2}{4}$ **(two fourths)** of the pencils are colored.

Color 3 of the 4 parts.



$\frac{3}{4}$ **(three fourths)** of the rectangle is colored.

Color 4 of the 4 cats.



$\frac{4}{4}$ **(four fourths)** of the cats are colored.

Objective

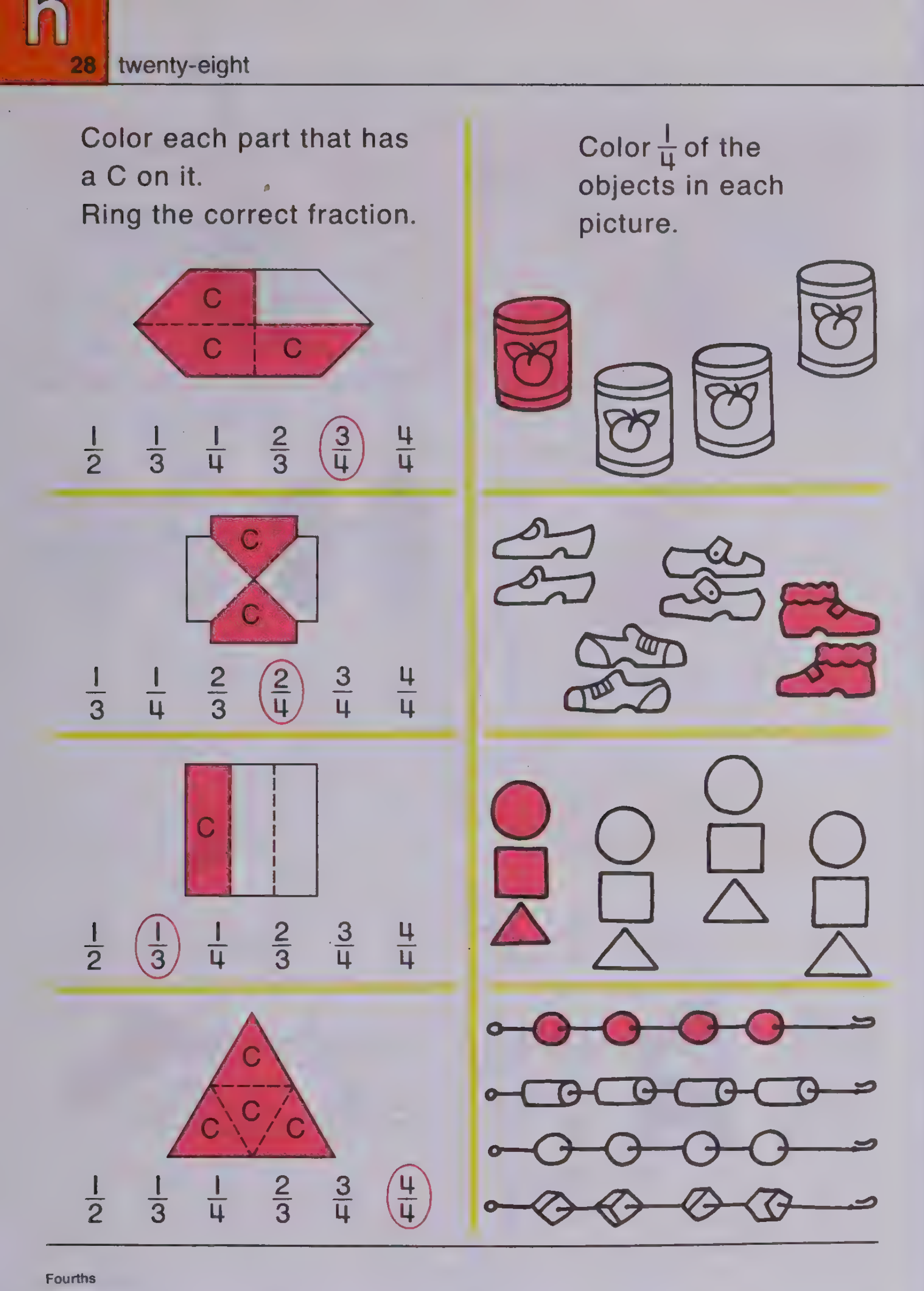
Given a region or set which has one half, one third, two thirds, three thirds, one fourth, two fourths, three fourths, or four fourths shaded or grouped, the child can select the correct fraction to describe the shaded or grouped parts.

Pre-Book Activity

Materials

a strip of paper approximately 10-by-30-cm to each group of four students
sets of counters which have 12 or 16 items

Assign the children into groups of four. Explain that you would like them to do an activity similar to those of the previous two lessons. Give some of the groups the 10-by-30-cm strips of paper and other groups sets of objects having 12 or 16 items. Then ask the children if they can divide whatever they have, either the strip or the set, so that each person in their group is given the same amount or the same number of objects. You might ask those who are working with the strip: "Into how many parts did you cut the strip?" Refer to the phrase *one out of four parts*. As you observe the children who are working with the objects, you might ask them: "Do all of the members of the group have an equal number of objects?" "Has the whole set been divided so that each of the four of you has the same number?" When children are sure of this, help them see that each of them has one fourth of the set.



h 28 twenty-eight

Color each part that has a C on it.
Ring the correct fraction.

C C C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{4}$

C C

$\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{2}{4}$ $\frac{3}{4}$ $\frac{4}{4}$

C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{4}$

C C C C

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{4}$

Color $\frac{1}{4}$ of the objects in each picture.

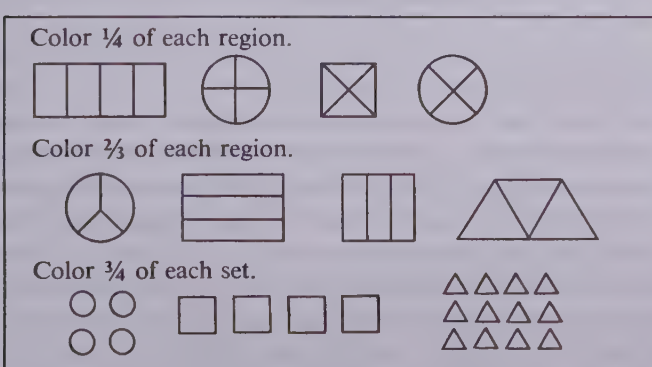
Fourths

Teaching
Page h-28

Explain to the children that on the left side of the page they will be working with regions that have been divided into halves, fourths, or thirds. First they should color each part that has a "C" on it. Then they should ring the fraction underneath the region which explains what part of the region they have just colored. Explain that the right-hand side of the page is similar to previous pages, but this time that they are interested in one fourth of each set illustrated. You might suggest that they think of how many objects would be in one fourth of the set and that they color that number of objects. Again, children who have difficulty might benefit from work with actual objects related to the pictures on the printed page.

Follow-up

Provide a worksheet of regions and sets to be colored according to instructions.



Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB, Cards 72–75, Responsive Environments Corp.

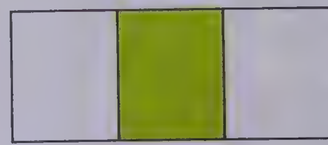
MATH ACTIVITIES, "Rod Comparison," Game 6–8, pp. 217–218, Allyn and Bacon

Teaching
Page h-29

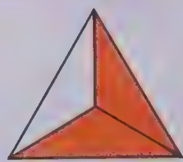
Although this page might be used as an evaluation instrument, you might choose to work through it with the children. Remind them to carefully study each region or set shown in each frame and then to think what fraction of the whole region or set is shaded. Then they should circle that fraction.

Show you know
Ring the fraction for the colored part.

$\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{2}{3}$ $\frac{4}{4}$



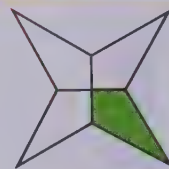
$\frac{3}{4}$ $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{1}{3}$ $\frac{3}{3}$



$\frac{1}{3}$ $\frac{2}{3}$ $\frac{1}{4}$ $\frac{3}{4}$ $\frac{1}{2}$ $\frac{3}{3}$

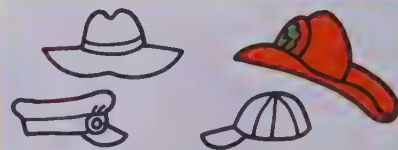
$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{2}{2}$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{3}{3}$



$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{4}$

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{3}{3}$



$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{4}$

Objective

The child will demonstrate his ability to work with the concepts developed in this module.

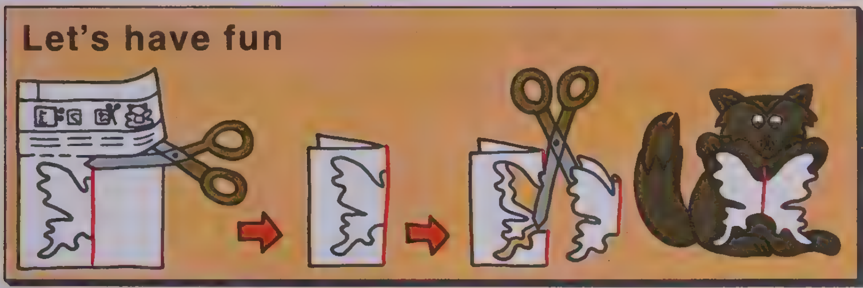
Pre-Book Activity

It would be helpful to review with the children all of the fractions which they have studied. For example, you might use demonstrations with felt objects on the flannelboard to show parts of a region or use shaded areas shown in figures such as they will be using on page h-29. Also display sets of concrete objects to demonstrate the fractions. Place particular stress on the fractions $\frac{2}{3}$ and $\frac{3}{4}$. For example, for the fraction three fourths, you might show cutouts of four small racing cars. Point to one racer and explain that we can think of one racer as one fourth of the group and we can think of the other three racers altogether as three fourths of the group. Ring three cars with yarn and show that we write the symbol $\frac{3}{4}$ for three fourths. Work similarly through a set for the fraction $\frac{2}{3}$.

Follow-up

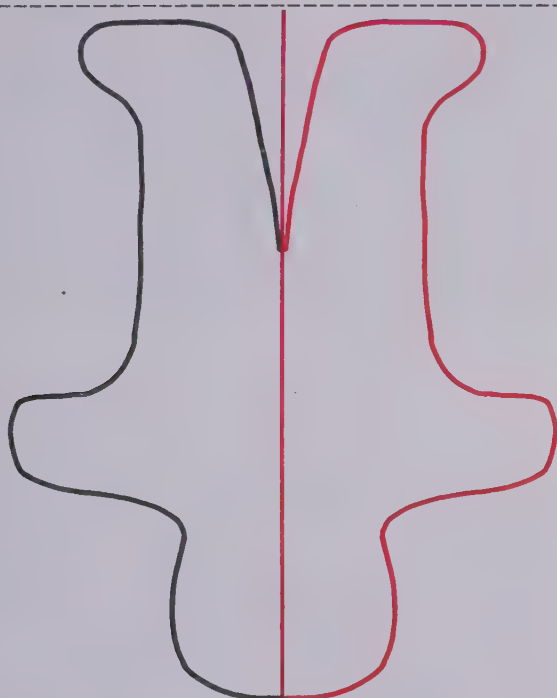
If you have some favorite activities which involve symmetric figures, it would be appropriate to introduce them at this time. For example, you might duplicate one half of the face of a clown on art paper and instruct the children to very heavily color that half of the face. Then they should fold the paper vertically in half and using a ruler or another stiff object apply pressure on the outside of the page to try to transfer the crayon from one side to the other. That is, have them take the colored half and fold it over the uncolored half and using a ruler go

Let's have fun



Cut off the bottom of this page. Fold along the red line. Cut out the figure. Unfold.

A symmetrical figure is formed. What does it look like?



Symmetry

TEACHING
Page h-30

This change of pace page gives children an opportunity to work with a symmetric figure. Explain that they should simply cut along the dotted line to cut the bottom half of the paper off, then fold the paper on the red line and cut on the black curved line. Stress with them that they cut only on the black lines and fold along the red line.

back and forth vertically and horizontally slowly and with much pressure. Then when they open up the page, the imprint of the colored half of the page will appear at least lightly on the other half of the page. They can then finish the coloring of this half following the outlines formed by the transferred crayon. If you choose, you might replace the medium of crayon with that of paint; however, here the two halves must be folded upon one another while the paint is still wet and yet not wet enough to cause smearing.

RESOURCES FOR ACTIVE LEARNING

Symmetry—reflection:

DEVELOPMENTAL MATH CARDS, $C^3 19$, $D^3 3$, Addison-Wesley

Franklin Series: MIRROR MAGIC, pp. 58–119, Lyons and Carnahan

MATHEMATICS IN MODULES: SPATIAL KNOWLEDGE • SK2 • Plane Shapes, pp. 8–11, Addison-Wesley

MATHEX: Geometry No. 4, pp. 30–34, Encyclopaedia Britannica Publications Ltd.

MIRROR CARDS, Webster, McGraw-Hill

Nuffield Project: BEGINNINGS 1, pp. 81–92; SHAPE AND SIZE 2, p. 45, Wiley

THINK AND COLOR, pp. 88–92, Educational Science Consultants

LIGHT GREEN MODULE, UNIT H

Addition and Subtraction with Regrouping – Power Skills

Pages h-31 to h-38

General Objectives

To develop concepts related to the addition and subtraction of two-digit numbers

To introduce power skills for the addition and subtraction of two-digit numbers with regrouping

The purpose of this module is to introduce the child to power-skill methods for the addition and subtraction of two-digit numbers in which regrouping is necessary. The regrouping algorithm is not presented as such, rather the child is encouraged to figure out such exercises by using the power-skill methods suggested. That is, the child is expected to use sets, strips, the number line, counting, etc. to find the sums and differences. The algorithm for regrouping will be studied in the following two modules.

Mathematics

This module deals strictly with the power skill in finding sums of two-digit numbers involving the regrouping concept; however, this regrouping is not performed as an algorithmic skill in this module. That is, the module focusses primarily upon finding these sums through various means such as simply counting. This counting process is accomplished through use of sets and number lines. When sets are used, the attention should be focussed upon the regrouping involved when the objects used for the units are put together. For example, in finding a sum such as 27 and 38, the children should see two bundles of ten and seven extras, along with three bundles of ten and eight extras. Then when the two sets are combined, the children should have attention focussed upon the fact that the seven units and the eight units together give one ten and five units besides. This regrouping at the power-skill stage or the set stage is important for future understanding of the regrouping in the more abstract algorithmic form.

Teaching Light Green Module, Unit H

Approximate Time: 4 to 6 days

Materials

counters or other concrete objects which the children can use individually

demonstration number line (which can be used with two-digit addition; a chunk or lattice number line may be obtained from an ADDISON-WESLEY supplier)

orange strips and white strips for each child

sticks, pipe cleaners, tongue depressors or other objects which can be grouped in bundles of ten

Vocabulary

regrouping

The emphasis in this module should be on the power-skill methods which utilize the concrete materials listed above. That is, the activities with the materials should stress understanding the concepts involved, rather than the numerical solution to a particular exercise.

Evaluation of Progress

It is not expected that each child master the skill of adding and subtracting two-digit numbers, even with the use of these materials. Hopefully, however, each child should be able to use at least one method involving concrete materials to enable him to solve these problems. Your evaluation of the children's work should not be limited to their performance on page h-37. You will be able to evaluate their understanding of the concepts as you observe their work throughout the module.

Resources for Active Learning

General Activities:

Computation with regrouping:

CHIP TRADING ACTIVITIES – SET I, Cards 17–28; SET II, Cards 8–15, Scott Scientific

Nuffield Project: COMPUTATION AND STRUCTURE 2, "Addition," pp. 74–77, Wiley

DIENES MULTIBASE ARITHMETIC BLOCKS – Tasks, Cards 1–12, Herder and Herder

NUMBER-BLOX, Creative Publications

Manipulative Devices:

Abacus or abacus boards (Educational Teaching Aids; Selective Educational Equipment)

Chips with hole in centre (Educational Teaching Aids; Selective Educational Equipment)

Multi-base arithmetic blocks (Educational Teaching Aids; Herder and Herder)

Primary tile and abacus board (Educational Teaching Aids; Math Media)

Investigation
Page h-31

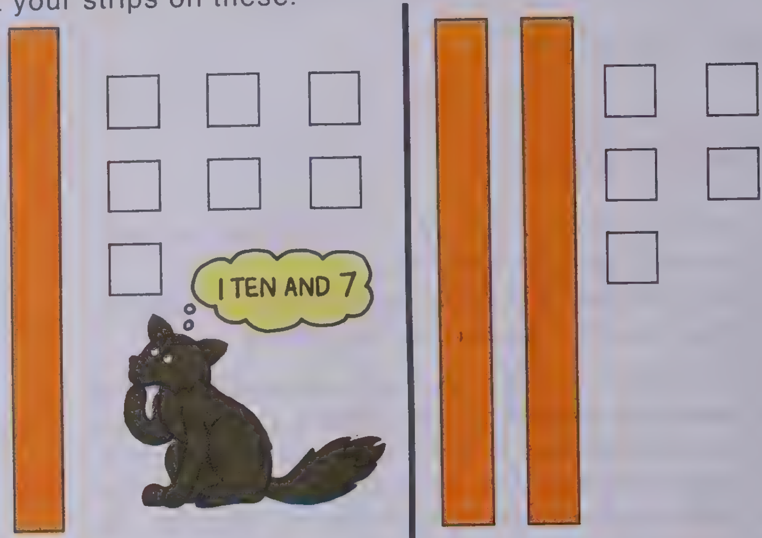
It will be important to guide the children through this investigation. Explain first that they should put their strips on top of those illustrated. Then they should write the numbers which these two sets of strips represent. They should then combine these two sets of strips. In order to answer the question: "Now how many?" It would be helpful to suggest that they first think, "How many tens do I have?" Encourage the children to continue counting their strips until they count up to the appropriate number, 42. Read the actual investigation question at the bottom of the page with them. Be sure that they understand that they should only be using the orange strips and the white strips; therefore, the answer to the investigation question should be four orange and two white strips. In order to show this number they will have to replace the ten white strips with one orange strip. This is the most important experience of the investigation, as it is the very essence of the regrouping process. Work through several other examples by putting directions similar to the following on the chalkboard:

Use only orange and white strips to show these pairs of numbers:

1) 26 and 49
2) 45 and 18

Let's do

Put your strips on these.

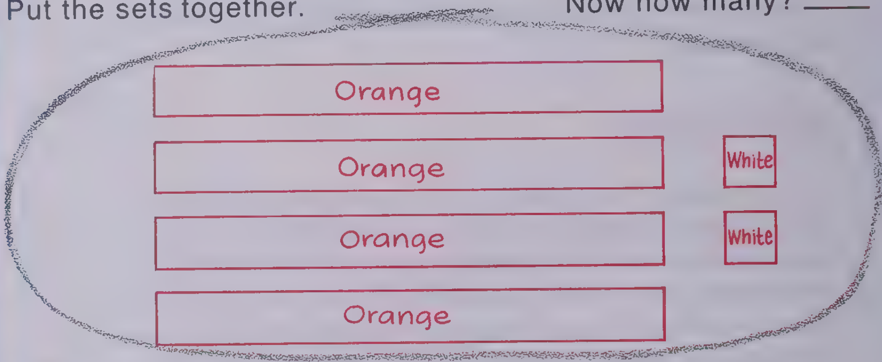


How many? 17

How many? 25

Put the sets together.

Now how many? 42



Can you show this number with the fewest possible number of strips?

Introduction to addition with regrouping

Purposes

To provide readiness for regrouping of two-digit numbers

To introduce some power-skill methods for the addition and subtraction of two-digit numbers

Preparation

Materials

one set of strips for each child

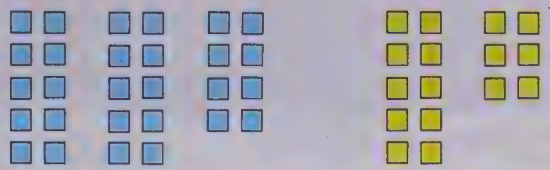
To prepare for this investigation, it would be helpful to review with the children how the strips can be used to represent two-digit numbers. For example, ask the children to use only the orange strips and the white strips to show several two-digit numbers. Stress with them that although a number, such as 27, might be shown by using two orange strips and a black strip, for purposes of developing the ideas in this module you would like them to use only orange strips and white strips. Thus, they should represent the number 27 using two orange strips and seven white strips. During the preparation, remind the children that the orange strip here represents the number ten, and you might even suggest that it could be replaced at anytime by a group of ten white strips.

Let's talk

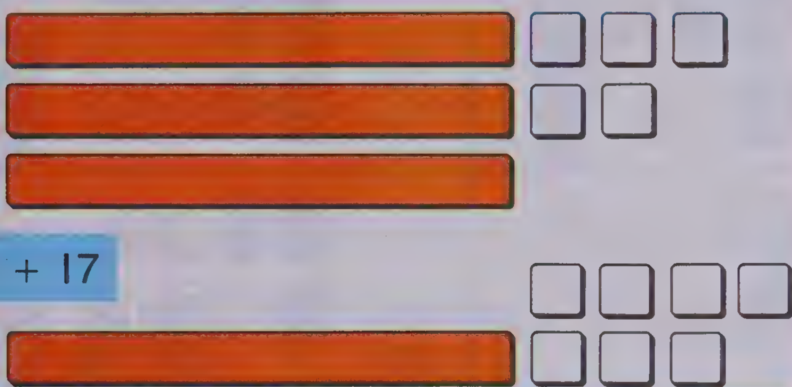
Explain how to find the sum.

Using sets.

28 + 16



Using strips.



35 + 17

Using a number line.

46 + 8

Using reasoning.

26 + 15

26	26	26	26	26
+ 2	+ 3	+ 4	+ 5	+ 15
28	29	30	31	41

Introduction to addition with regrouping

Discussion
Page h-32

The frames on this page give you an opportunity to introduce the various power methods which will be developed throughout the module. Although this page deals strictly with addition, you might also discuss each of these power methods in relation to subtraction. As you work through the discussion of these frames, it would be helpful to have the various materials which the children will be using available. For example, the top frame might be used with counters piled in groups of ten, or if you have sticks in bundles of ten, use these for discussion at this time. The children should be familiar with the method used in the second frame from the investigation activity. In the third frame, point out that the number line may again be used as a counting device. In the last frame, you might simply point out how they can think of adding 1 or 2 to 26, then adding 3, 4, and watching the sums increase. For example: 26 plus 3 will give them the sum of 29. So, 26 plus 3 and one more will give them the sum of 30. Similarly, 26 plus 5, or 26 plus 4 and 1 more, will give them the sum of 31. Since they are adding 26 and 15, and they know that 15 is ten more than 5, they can simply add ten more to 31 to get their final sum of 41. Since this is the most abstract power method presented, do not expect all the children to understand it.

Follow-up

Ask the children first to show the number 32 with their strips. Then explain that you want them to take away five white strips. In order to do this, the children will have to replace their orange strip with ten white strips. Then they will be able to remove five white strips. Then ask them to record the number representing the strips which they have left.

Sample recording for 32 − 5.

3 orange 2 white → 2 orange 12 white

take away _ orange 5 white

I now have 2 orange 7 white

Many children will need direct guidance through examples such as this. A worksheet such as that suggested in the next column might be helpful.

Use only orange and white stripes			
Show	Take away	Regroup	Record
41	5 W	3 O 11 W	3 O 6 W
34	9 W	2 O 14 W	2 O 5 W
48	6 W	4 O 8 W	4 O 2 W

Resources for Active Learning

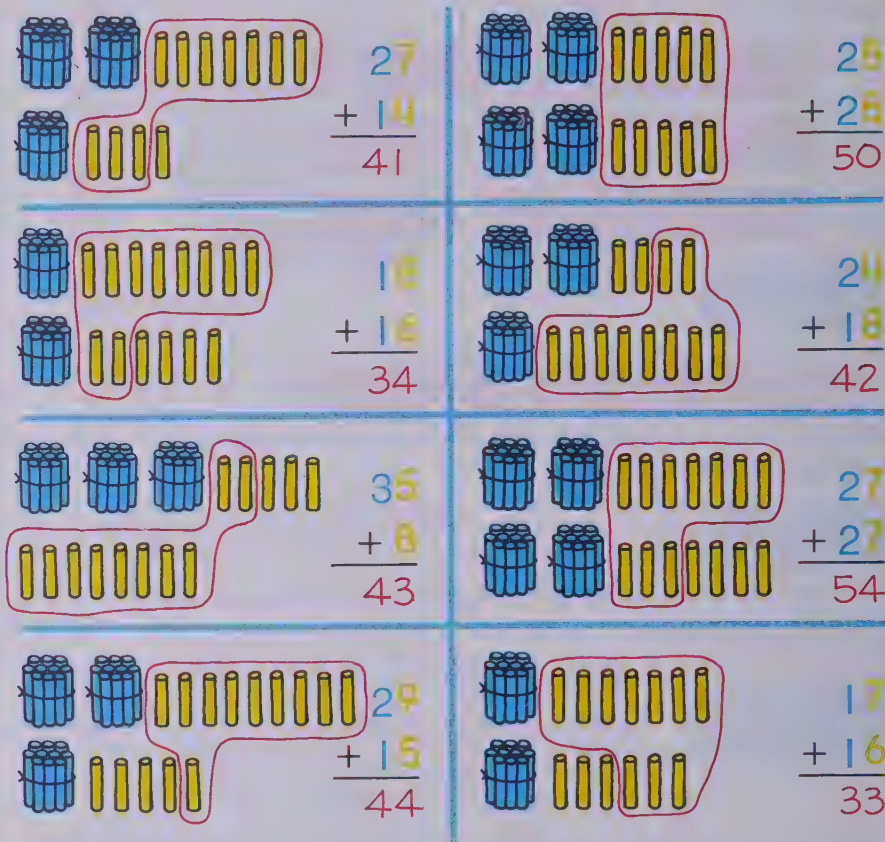
A CLOUDBURST, Vol. 1, No. 2271, Midwest Publications

DEVELOPMENTAL MATH CARDS, "Niner," D‡16, Addison-Wesley

Teaching
Page h-33

Read the directions at the top of the page with the children. It would be helpful to work through some of the frames with them. In the first frame ask them to explain how the numeral 27 relates to the illustration. Then direct them to ring a group of ten sticks as the fox in the demonstration art is doing. Help those children who are having difficulty to depend on the set illustrations and first ring ten sticks before they count to find the sum. It would be helpful to have children work in small groups with actual bundles and share their ideas about how to solve these problems.

Circle ten yellow sticks.
Can you use the sets to tell how many sticks?



Addition with regrouping—sets

Objective

Given addition problems involving two-digit addends, the child will be able to find the sums by studying set illustrations or using discrete objects.

Given a subtraction problem in which a one-digit number is subtracted from a two-digit number, the child will be able to find the difference by referring to set illustrations or using concrete objects.

Pre-Book Activity

Materials

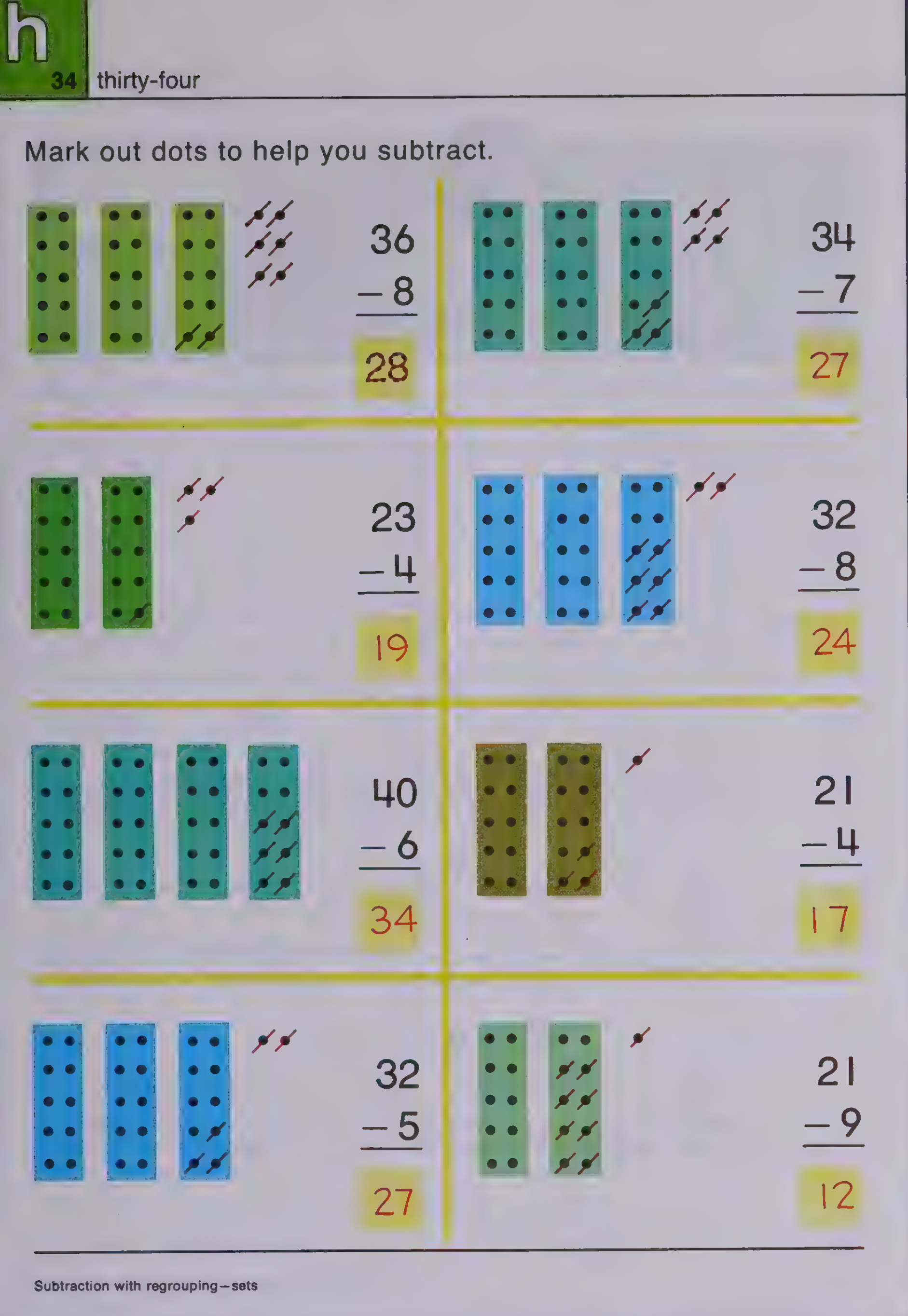
counters or other concrete objects which the children can use individually

sets for demonstration, such as sticks and sticks grouped in bundles of 10

Use individual sticks and sticks in bundles of ten to display two numbers such as 37 and 25. On the chalkboard write the vertical notation showing

$$\begin{array}{r} 37 \\ +25 \\ \hline \end{array}$$

Explain to the children that the problem is to figure out how many sticks there are altogether. Remind the children that a basic way of finding out how many is to count objects. Talk about ways of counting the two sets. Suggest that they begin with one of the sets, by taking three bundles of ten and seven sticks for the 37 and then start counting until they have added on the other 25 sticks. Remind them that every group of ten sticks should be bundled together and thought of as one ten.


h

34 thirty-four

Mark out dots to help you subtract.

36 − 8 = 28	34 − 7 = 27		
23 − 4 = 19	32 − 8 = 24		
40 − 6 = 34	21 − 4 = 17		
32 − 5 = 27	21 − 9 = 12		

Subtraction with regrouping—sets

Teaching
Page h-34

It will be necessary to work through at least the first few frames with the children. Remind them that subtraction means taking away. On this page the set of dots corresponding to the first two-digit numeral in the problems is shown. The children should mark out the number of dots indicated by the second numeral. In order to find how many are left, they might simply count those dots that have not been marked out. Capable children might count backwards as they mark out the dots. The children might use the illustrations with your guidance to prepare them for the regrouping which will be studied in more detail in the following module when the algorithmic method is introduced.

In the first example, the children might mark out six dots first since there are six single dots. Then they must think "How many more dots do I have to mark out before I have marked out eight?" Here they must go to a set of ten, think of the single dots in the group of ten, and mark out two more. Then in order to write down the difference that remains they can no longer think of three groups of ten, but they now have two groups of ten and eight single dots. Since this is a power method using counting to find differences they may use objects such as counters, if they wish. Most children, however, will be able to work directly with the illustrations of the dots.

You might also introduce another counting technique. Have them start with 37 and then count by ones, 38, 39, 40, 41, 42. When they see that they have only bundles and no more single sticks to count, talk about counting by tens. If they had 42, ten more would be 52. Finally, ten more gives them a total of 62. Thus, 37 + 25 is 62. After you have worked through a few examples of this type, children might choose to count the bundles of ten first, and then count the single objects. Let them proceed with whichever method they prefer, but stress that here is a counting technique which uses sets to help them find sums for two-digit numerals. If you prefer an activity with the strips, suggest that the children use orange strips and white strips in place of the bundles and sticks.

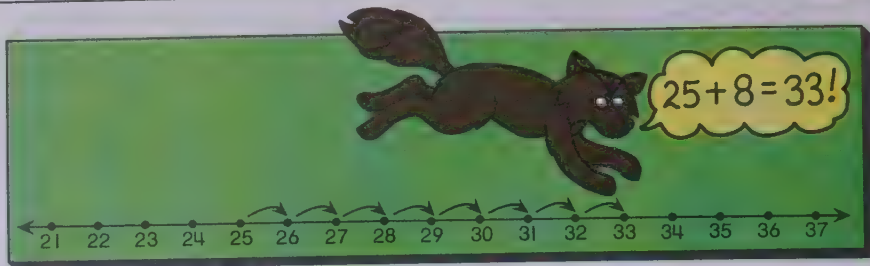
Follow-up

If necessary, give the children an opportunity to review basic facts. For this purpose, you might give them a timed test on the basic addition facts and help them identify any facts which they consistently do incorrectly. To accomplish this prepare a worksheet like the following:

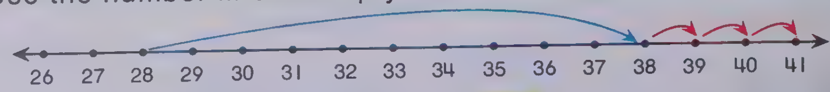
7 +9	8 +6	4 +7	5 +8	6 +9	8 +9	3 +8
4 +9	5 +9	4 +8	6 +7	8 +8	7 +7	9 +9
3 +9	5 +7	7 +8	3 +7	6 +6	6 +5	5 +5

Teaching
Page h-35

Explain to the children that the number line illustrations are given for each addition and subtraction problem, thus they will be able to work their problem directly on the number line shown in the text. Point out that in the first frame, the first jump has been a jump of 10 followed by 3 single jumps. After the children have worked the exercises, it would be helpful to have them show how they solved each exercise on the demonstration number line. Point out that exercises at the bottom of the page are subtraction exercises instead of addition exercises.

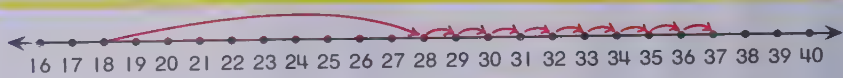


Use the number line to help you find the sums and differences.

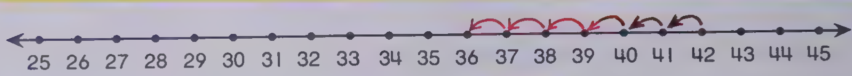


28 + 13 = 41

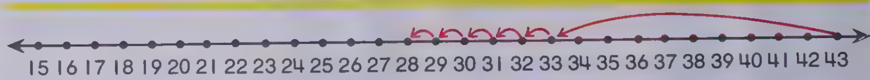
35 + 7 = 42



18 + 19 = 37



42 − 6 = 36



43 − 15 = 28

Objectives

Given addition and subtraction exercises involving two-digit numbers which require regrouping, the child will be able to find the sums or differences by using a number line.

Given a sequence of addition or subtraction exercises, the child will be able to find sums and differences of those exercises which involve regrouping by studying a patterned sequence.

Pre-Book Activity

Materials

a demonstration number line which can be used with two-digit addition (a chunk number line or hundred board)

Since a hundred board might be thought of as a chunk number line, you might use it for this lesson. Write an addition exercise such as 56 plus 17 on the chalkboard. Ask a child to find 56 on your demonstration number line or the hundred board and to show how he would use these materials to add 17. He might simply take 17 single jumps, which is acceptable; however, you might encourage the children to learn some shortcuts. For example, a child might take a jump of 10 to arrive at 66 and then take 7 single jumps. Work through other examples such as the following: Write 29 + 54 on the chalkboard. Again ask a child to begin at 29 and then to think what would be the best way to use the number line to add 54. Lead the children to see how jumping by 10's might be used. A child might begin at 29, take 4 single jumps and then 5 jumps of 10 each. Or, he might begin at 29, take 5 jumps of 10 and then 4 single jumps.

Find the sums and differences.

36 + 2 = 38	36 + 3 = 39	36 + 4 = 40	36 + 5 = 41	36 + 6 = 42	36 + 7 = 43
44 + 3 = 47	44 + 4 = 48	44 + 5 = 49	44 + 6 = 50	44 + 7 = 51	44 + 17 = 61
25 + 12 = 37	25 + 13 = 38	25 + 14 = 39	25 + 15 = 40	25 + 16 = 41	25 + 17 = 42
45 − 3 = 42	45 − 4 = 41	45 − 5 = 40	45 − 6 = 39	45 − 7 = 38	45 − 8 = 37
56 − 14 = 42	56 − 15 = 41	56 − 16 = 40	56 − 17 = 39	56 − 18 = 38	56 − 19 = 37
32 − 10 = 22	32 − 11 = 21	32 − 12 = 20	32 − 13 = 19	32 − 14 = 18	32 − 15 = 17

Addition and subtraction with regrouping – reasoning

Teaching
Page h-36

It will be helpful to work through much of this page with the children. Explain to them that each row shows six exercises which follow certain patterns. For example, in the first row all of the two-digit addends are 36. The first number added to 36 is 2, the second number added to 36 is 3, then 4, 5, 6, and 7. Point out that they know how to add 36 plus 2, and 36 plus 3. To help them add 36 plus 4, ask them how much more they are adding then when adding 36 plus 3. Help them see that since they are adding one more than 3 their sum should be one more than the sum in the previous problem. Work through this type of reasoning with all of the addition problems. Stress that in this set they can find their sums quickly if they realize the relationship between each sum and the one that went before it.

In a similar manner, help the children work through the subtraction exercises.

Also, use the demonstration number line to work through some subtraction examples. Again, encourage the children to think of the number that they are taking away if it is below 10, simply in terms of single jumps, but if it is above 10 to think of it in terms of 10 and so many ones. For example, they might subtract 8 from 36 by beginning at 36 and taking 8 single steps back. But, to subtract 28 from 36 they would begin at 36, take 8 single steps back and then take two jumps of 10 back.

Follow-up

Exercises using inequalities will help capable children think about regrouping.

Put the correct sign, < or >, in each blank:	
9 + 7 ○ 20	35 + 47 ○ 80
48 + 4 ○ 50	62 + 24 ○ 90
86 + 6 ○ 90	58 + 17 ○ 70

Many children will benefit from further work with the strips. Patterned exercises such as the following should be helpful to those having difficulty with the idea of regrouping:

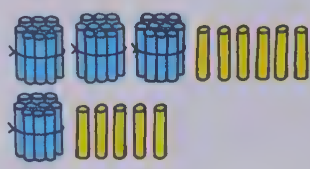
Build	Combine and record	Regroup
25 and 14	3 O 9 W	3 O 9 W
25 and 15	3 O 10 W	4 O 0 W
3 and 36	3 O 9 W	3 O 9 W
4 and 36	3 O 10 W	4 O 0 W

Teaching
Page h-37

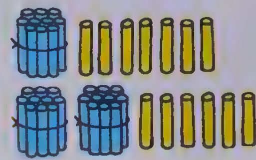
Explain to the children that they have already worked exercises like the ones shown on this page. You might want to use the page as review and work through the exercises with the children; however, if you think the children are capable, encourage them to do the work independently. Remind them that they might use counters or bundles of sticks for the first four frames. Also remind them that they might show their solutions on the number lines illustrated. Keep in mind that the purpose of this module is not to develop skill with any *algorithm*. The most important point is that the children understand how the various methods studied in the module can be applied. They should not be expected to work any of these exercises by an algorithmic method. You should allow them sufficient time to work through each problem carefully using the power methods suggested.

Show you know

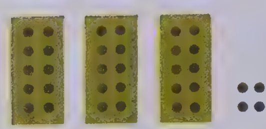
Solve.



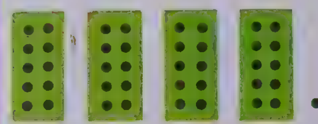
$\begin{array}{r} 36 \\ +\ 15 \\ \hline 51 \end{array}$



$\begin{array}{r} 17 \\ +\ 26 \\ \hline 43 \end{array}$



$\begin{array}{r} 34 \\ -\ 6 \\ \hline 28 \end{array}$



$\begin{array}{r} 41 \\ -\ 5 \\ \hline 36 \end{array}$

17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40

19 + 16 = $\boxed{35}$

12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35

32 − 17 = $\boxed{15}$

$\begin{array}{r} 26 \\ +\ 12 \\ \hline 38 \end{array}$ $\begin{array}{r} 26 \\ +\ 13 \\ \hline 39 \end{array}$ $\begin{array}{r} 26 \\ +\ 14 \\ \hline 40 \end{array}$ $\begin{array}{r} 26 \\ +\ 15 \\ \hline 41 \end{array}$ $\begin{array}{r} 26 \\ +\ 16 \\ \hline 42 \end{array}$ $\begin{array}{r} 26 \\ +\ 17 \\ \hline 43 \end{array}$

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Materials

a demonstration number line
objects to be used in demonstrations of sets
a set of strips for each child

It would be helpful to review the various power methods for addition and subtraction developed in the previous lessons. For example, write an addition problem with two-digit addends which requires regrouping on the chalkboard and ask children to suggest various ways in which this problem could be solved. Review with the children the technique of counting by tens used with both the number line and with bundles of ten sticks. Remind the children either that they might start counting with tens or that they might start counting with ones and then add on the tens.

Let's have fun

0 4 8 12 16 20

Can you give the next 3 numbers in each row?

0	1	2	3	4	5	6	7
0	2	4	6	8			
1	3	5	7	9			
0	3	6	9	12			
0	5	10	15	20			
10	20	30	40	50	60	70	80
5	15	25	35	45			
7	12	17	22	27	32	37	42
23	21	19	17	15			
1	3	6	10	15	21	28	36

Sequences

Teaching
Page h-38

This change of pace page gives children an opportunity to work with skip counting. You might suggest that they use a number line to help them fill in the blanks. Notice that the last sequence of numbers is not skip-counting because the number added is increased by one each time. Do not expect many children to succeed with this last exercise, but let those who do explain it to the rest of the class.

Follow-up

The following game may help increase the children's understanding of regrouping ideas. Begin by saying: "I'm thinking of the difference 12 minus 9." When the correct answer is given, continue with other combinations involving different decades, such as, 22 − 9, 32 − 9, 42 − 9 and so on. Give three or four problems in each sequence to make the children think about the pattern involved. Possible sequential problems are suggested below.

1. $10 - 3 = \square$
   $20 - 3 = \square$
   $30 - 3 = \square$
   $40 - 3 = \square$
2. $15 - 8 = \square$
   $25 - 8 = \square$
   $35 - 8 = \square$
   $45 - 8 = \square$
3. $12 - 7 = \square$
   $22 - 7 = \square$
   $32 - 7 = \square$
   $42 - 7 = \square$
4. $13 - 6 = \square$
   $23 - 6 = \square$
   $33 - 6 = \square$
   $43 - 6 = \square$

Addition with Regrouping

Pages h-39 to h-48

General Objectives

To develop understanding of the standard algorithm used for two-digit addition problems that require regrouping

To maintain previously developed concepts and skills of addition

The content of this module centres on the development of regrouping in addition. It is intended to be used as an optional module. The first lesson introduces addition with the use of the nomograph and provides exercises to be discussed. Following this introduction, a three-step process is presented for solving exercises involving regrouping. The children find the sum of the numbers in the ones' place and write it down. Then below the first sum, they write the sum of the numbers in the tens' place. Finally the children find the sum of these two numbers. This process leads to the more formal procedure of addition involving regrouping. This approach helps children discover and understand the ideas used in doing problems in the standard short method of addition.

Mathematics

The algorithms presented in this unit are not merely isolated, efficient rules and techniques for finding sums. Rather, as extensions of previously developed basic concepts, the algorithms of arithmetic are an integral part of the total structure.

Place value is a key idea for the regrouping algorithm, the main algorithm of this unit. This algorithm receives its name from an idea involving place value.

To investigate the mathematics of a simple exercise in regrouping, consider the following example.

$$\begin{array}{r} 28 \\ +64 \\ \hline 92 \end{array} \qquad \begin{aligned} 28 + 64 &= (20 + 8) + (60 + 4) \\ &= (20 + 60) + (8 + 4) \\ &= 80 + 12 \\ &= 80 + (10 + 2) \\ &= 90 + 2 \\ &= 92 \end{aligned}$$

Up to the expression $80 + 12$, we used the same procedures as we did for sums like $23 + 64$. For $80 + 12$, we write 12 as $10 + 2$ and regroup to complete the problem.

In developing a short form for adding numbers such as 28 and 64, an intermediate step is introduced. The example that follows shows how the two processes are related. This relationship illustrates how children are lead to understand the final algorithm. The hand-lettered numerals illustrate the relation between the processes.

Intermediate Step	Final Algorithm
$\begin{array}{r} 28 \\ +64 \\ \hline 12 \\ 80 \\ \hline 92 \end{array}$	$\begin{array}{r} 28 \\ +64 \\ \hline 92 \end{array}$

Teaching Dark Green Module, Unit H

Approximate Time: 5 to 8 days

Materials

demonstration nomograph for the overhead projector or a demonstration nomograph drawn on chalkboard
felt strips marked into 10 felt units (3 cm by 30 cm)
felt squares (3 cm²)
flannelboard
objects easily grouped by tens for set demonstrations
rulers or straight edge, 1 per child

Vocabulary

regrouping

Although the use of concrete materials is not emphasized in this module, materials for demonstrations would still be helpful. The flannelboard with felt strips for tens and felt squares for units would be very useful to accompany algorithmic notation shown on the chalkboard. Set demonstrations with objects that can be easily bundled into tens would be helpful with these exercises also.

Evaluation of Progress

This module presents optional material. Not all children should be expected to master the addition algorithm. Keep in mind that mastery of an algorithmic method does not necessarily mean an understanding of the regrouping ideas. Thus, in your discussion with individual children two things should be considered:

1) Has the standard algorithm been mastered? (Is the child able to work the exercises involving regrouping?)
2) Are the ideas involved in regrouping understood?

Resources for Active Learning

General Activities:

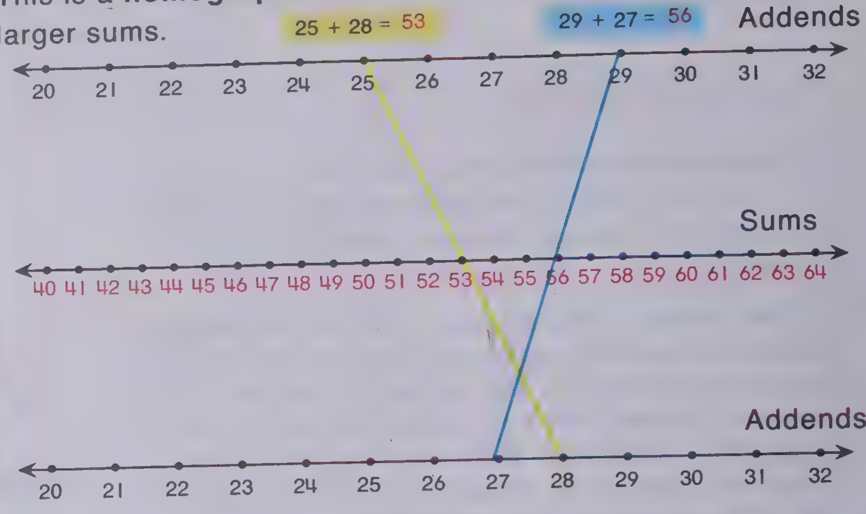
For activities involving computation with regrouping, refer to the introduction to the light green module of Unit H.

Investigation
Page h-39

Explain to the children that the nomograph on page h-39 is a nomograph to be used for finding larger sums. Point out how the equation 25 + 28 = 53 is related to the yellow line and the equation 29 + 27 = 56, to the blue line. When you are sure children understand how to work with a nomograph and that each child has a straight edge or ruler to use, explain the exercises at the bottom of the page. First, they are asked to find some sums using the nomograph. Then they are asked to find some other pairs of numbers whose sum is 53. In this case, they are given the sum and they are to find two points which are in line when their ruler goes through the sum. Then the children are asked to use the nomograph to find as many pairs of numbers as they can whose sum is 52. Encourage the children to work independently, but be prepared to help any who have difficulty.

Let's do

This is a **nomograph** for finding larger sums.



Find the sums. Use the nomograph.

24	22	26	25	31	28	29
+ 27	+ 29	+ 31	+ 27	+ 26	+ 23	+ 25
51	51	57	52	57	51	54

Find some pairs of numbers that add to the sums shown. Answers will vary. Examples are given.

27	25	22	28	32	30	29
+ 26	+ 28	+ 31	+ 24	+ 20	+ 22	+ 23
53	53	53	52	52	52	52

Readiness for skills with regrouping

Purpose

To provide an interesting introduction to a study of the addition of two-digit numbers.

Preparation

Materials

a demonstration nomograph for the overhead projector or demonstration nomograph drawn on the chalkboard
ruler or straight edge, one per child

Prepare your demonstration nomograph to show the numbers 0 through 12 on each of the two outside lines and 0 through 24 on the middle line. Use this nomograph to explain to the children how it is used. Have volunteers show how sums such as 9 plus 8, 10 plus 12, 5 plus 9, and so on might be found by placing the pairs of addends in a direct line and reading off the sum from the middle line. It would be helpful to write equations for each addition example. Note that such an introduction also serves as a review of the basic facts.

Let's talk

Which sums are more than 29?

24	24	24	24	24	24
+ 3	+ 4	+ 5	+ 6	+ 7	+ 8
27	28	29	30	31	32

Which sums are more than 49?

35	35	35	35	35	35
+ 10	+ 12	+ 15	+ 17	+ 13	+ 18
45	47	50	52	48	53

Which sums are more than 59?

23	23	25	25	25	23
+ 34	+ 38	+ 33	+ 36	+ 34	+ 39
57	61	58	61	59	62

Circle the correct sum.

32	38	21	27	18	14
+ 43	+ 47	+ 32	+ 36	+ 19	+ 13
75	75	53	53	27	27
85	85	63	63	37	37

27	22	41	46	12	19
+ 37	+ 32	+ 30	+ 35	+ 33	+ 36
54	54	71	71	45	45
64	64	81	81	55	55

Readiness for skills with regrouping

Discussion
Page h-40

It would be possible to have the children work in small groups to discuss and answer the questions on this page and then have the class together discuss the conclusions. Whether you are discussing conclusions or working the page with all the children, use the following suggestions for each frame. For the first frame, point out the sequence of the one-digit addends. Also, ask the children to explain when a new decade of ten is reached in the sum. Help them see how knowing this should enable them to find the sums of the remaining problems. In the second frame, discuss how the counting technique may be used. For example, in adding 35 plus 15, they might think "35 and 10 more is 45 and 5 more is 50," or in adding 35 plus 17, they might think "35 plus 10 more is 45," and then count 7 more, "46 . . . 52." You might work through the next frame similarly. For example, children can think of 23 plus 38 as "23 plus 10, plus 10 and plus 10 again" to give them 53 and then count 8 more. You might also suggest that those who need help begin with 23 plus 30, 23 plus 31, and so on. Explain that in the bottom two sections they should think of adding tens and then think of adding ones. Then they should ask themselves the question: "Do the ones give me another group of 10?" The answer to that question should help them recognize the correct sum.

Follow-up

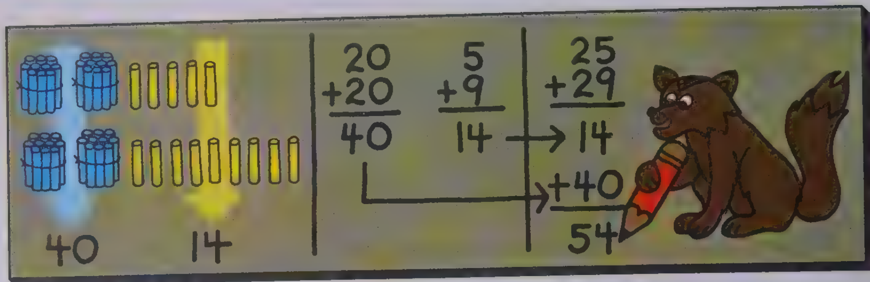
To prepare children to work with the addition algorithm studied in this lesson, it would be helpful to review the basic addition facts, particularly from 11 to 18. You might do so orally by playing "What's My Rule?" or "I'm thinking of a number," and say, for example: "I'm thinking of the sum of 7 plus 6." "What's my number?" or "I'm thinking of the sum of 9 plus 8." "What's my number?" A game such as Combo might also be used to review these basic facts.

To encourage children to find and complete patterns, duplicate a worksheet similar to the following.

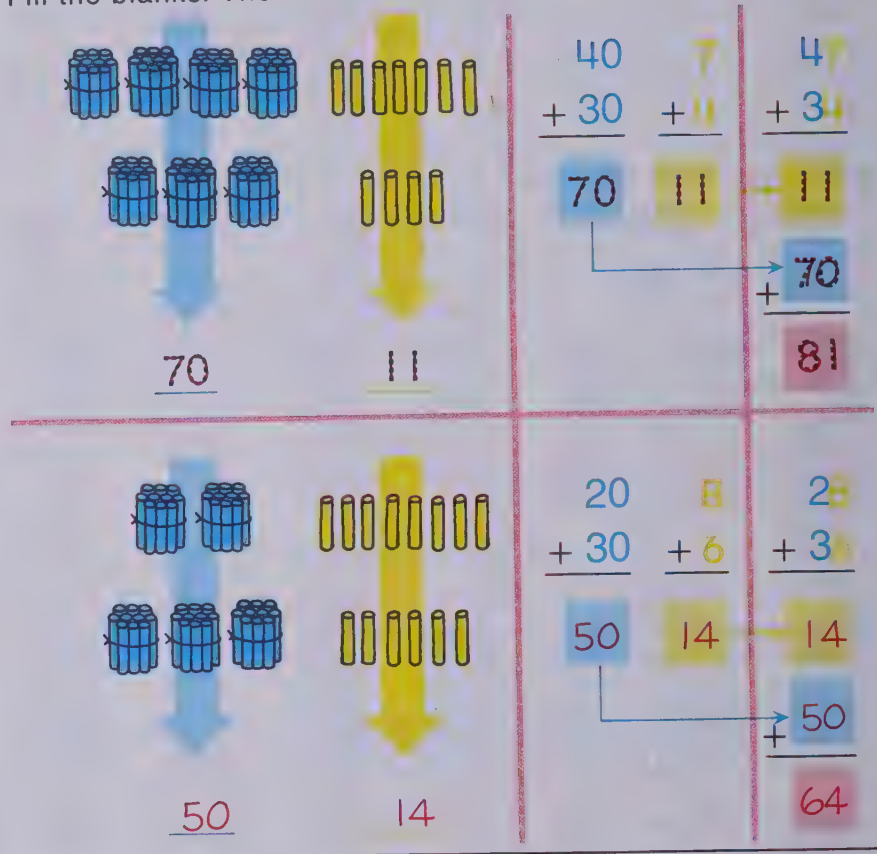
Use pairs of numbers to make each statement true.					
□ + △ = 21		△ − □ = 9		4 + □ = △	
□	△	△	□	□	△
14	7	20	11	3	7

Teaching
Page h-41

The algorithm taught on this page is a preliminary algorithm taught both for its own sake and as preparation for understanding the shorter standard algorithm. Some children may be able to do this algorithm and not the other; however, do not stress practice with this algorithm because for most children it will simply be a lead-in to the shortcut method. Use this page to relate the pre-book activity to the written notation. Read the directions with the children and then ask how many tens in all the blue arrow shows. Comment that we write 7 tens as 70. Ask the children to trace the dashed 70. Then ask how many ones the yellow arrow shows. After someone gives 11 as the sum of 7 and 4, ask the children to trace the numeral. Call attention to the next frame at the top and point out the color coding which relates the numerals to the illustrated bundles and arrows. Ask the children to add 7 plus 4 and trace the numeral, and then add 40 plus 30 and trace the numeral. Continue the explanation by pointing out that the third frame shows that we can find the sum 7 plus 4 and 40 plus 30 and write the answers in vertical form as shown here. This makes it easy to find the final sum by adding the ones and the tens. Repeat this same type of development by working through the last example with the children.



Fill the blanks. Then find the sums.



Objective

Given an addition problem involving two-digit numerals, the child will be able to find the sum by using a preliminary method that will lead to the standard addition algorithm.

Pre-Book Activity

It would be helpful at this point to relate the methods used with the concrete materials in the previous module to the algorithmic methods studied here. For example, use sets of sticks to provide a demonstration to review grouping. You might write the problem 53 + 28 on the chalkboard. Then take sets of the sticks to show 53 and 28. Group these in terms of five sets of ten and 3, and two sets of ten and 8. Ask a child to group the tens together and to group the single sticks together. Then display the problems 30 plus 20 and 3 plus 8 in vertical notation. Beside them show the addition:

$$\begin{array}{r}50\\+20\\\hline 70\end{array}\qquad\begin{array}{r}3\\+8\\\hline 11\end{array}\qquad\begin{array}{r}53\\+28\\\hline 11\\+70\\\hline 81\end{array}$$

Then ask the children to count up all the single sticks and to bundle together the new group of ten and then place that new bundle of ten with the 5 and the 2. Thus, children see that the ones form a new group of ten which should be added to the other groups of ten already present. Repeat the entire procedure several times with new problems that require regrouping and encourage the

Find the sums.

38	52	33	26
+ 45	+ 29	+ 29	+ 8
13	11	12	14
+ 70	+ 70	+ 50	+ 20
83	81	62	34

25	36	27	27	34
+ 48	+ 14	+ 44	+ 36	+ 8
13	10	11	13	12
+ 60	+ 40	+ 60	+ 50	+ 30
73	50	71	63	42

52	17	38	64	28
+ 29	+ 35	+ 7	+ 16	+ 24
11	12	15	10	12
+ 70	+ 40	+ 30	+ 70	+ 40
81	52	45	80	52

Addition with regrouping

Teaching
Page h-42

Point out to the children that the shading in the problems indicates that they are to add the ones' digits first, the tens' digits next, and then add the two sums to get the final sum. Work through the first problem with the children and ask them to trace the dashed numerals. Encourage the children to do the second problem by themselves and then explain each step so that the children can check their work. Next, instruct them to work the last two problems in the first row and check them. Help those who need further explanation and urge those who seem to know what to do to complete the page independently. Remember that the intent of this algorithm is to lead children into the shortcut method presented in the next lesson; therefore, excessive emphasis should not be placed on this algorithm.

children to participate in the demonstration. Continue to stress that the ones added form a new group of tens which can then be added to the tens already present.

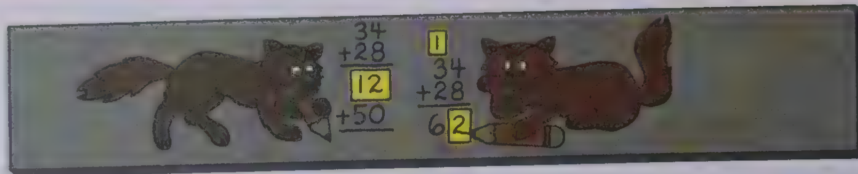
Follow-up

Some children might benefit from using the strips to help them add two two-digit numbers and then recording the strips they used. For this purpose, you might want to duplicate a worksheet such as the following. Many children will be able to work with the strips and the algorithm, without using the middle column suggested here.

Show	Combine and Record	Add
18 and 25	3 O 13 W Regroup: 4 O 3 W	18 +25 13 30 43
27 and 19	3 O 16 W Regroup: 4 O 6 W	27 +19 16 30 46

Teaching
Page h-43

Point out to the children that they can work each problem on this page in two ways. Demonstrate by working through both methods in the first frame. Then ask the children to trace the dashed numerals. If necessary, work through another example with them; however, urge the children to complete the page by themselves as soon as you think they are able.



Find the sums.

$$\begin{array}{r} 58 \\ +\ 25 \\ \hline 13 \\ +\ 70 \\ \hline 83 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 58 \\ +\ 25 \\ \hline 83 \end{array} \qquad\qquad \begin{array}{r} 49 \\ +\ 36 \\ \hline 15 \\ +\ 70 \\ \hline 85 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 49 \\ +\ 36 \\ \hline 85 \end{array}$$

$$\begin{array}{r} 27 \\ +\ 5 \\ \hline 12 \\ +\ 20 \\ \hline 32 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 27 \\ +\ 5 \\ \hline 32 \end{array} \qquad\qquad \begin{array}{r} 66 \\ +\ 24 \\ \hline 10 \\ +\ 80 \\ \hline 90 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 66 \\ +\ 24 \\ \hline 90 \end{array}$$

$$\begin{array}{r} 78 \\ +\ 16 \\ \hline 14 \\ +\ 80 \\ \hline 94 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 78 \\ +\ 16 \\ \hline 94 \end{array} \qquad\qquad \begin{array}{r} 55 \\ +\ 29 \\ \hline 14 \\ +\ 70 \\ \hline 84 \end{array} \qquad \text{shortcut} \quad \begin{array}{r} 1 \\ 55 \\ +\ 29 \\ \hline 84 \end{array}$$

Objective

Given two two-digit addends, the child will be able to find the sum by using the standard addition algorithm.

Pre-Book Actitity

To give the children an introduction to the shortcut method, first review the algorithm introduced in the previous lesson. Then, referring to an example you have written on the chalkboard, ask children to suggest how they might combine some of the steps and write three lines rather than five. Ask questions such as: "What do we add first?" "Which part of this answer is in the final answer?" "Could we possibly remember the other parts of the answer or write it down somewhere else?" "Might we then write the final answer directly underneath the addends?" Work through several examples with questions such as these and show children the shortcut method as presented on page h-43. Lead the children to see that the purpose of writing down the 1 above the tens' place in the shortcut method is that it serves as a reminder to add it to the tens. Stress that this is not changing the problem; it is simply changing the written notation of the problem and finding a shorter method of writing it down.

Follow-up

The worksheet in the next column might be used to practice the shortcut method:

Find the sums.

A

16	34	45
+ 25	+ 27	+ 17
41	61	62
19	24	37
+ 13	+ 28	+ 26
32	52	63

B

48	68	19
+ 23	+ 12	+ 25
71	80	44
26	37	46
+ 16	+ 13	+ 28
42	50	74

C

16	28	75
+ 48	+ 67	+ 19
64	95	94
52	27	18
+ 36	+ 67	+ 71
88	94	89
65	53	36
+ 18	+ 24	+ 49
83	77	85

D

17	68	19
+ 29	+ 7	+ 36
46	75	55
7	18	6
+ 48	+ 18	+ 39
55	36	45
47	39	68
+ 32	+ 19	+ 9
79	58	77

Practice—addition with regrouping

TEACHING
Page h-44

Since the exercises in the first two frames, A and B, are relatively easy, you might wish to choose an example from frames C or D to work with those children who still are having difficulty. Explain that again they should find the sums. Explain also that they may work these problems in either of the two ways shown on the previous page. For class participation, you might have various children present the exercises on the chalkboard, and discuss the ideas involved. The problems in frame C are a bit harder than in frames A and B although some of them do not require regrouping. Note that the problems in D also include problems with both two-digit addends and one-digit addends. If some of the children can do these exercises using the three-step method, but have difficulty doing them using the standard algorithm, permit them to use the former method. It is preferable to let the children work these exercises with ease using the three-step method, rather than have them memorize and use an algorithm they do not understand. As the children work, move around the class to assit those who may need help.

Find the sums.
Show the shortcut method for each problem.

37	37	28	28	45	45
+25	+25	+65	+65	+19	+19
12		13		14	
+50		+80		+50	
26	26	35	35	61	61
+54	+54	+49	+49	+39	+39
10		14		10	
+70		+70		+90	

For more able children, extend the concept of regrouping by giving them exercises like the following. Remind them to add the ones' column first and suggest that they write down the extra tens if they cannot remember them. Allow them to use either addition method.

Find the sums.

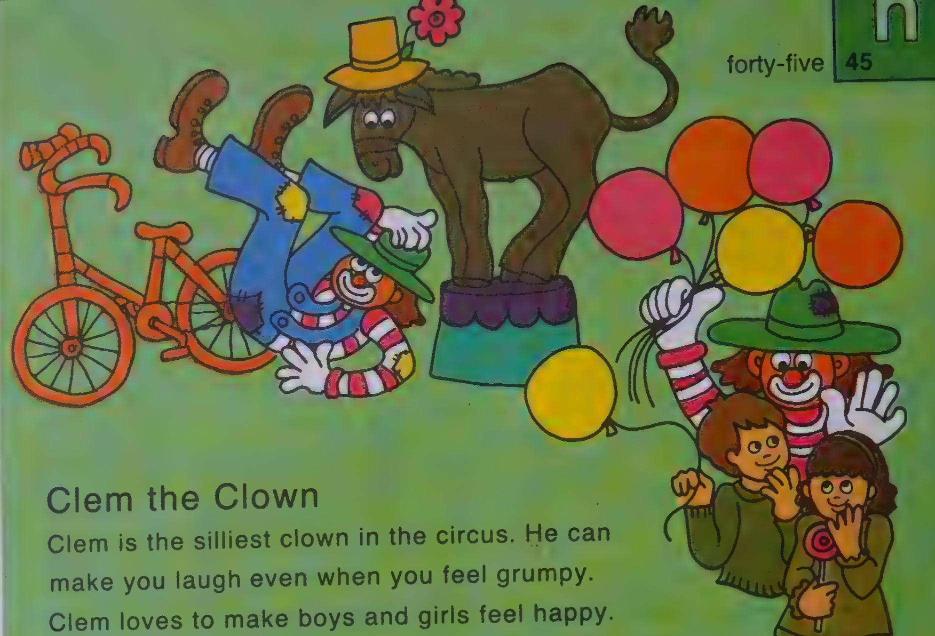
14		ʹ13	4	19	91
8		8	5	12	7
+15		+16	+12	+ 7	+ 8
17	or	37			
20					
37					
36		42	63	29	15
13		25	17	18	19
+27		+18	+14	+42	+39

RESOURCES FOR ACTIVE LEARNING

CHIP TRADING ACTIVITIES—SET II, Cards 16–19, Scott Scientific

Teaching
Page h-45

Read the top paragraph with the children. If necessary, read through all of the problems together also. In any case, do at least a few of the problems with the children. They might need help in understanding various phrases. For example, in the first problem, they must realize that they are trying to figure out how long both of Clem's shoes are, when placed end to end. Thus, they have to use the number 47 twice. Remind the children that when solving problems they should first think: "What information am I given?" second: "What information do I have to find?" third: "What arithmetic can I use to solve the problem?" and fourth: "Does my answer make sense?" It would be helpful to use these questions as guides as you work through the problems with the children. It is not necessary that they write an equation for each problem, but you might prefer that they do so.



Clem the Clown

Clem is the silliest clown in the circus. He can make you laugh even when you feel grumpy. Clem loves to make boys and girls feel happy. You can learn more about Clem in these problems.

1. Each of Clem's shoes is 47 centimetres long. How long are they together, placed end to end? 94 cm

2. Clem has 14 patches on his shirt and 17 on his pants. How many patches? 31

3. It takes Clem 25 minutes to put on his funny clothes and 15 minutes to put on his silly face. How long to get ready? 40 minutes

4. 37 boys and 39 girls from Lake School went to see Clem. How many children? 76

5. Clem tripped and fell on his nose 18 times. He fell on his seat 14 times. How many falls? 32

6. Clem gave away 36 red balloons and 28 blue balloons. How many? 64

7. Clem fell off his bike 9 times. He fell off the donkey 14 times. How many falls? 23

Story problems

Objective

Given word problems which require the use of regrouping for the addition of two-digit addends, the child will be able to solve the problems.

Pre-Book Activity

Give the children various problem situations which involve the addition of two-digit numerals. For example, you might ask them to help you figure out how many children there are in two or three rooms of your school. For example, if you have three rooms in one section of the school, you might ask the children to help you figure out how many students there are altogether in those three classes. However, remember to do the addition by adding the number of two classes at a time. Then help them with the second addition in which the third number is added on. Also, you might ask them to help you figure out how many books you have altogether in two sets. For example, explain that you have 42 science books and 27 music books; you want to know how many books there are altogether. Sample word problems are suggested below, but problems which are more relevant to your particular group of children are preferred. Note that the sums must remain less than 100.

Sample problems:

1. There are 26 children in Bonnie's class. She brought 15 chocolate and 15 cherry cupcakes for her birthday. Does she have enough cupcakes for everyone in her class?
2. Billy and Jill collect miniature cars. Billy has 42 and Jill has 29. How many cars do they have in all?

Find the sums.

16 + 35 = 51	43 + 29 = 72	14 + 58 = 72	28 + 48 = 76	36 + 38 = 74	79 + 6 = 85
67 + 6 = 73	25 + 32 = 57	8 + 76 = 84	57 + 18 = 75	49 + 47 = 96	8 + 89 = 97
27 + 25 = 52	39 + 44 = 83	58 + 22 = 80	14 + 35 = 49	39 + 49 = 88	55 + 18 = 73

Complete each row.

6	+ 6	12	+ 6	18	+ 6	24	+ 6	30	+ 6	36
7	+ 7	14	+ 7	21	+ 7	28	+ 7	35	+ 7	42
8	+ 8	16	+ 8	24	+ 8	32	+ 8	40	+ 8	48
9	+ 9	18	+ 9	27	+ 9	36	+ 9	45	+ 9	54

Practice—addition with regrouping

Teaching
Page h-46

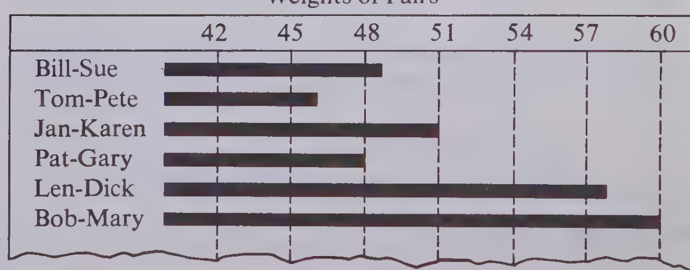
Read the directions at the top of the page with the children. Explain that they may use either addition method; however, for those who understand it, the shortcut method would probably allow them to work with more speed. Give careful directions for the bottom section of the page. Explain that they should read from left to right and add the numbers, putting their answers in the colored spaces. Once they have written an answer, they should use it to add to the next number to get their next answer and so on.

Encourage the children to discuss how each problem would be solved.

Follow-up

To provide the children with interesting practice for situations of adding two-digit numbers, suggest that they find a partner. Then have them weigh themselves and add the weights together. Then, it would be helpful to use a large piece of chart paper or an area on the chalkboard to graph the sums for each pair of children. To make this even more interesting, you might challenge them to see if they can find another two children in the classroom whose weights when added together will give the same sum as the sum of their weights.

Weights of Pairs



Resources for Active Learning

Connecting dots:

A CLOUDBURST, Vol. 1, No. 8231, Midwest Publications

ENRICHMENT OF ARITHMETIC, pp. 1/35–28, Webster, McGraw-Hill

Teaching
Page h-47

Read the directions at the top of the page with the children. Then explain to them that in finding the sums, they may use either method that they have studied in the module. That is, they may use the two-step method of writing down partial sums and adding them to get the final sum, or they may use the shortcut method of showing the regrouping by writing 1 on top of the tens' columns. Help any children who have difficulty reading the word problems, but encourage them to figure out how to solve them by themselves.

Show you know

Find the sums.

26 + 35 = 61	38 + 44 = 82	42 + 19 = 61	38 + 25 = 63	57 + 17 = 74	26 + 67 = 93
25 + 27 = 52	13 + 27 = 40	48 + 15 = 63	39 + 14 = 53	26 + 30 = 56	48 + 24 = 72
71 + 18 = 89	8 + 25 = 33	35 + 9 = 44	37 + 38 = 75	48 + 39 = 87	67 + 3 = 70
67 + 18 = 85	46 + 36 = 82	27 + 59 = 86	6 + 89 = 95	19 + 19 = 38	36 + 47 = 83

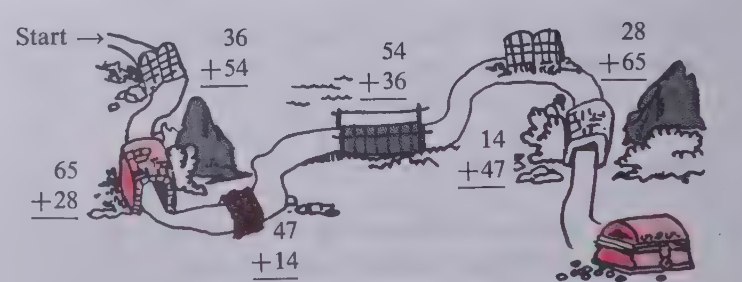
Jan saved 47 cents. Then she earned 25 cents. How much now? 72¢

Jay scored 28 points in the first game and only 17 in the second. How many points? 45

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.



Pre-Book Activity

As a review of addition with two-digit numbers, draw on the chalkboard or prepare beforehand on a duplicating master, a road leading to a treasure chest. Along the road, have stops consisting of problems with two-digit addition which require regrouping. Include among these numbers pairs which are commutative so that the top addend of one problem is the second addend of another, as shown in the illustration.

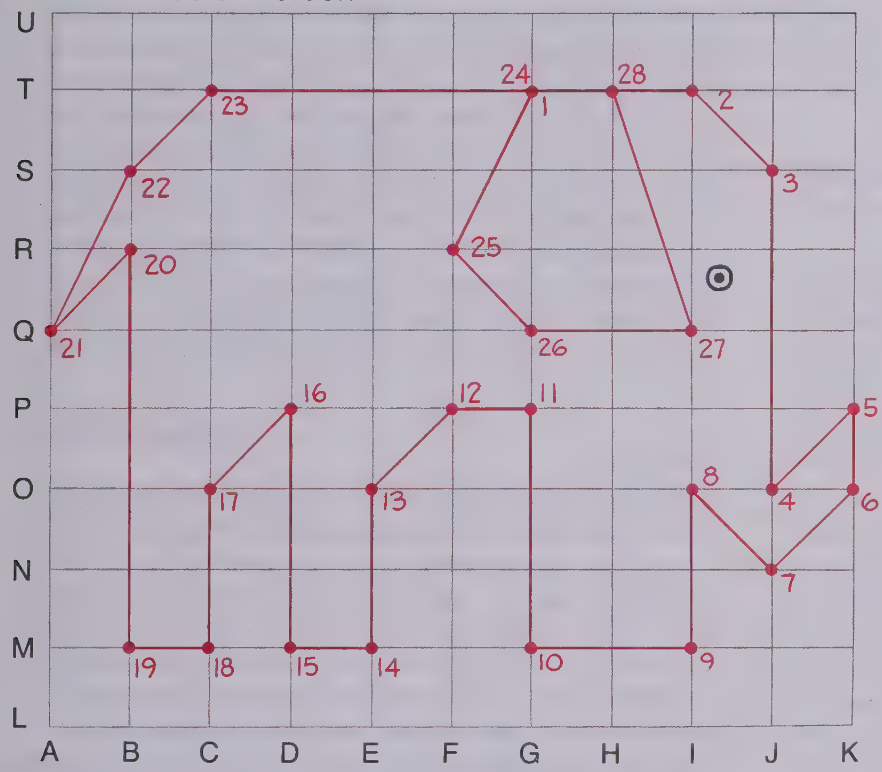
Challenge the children to see who can be first to get to the treasure. Tell them all when to start. The first child who gets to the treasure will probably have discovered the use of the commutative property. Although it is not essential to discuss this property, this type of review will provide motivation for practice with two-digit addition as well as remind the children that addends may be grouped in any manner.

Let's have fun

Put a dot where these lines cross.	G T	I T	J S	J O	K P	K O	J N	I O	I M	G M	G P
Put this number beside the dot.	1	2	3	4	5	6	7	8	9	10	11

F P	E O	E M	D M	D P	C O	C M	B M	B R	A Q	B S	C T	G T	F R	G Q	I Q	H T
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28

Connect the dots in order.



Graphing

Teaching

Page h-48

Explain to the children that in this change of pace lesson they will be able to draw their own dot picture. Point out the two rows at the top of the page. Explain that each pair of letters matches a "corner" or point where two lines on the graph meet. When they find this point, using the lettered lines as guides, they should write the number which appear in the box underneath that pair of letters. Help them position the first three or four dots and then encourage them to continue independently. Suggest that they color the picture after they have connected the dots in order. Some may wonder about numbers 1 and 24 being at the same point but assure them that this matching is correct.

Follow-up

For more able children, provide an extra work sheet with problems containing three and four numbers that require regrouping to the next place. For example:

Solve.

9 7 +5	8 8 +8	14 38 +29	36 57 +19	28 65 +17	49 29 +19
8 6 9 +8	9 5 9 +7	9 28 14 +49	17 18 46 +25	29 19 8 +27	38 17 9 +33

Resources for Active Learning

Connecting dots:

A CLOUDBURST, Vol. 1, No. 8231, Midwest Publications

ENRICHMENT OF ARITHMETIC, pp. 1/35–38, Webster, McGraw-Hill

Subtraction with Regrouping

Pages h-49 to h-62

General Objectives

To introduce two-digit subtraction problems that require regrouping

To develop understanding of the algorithm for subtraction of two-digit numbers which require regrouping

The chief emphasis in this optional module is on the development of the algorithm for two-digit subtraction with regrouping. After the introductory lesson in which the basic problem showing the need for regrouping in subtraction is presented, the children are introduced to regrouping of two-digit numbers by solving equations such as $63 = 50 + \square$. Next the idea of regrouping is related to subtraction exercises. Finally, a shortcut method of notation for regrouping in subtraction exercises is introduced. Practice with regrouping and additional work with word problems complete the module.

Mathematics

In this unit, as in the previous one, no new mathematical concepts are introduced. The algorithms presented here utilize several concepts brought out earlier. Again it is important that you think of these algorithms not only as efficient techniques for finding differences, but also as extensions of previously developed concepts.

Place value is a key concept for the main algorithm. The regrouping idea for subtraction that involves place value is, in a sense, a reversal of the regrouping process in addition. That is, instead of regrouping one ten into the tens' column, one ten is regrouped for use in the ones' column. It is in this sense that the word regroup has become associated with both the addition and subtraction algorithms.

We offer here a discussion of a simple exercise involving regrouping.

$$\begin{array}{r} 34 \\ -\ 6 \\ \hline 28 \end{array} \qquad \begin{aligned} 34 - 6 &= (30 + 4) - 6 \\ &= (20 + 14) - 6 \\ &= 20 + (14 - 6) \\ &= 20 + 8 \\ &= 28 \end{aligned}$$

In previous subtraction problems, ones were subtracted from ones; this is impossible in this problem because 6 is greater than 4. So $30 + 4$ is regrouped as $20 + 14$, and 6 is subtracted from 14.

Regrouping often occurs when one two-digit number is subtracted from another, but only one further step is needed. That step involves subtracting in the tens' column. This regrouping is shown by the hand-lettered numerals below.

$$\begin{array}{r} {\scriptstyle 5\ 1} \\ 67 \\ -28 \\ \hline 39 \end{array}$$

Teaching Blue Module, Unit H

Approximate Time: 7 to 10 days

Materials

felt unit strips: 3-by-30-cm strips marked into 10 units and single units 3 cm²
flannelboard
groups of objects which can be grouped by tens
set of orange and white strips for each child

A variety of materials will help the children understand the regrouping process involved in the algorithm developed in this module. Children can use their orange strips and white strips individually. Activities involving these strips are suggested in the teacher notes. It would also be helpful to use set materials, felt unit strips, and felt single units on the flannelboard; or pipe cleaners, pencils, and other objects bundled into tens to illustrate examples involving the algorithm. Use these set materials to go through the steps in the regrouping process shown on pages h-53 to h-56.

Evaluation of Progress

In this module the children's performance should be evaluated in two areas. First, their understanding of regrouping should be revealed in daily evaluations. Second, their ability to compute efficiently should be measured at the conclusion of the unit. Keep in mind, however, that this module should be considered optional material, and mastery should not be expected from all the children.

Resources for Active Learning

General Activities:

For activities involving computation with regrouping, refer to the Introduction to the light green module in Unit H.

Investigation
Page h-49

Read the directions at the top of the page with the children. Explain that each frame contains a pair of exercises. One of the exercises can be worked out as a subtraction exercise, the other cannot. Work through the first frame as an example. Help the children see that 5 cannot be subtracted from 3, but 5 can be subtracted from 13. It would also be helpful to work through the next frame. Again help them realize that they cannot subtract 7 from 1, but they can subtract 7 from 11. Encourage them to continue to do the problems independently. Explain to them that at the bottom of the page they are given a challenge to make up some subtraction problems that cannot be done. Note that this investigation places emphasis on the correct interpretation of the written notation for subtraction exercises. One of its purposes is to help children to overcome the incorrect reading of the subtraction exercises which causes them to immediately respond by subtracting the smaller number from the larger number, no matter where it appears in the problem. Urge them to think of the whole problem rather than just examining the ones' column. Such an activity has as its objective getting the children ready for the subtraction algorithm by enabling them to readily identify when they can or cannot subtract without regrouping. Do not stress or teach how to find these differences, just discuss why they can or cannot be done.

Let's do

4 −7	14 −7 7

Find as many differences as you can. Put a ring around the ones "cannot be done."

3 − 5	13 − 5 8	1 − 7	11 − 7 4	12 − 6 6	2 − 6
15 − 6 9	5 − 6	4 − 5	14 − 5 9	13 − 4 9	3 − 4
16 − 8 8	6 − 8	5 − 7	15 − 7 8	12 − 8 4	2 − 8

Can you make up some subtraction problems that "cannot be done"? Answers will vary. Examples are given.

2 − 4	3 − 5	4 − 8	6 − 8	7 − 8	9 − 10

Introduction to skills for subtraction with regrouping

Purposes

To stimulate interest in studying the subtraction algorithm

To introduce subtraction of two-digit numbers

Preparation

A brief review of the basic subtraction facts would be helpful as a preparation for this lesson. For example, you might play "What's My Rule?" with the children. Ask the children to give you a number between 11 and 18. Then think of a rule such as subtract 8, and respond with the difference. For example, if a child gives you the number 13, you will respond 5. Explain to the children that as soon as they have discovered your rule, they should fold their arms. Then call on a child whose arms are folded, give him a number between 11 and 18, and ask him to respond with the answer. After several children have discovered the rule, change the rule and announce "New Game." You might remind the children how helpful it is to think of a related addition equation if they do not know the subtraction fact.

Let's talk

Which differences are less than 30?

35	35	35	35	35	35
− 2	− 3	− 4	− 5	− 6	− 7
33	32	31	30	29	28

Which differences are less than 40?

46	46	46	46	46	46
− 4	− 5	− 6	− 7	− 8	− 9
42	41	40	39	38	37

One answer is correct. Circle the correct difference.

34	55	63	74	26	33
− 6	− 7	− 8	− 6	− 9	− 5
32	52	55	68	23	32
or	or	or	or	or	or
28	48	65	72	17	28

24	35	43	37	56	55
− 7	− 7	− 6	− 8	− 9	− 8
23	28	37	31	47	53
or	or	or	or	or	or
17	22	43	29	43	47

Introduction to skills for subtraction with regrouping

Discussion
Page h-50

The exercises presented on this page should not be accompanied by an explanation of the subtraction algorithm. Rather the exercises are presented to help children study patterns for the purpose of understanding when they cannot subtract in a problem and will need to regroup. Help the children see the patterns shown in the first two frames. Point out, what happens when they subtract 5 from 35 compared to what happens when they subtract 6 from 35. In the exercises at the bottom of the page, help the children discuss spanning the decade: for example, point out that since the number of ones which they are taking away is greater than the number of ones which they have in the top number, their answer will not be in the same decade as the number they are subtracting from. Thus, in the problem 34 minus 6, since 6 is greater than 4, their answer cannot be in the same decade as 34. Encourage the children to give reasons for the answers which they choose as correct. If necessary, refer to the power skills developed in the light green module; it is not intended that you begin to teach the subtraction algorithm at this time. Instead, use these exercises to introduce the need for a method of regrouping.

Follow-up

Practice on subtraction tables may help increase the children's efficiency with subtraction facts so that they can concentrate on the algorithm being presented on the following pages.

Sample tables:

−6	
16	10
9	
15	
8	
12	
10	
13	
11	
14	

−7	
12	5
15	
10	
13	
11	
17	
14	
9	
16	

−8	
17	9
9	
16	
8	
15	
14	
10	
12	
13	

−9	
13	4
10	
12	
17	
11	
18	
15	
16	
14	

Teaching
Page h-51

Introduce this page by discussing equations on the chalkboard and showing sets to illustrate their regrouping. For example, display the equation 44 = 30 + □. Separate a set of 44 into two sets, one of 30, the other of 10 plus 4 or 14 and complete the equation, 44 = 30 + 14. Use other simple examples to stress that regrouping tens is the way to arrive at the solution for the equation. The extent of your work with demonstrations such as this will depend on the understanding of the children. You might relate your demonstration to the pre-book activity by using the flannelboard and felt ten-strips and units. As you present the text page, direct the children's attention to the "Add 10" table and ask: "If you add 10 to 40, what number results?" Instruct the children to trace the dashed numeral 50 and then complete the table. Next, point out that the directions say to complete the rest of the page by solving the equations. Stress that they should think of regrouping tens. Point out that the equations on the bottom half of the page are patterned so that the children can work from a less difficult problem to a more difficult one. Encourage the children to complete the page by themselves.

ADD 10	
10	20
30	40
70	

Complete the table.

Add 10	
40	50
20	30
60	70
50	60
80	90

Solve the equations.

50 = 40 + 10

30 = 20 + 10

70 = 60 + 10

60 = 50 + 10

Solve the equations.

50 = 40 + 10

52 = 40 + 12

56 = 40 + 16

30 = 20 + 10

31 = 20 + 11

34 = 20 + 14

70 = 60 + 10

73 = 60 + 13

78 = 60 + 18

60 = 50 + 10

64 = 50 + 14

65 = 50 + 15

Place value concepts for regrouping

Objective

Given a two-digit numeral, the child will be able to regroup it into a different number of tens and ones.

Pre-Book Activity

Materials

a set of strips for each child

Ask the children to use their strips to show a two-digit number, such as 26. Then ask them to replace one of their ten strips with ten white strips and record the new set of strips that they have. Thus, for 26, they would record 10 plus 16. You might draw the following chart on the chalkboard and suggest that the children use their strips to complete it.

Build	Regroup	Record
26	10 and 16 W	10 + 16
32	20 and 12 W	20 + 12
21	10 and 11 W	10 + 11
30	20 and 10 W	20 + 10

Complete the matching.

50 + 4	50 + 12
60 + 2	60 + 11
80 + 5	40 + 14
70 + 1	30 + 10
40 + 0	70 + 15

Solve the equations.

43 = 30 + 13	32 = 20 + 12
36 = 20 + 16	47 = 30 + 17
52 = 40 + 12	62 = 50 + 12
61 = 50 + 11	73 = 60 + 13
74 = 60 + 14	80 = 70 + 10
45 = 30 + 15	54 = 40 + 14
93 = 80 + 13	91 = 80 + 11
70 = 60 + 10	75 = 60 + 15

Place value concepts for regrouping

Teaching
Page h-52

You might wish to use this page as a separate day's activity, preceded by set or strip demonstrations for regrouping the tens. It would also be possible to have the children work with their strips to complete the matching at the top of page h-52. For example, for the first sum, 50 + 4, they would show five orange strips and four white strips. Then ask them to replace one of their orange strips with 10 white strips and figure out which sum on the right matches their new set of strips. Direct the children to complete the equations at the bottom of the page. If they wish to use the strips as aids, allow them to do so, but gradually encourage them to think about the regrouping and wean them away from the concrete materials. You might want to work through several of the equations together with the children.

Follow-up

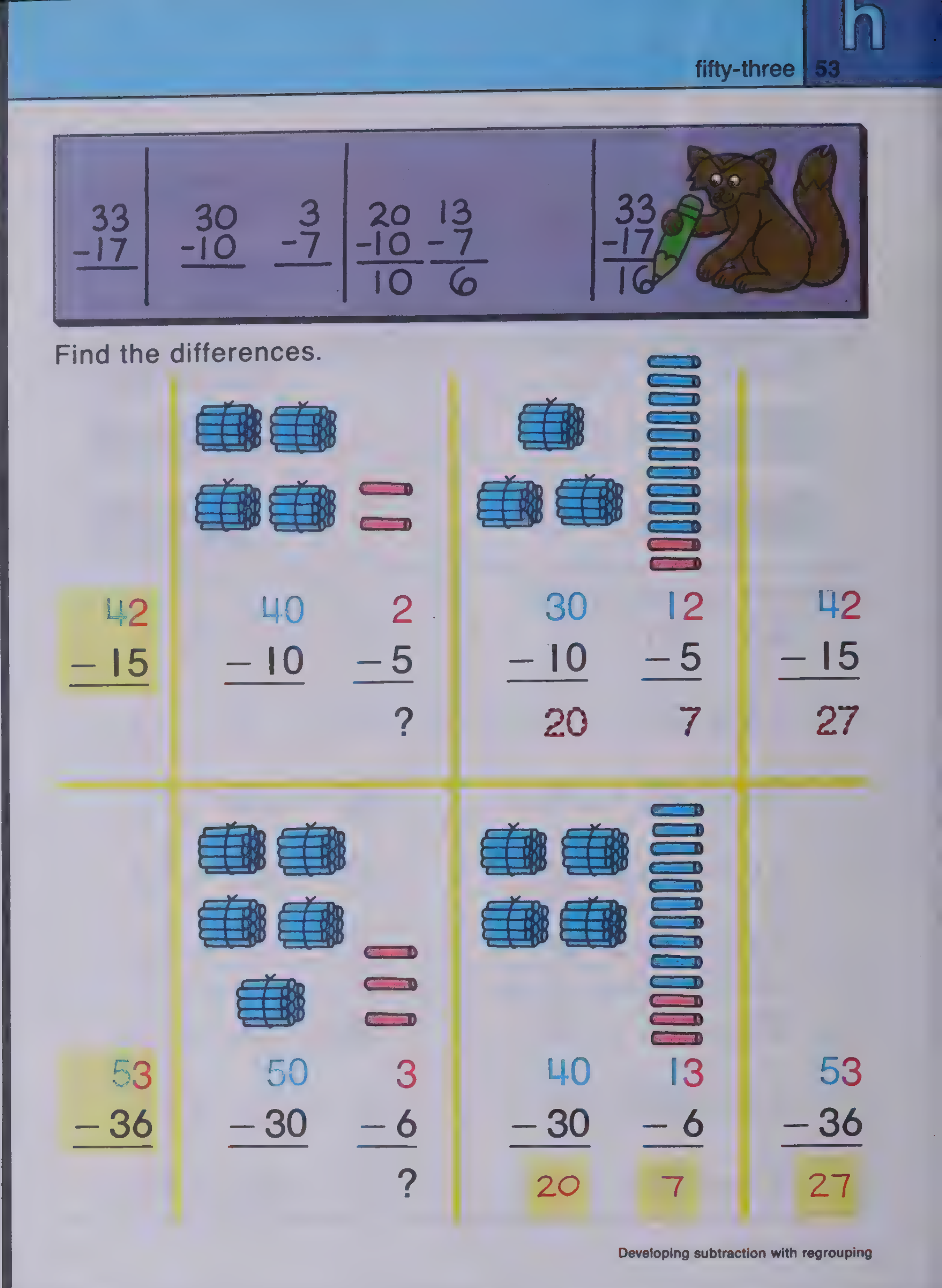
For more practice in regrouping, prepare a worksheet such as the following. Allow children to use the strips if they choose to.

Find the number for each □.	
33 = 20 + □	68 = □ + 18
89 = 70 + □	76 = □ + 16
94 = 80 + □	42 = □ + 12
55 = 40 + □	31 = □ + 11

Teaching
Page h-53

Work through the material on this page as a sequel to the pre-book activity. If the children want to find the differences 40 − 10 and 50 − 30, let them, but point out that they cannot find the other differences, 2 − 5 and 3 − 6. Remind the children that they have learned from regrouping the tens that 40 + 2 becomes 30 + 12 and that this fact can help complete the subtraction. Now ask the children to trace the dashed numerals.

As you work through the second exercise, explain the regrouping procedure again. Ask the children to find the differences and write them in the answer boxes.



Objective

Given two-digit subtraction exercises patterned to show regrouping, the child will be able to find the differences.

Pre-Book Activity

Provide the children with a set demonstration of regrouping. For example, exhibit a set of 34 objects, such as pipe cleaners. Group these objects by tens, so that the children can clearly see 3 tens and 4. Tell them to subtract 17 and to begin by subtracting 7 objects. Observe that to subtract 7, they can take 4 away, and then break up one of the sets of ten to remove 3 more. That is, they will subtract 7 from 4 + 10, or 14. Now observe that in order to complete the subtraction of 17, they must remove another 10. Ask a child to remove the additional set of 10 from the sets. Then show the problems in vertical notation on the chalkboard:

$$\begin{array}{r} 34 \\ -17 \\ \hline \end{array} \qquad \begin{array}{r} 20 \\ -10 \\ \hline \end{array} \qquad \begin{array}{r} 14 \\ -\ 7 \\ \hline \end{array}$$

Repeat this demonstration set for another pair of numbers, and show the corresponding symbols on the chalkboard. If you organize your work to look like that presented on text page h-53, you can color code it for clarity. Repeat this demonstration using other sets until you think the children understand the ideas presented.

Find the differences for each ■.

74 − 36	70 − 30	4 − 6 ?	60 − 30 $\boxed{30}$	14 − 6 $\boxed{8}$	74 − 36 $\boxed{38}$
92 − 17	90 − 10	2 − 7 ?	80 − 10 $\boxed{70}$	12 − 7 $\boxed{5}$	92 − 17 $\boxed{75}$
60 − 23	60 − 20	0 − 3 ?	50 − 20 $\boxed{30}$	10 − 3 $\boxed{7}$	60 − 23 $\boxed{37}$
53 − 5	50 − 0	3 − 5 ?	40 − 0 $\boxed{40}$	13 − 5 $\boxed{8}$	53 − 5 $\boxed{48}$
81 − 25	80 − 20	1 − 5 ?	70 − 20 $\boxed{50}$	11 − 5 $\boxed{6}$	81 − 25 $\boxed{56}$

Developing subtraction with regrouping

Teaching
Page h-54

Ask the children to read the directions, and call their attention to the first problem. Point out that the original problem has been rewritten, and that they cannot subtract 6 from 4. Suggest, however, that the children regroup 70 and 4 to 60 and 14 and then find the differences. Direct them to complete the subtraction and write the differences in the answer boxes. Point out that they should complete the page by thinking through the regrouping to find the differences.

Note: this method of subtraction is intended only as a preparation for the algorithm in the next lesson. It is not intended that it be practiced to any great extent.

Follow-up

Suggest to any children who had difficulty understanding the lesson that they work with actual sticks. They might regroup bundles of sticks as they work through exercises similar to those in the text. Worksheet problems such as the ones in the next column will help to reinforce regrouping skills in subtraction.

Find the number for each □.

51 − 25	→	40 − 20 □	11 − 5 □	→	51 − 25 □
33 − 18	→	20 − 10 □	13 − 8 □	→	33 − 18 □

Teaching
Page h-55

You might want to spend one day on each page of this lesson. Read the directions with the children. Work through the first two exercises with them, as you did in the pre-book activities. Urge the children to try to do the last few exercises independently, but continue to guide them as long as necessary.

You may wish to use the exercises suggested in the follow-up at this time, so that you will be sure that the children have a fair understanding of the regrouping necessary for subtraction.

Write the numeral for each ▢.

To think of 63 as 50 + 13, we write $\overset{5}{\cancel{6}}\,\overset{13}{\cancel{3}}$.

To think of 46 as 30 + 16, we write $\overset{3}{\cancel{4}}\,\overset{16}{\cancel{6}}$.

To think of 34 as 20 + 14, we write $\overset{2}{\cancel{3}}\,\overset{14}{\cancel{4}}$.

To think of 82 as 70 + 12, we write $\overset{7}{\cancel{8}}\,\overset{12}{\cancel{2}}$.

To think of 57 as 40 + 17, we write $\overset{4}{\cancel{5}}\,\overset{17}{\cancel{7}}$.

To think of 75 as 60 + 15, we write $\overset{6}{\cancel{7}}\,\overset{15}{\cancel{5}}$.

To think of 31 as 20 + 11, we write $\overset{2}{\cancel{3}}\,\overset{11}{\cancel{1}}$.

Notation for regrouping

Objective

Given a subtraction exercise which requires regrouping, the child will be able to find the difference by using the standard subtraction algorithm.

Pre-Book Activity

Use a set demonstration or demonstration strips to work through a subtraction exercise. Write the subtraction exercise on the chalkboard. Then have the children examine the exercise and write it in two-step notation as in the previous lesson. Ask them if they can suggest a way to do the problem so that they will not have to write two problems. Allow children an opportunity to show their suggestions on the chalkboard. If the traditional shortcut algorithm is not suggested, suggest crossing out the tens' place and writing the regrouping above the number as shown on page h-55. When children see the reason for the shortcut method, give several examples, stressing the written notation for the regrouping. Practice just the method for showing regrouping at first. Do not use it to find differences requiring regrouping until the children understand it well.

Find the difference for each ☐.

	60 + 14		40 + 12
74 − 26	$\overset{6\ 14}{\cancel{74}}$ − 26 = 48	52 − 14	$\overset{4\ 12}{\cancel{52}}$ − 14 = 38
50 − 37	$\overset{4\ 10}{\cancel{50}}$ − 37 = 13	65 − 7	$\overset{5\ 15}{\cancel{65}}$ − 7 = 58
83 − 44	$\overset{7\ 13}{\cancel{83}}$ − 44 = 39	46 − 37	$\overset{3\ 16}{\cancel{46}}$ − 37 = 9

$\overset{5\ 17}{\cancel{67}}$ − 28 = 39	$\overset{7\ 13}{\cancel{83}}$ − 37 = 46	$\overset{5\ 10}{\cancel{60}}$ − 16 = 44	26 − 14 = 12	$\overset{6\ 14}{\cancel{74}}$ − 8 = 66	$\overset{7\ 18}{\cancel{88}}$ − 29 = 59
$\overset{6\ 11}{\cancel{71}}$ − 15 = 56	64 − 21 = 43	$\overset{2\ 13}{\cancel{33}}$ − 16 = 17	$\overset{3\ 17}{\cancel{47}}$ − 29 = 18	$\overset{7\ 12}{\cancel{82}}$ − 34 = 48	$\overset{4\ 14}{\cancel{54}}$ − 26 = 28

Development of subtraction algorithm

Teaching
Page h-56

Read the directions with the children. Work through the first problem with them and explain each step. For example, say: "To subtract 26 from 74, think of 74 as 60 + 14." Point out the little "think cloud." Instruct the class to use the method that they learned on page h-55 to rewrite this regrouping, and then to find the difference. Suggest that they begin with the ones, and then do the tens to complete the problem.

Go on to the second exercise. Draw as much of the explanation from the children as possible. If they can, ask them to do the next four problems by themselves. Then check the work with them.

Move to the last two rows of problems. Suggest that the children use the new method of remembering regrouping if they need to. Work through the first example with them, and then suggest that they finish the page independently. For further practice, provide exercises similar to those on page h-58.

Follow-up

For further practice in regrouping, duplicate a work sheet or write exercises such as the following on the chalkboard.

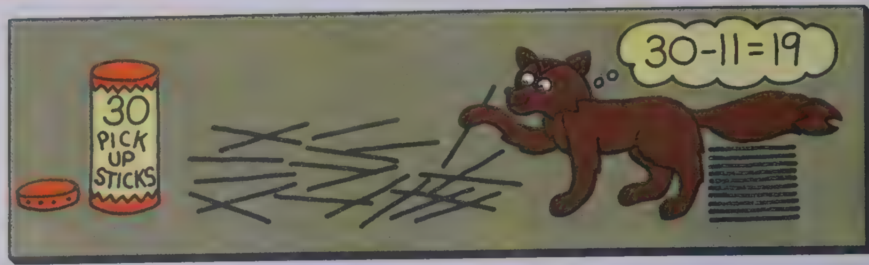
To think of	
72 as 60 + 12, we write	$\overset{\boxed{6}\boxed{12}}{\cancel{7}\cancel{2}}$
55 as 40 + 15, we write	$\overset{\boxed{4}\boxed{15}}{\cancel{5}\cancel{5}}$
86 as 70 + 16, we write	$\overset{\boxed{}\boxed{}}{\cancel{8}\cancel{6}}$
44 as 30 + 14, we write	$\overset{\boxed{}\boxed{}}{\cancel{4}\cancel{4}}$
91 as 80 + 11, we write	$\overset{\boxed{}\boxed{}}{\cancel{9}\cancel{1}}$

Resources for Active Learning

CHIP TRADING ACTIVITIES—SET II, Cards 21–24, Scott Scientific

TEACHING
Page h-57

Depending upon the ability of the children, you may either want to encourage them to work these pages independently or you may discuss the exercises with them. When they have finished, have some children show their work on the chalkboard. Encourage the children to ask questions about any problem that they miss.



Find the differences.

31	54	42	62	51	40
− 14	− 38	− 24	− 42	− 23	− 13
17	16	18	20	28	27
63	25	91	72	27	32
− 46	− 17	− 36	− 5	− 16	− 13
17	8	55	67	11	19
34	52	43	24	41	55
− 17	− 46	− 17	− 6	− 15	− 23
17	6	26	18	26	32
23	26	80	61	22	44
− 14	− 18	− 52	− 32	− 7	− 25
9	8	28	29	15	19
56	64	33	45	53	65
− 37	− 39	− 8	− 26	− 25	− 38
19	25	25	19	28	27

Practice—subtraction with regrouping

OBJECTIVE

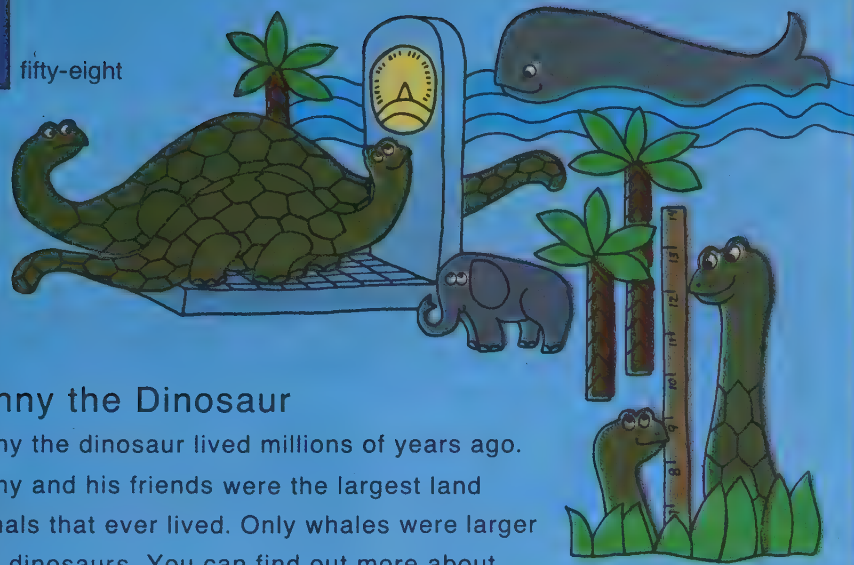
Given two-digit subtraction problems which require regrouping, the child will be able to find the differences using the standard subtraction algorithm.

PRE-BOOK ACTIVITY

Discuss comparisons, for example compare the number of children in different classrooms in your school. Use comparisons such as the following and work through the algorithm.

34 children in class A.
28 children in class B.
How many more in class A?

$$\begin{array}{r} \overset{2}{\cancel{3}}\overset{14}{\cancel{4}} \\ -28 \\ \hline 6 \end{array}$$ children



Danny the Dinosaur

Danny the dinosaur lived millions of years ago. Danny and his friends were the largest land animals that ever lived. Only whales were larger than dinosaurs. You can find out more about Danny in these problems.

1. Danny was 17 metres long. His father was 25 metres long. How much longer was Danny's father? 8 metres
2. Danny was 9 metres tall. His father was 13 metres tall. How much taller? 4 metres
3. Danny and his father weighed 73 metric tons together. His father weighed 44 metric tons. How much did Danny weigh? 29 tons
4. Danny's tail was 13 metres long. His neck was 8 metres How much longer was his tail? 5 metres
5. Danny's father weighed 44 metric tons. A full-grown elephant weighs only 6 metric tons. How much more did Danny's father weigh? 38 tons
6. A full-grown whale can weigh 92 metric tons. Danny's father weighed 44 metric tons. How much more does the whale weigh? 48 tons
7. Danny could eat 2 metric tons of food a day. His father could eat 3 times that much. How many metric tons could his father eat? 6 tons

Story problems

Teaching
Page h-58

Read the paragraph at the top of the page with the children. Then help them work through each of the problems. If the children are capable, have three or four meet together in a small group and try to read and work through the problems. As they discuss the problems, move around the room to answer their questions and check their work. It is not necessary that they write an equation for each exercise, but you might encourage them to do so. Notice that problem 7 does not deal with subtraction and may be solved either by repeated addition or multiplication.

Follow-up

Short-story problems may increase the children's efficiency in applying the subtraction algorithm. Write examples like the following on the chalkboard:

1) Allowance was 25 cents. Bought a glider for 19 cents. Change?

2) Model car costs 77 cents. Piggy bank has 58 cents. Need?

3) Birthday money 75 cents. Necklace cost 69 cents. Change?

4) Had 25 cents, bought two 8¢ stamps. Change?

For more capable children, your questions should require more insight. For example:

5) Roller-coaster ride costs 35 cents, peanuts cost 15 cents. Alan has 40 cents. Could he ride the roller coaster? What would his change be? Could he just have peanuts? What would his change be? Could he have both a roller-coaster ride and peanuts? How much more does he need to have both a roller-coaster ride and peanuts?

Teaching
Page h-59

Point out to the children that on this page they have a combination of both addition and subtraction exercises. Ask them to begin the exercises independently. When all the children have completed the top two frames, it would be helpful to have children put some exercises on the chalkboard and show their solutions. Ask other children to check the answers against theirs. Point out that the exercises on the bottom are an extra challenge for anyone who wishes to try them.

Find the sums and differences.

16 + 27 = 43	45 + 36 = 81	54 + 18 = 72	62 − 27 = 35	41 − 16 = 25	73 − 34 = 39
63 + 27 = 90	75 + 14 = 89	27 + 7 = 34	81 − 23 = 58	65 − 42 = 23	94 − 85 = 9
18 + 47 = 65	9 + 43 = 52	34 + 29 = 63	82 − 8 = 74	56 − 29 = 27	42 − 17 = 25

Find the missing digits.

27 + 36 = 63	48 + 29 = 77	75 + 15 = 90	52 − 14 = 38	61 − 27 = 34	83 − 24 = 59

Practice—addition and subtraction with regrouping

Objective

Given two-digit addition or subtraction exercises requiring regrouping, the child will be able to find the sums or differences.

Pre-Book Activity

The "What's My Rule?" game would be an appropriate pre-book activity for this lesson. It might be interesting for the children to use the form suggested by the illustration in the next column. Write a number of rows or columns on the chalkboard and fill in one row with numbers less than 10. Then according to your rule, fill in a second row. Then either row may be used with a rule to form the third row.

Rule: Add 8	15	13	16	?	?	?	?	?
	7	5	8	4	6	2	9	1
Rule: Subtract 5 from top row	10	8	11	?	?	?	?	?

This activity should include rules for both addition and subtraction.

Lori and Bob

Lori and Bob are second graders. They are about the same size and age as you. You can learn more about them **(and maybe yourself)** if you work these problems.

1. Lori weighs 26 kilograms. Bob weighs 23 kilograms. How much do they weigh together? 49 kg

2. Lori is 96 centimetres tall. Bob's little sister is 58 centimetres How much taller is Lori? 38 cm

3. Lori's hand and wrist have 27 bones. How many bones in both her hands and wrists? 54

4. Bob's ankle and foot have 26 bones. How many bones in both his ankles and feet? 52

5. About 15 kilograms of Lori's body is water. About 17 kilograms of Bob's is water. How much water in both? 32 kg

6. Bob's muscles weigh about 12 kilograms. His father's muscles weigh about 35 kilograms. How much more do his father's muscles weigh? 23 kg

7. Lori's heart beats 82 times a minute. Bob's beats 78. How many more times does Lori's heart beat in a minute? 4

Teaching
Page h-60

Read the paragraph at the top of the page with the children. Then give the children the opportunity to discuss the problems. This may be done either by assigning the children to work in small groups or by guiding the discussion yourself. Point out that the problems will require either addition or subtraction. It would be helpful to remind them of guidelines which they might follow. For example, suggest that they first think: "What facts am I given?" second: "What question am I being asked?" third: "How can I use my facts to answer the question?" and, fourth, when they have finished their solutions, "Is it reasonable?" "Does my answer make sense?" These guidelines should be used as you work with the children in discussing the exercises; children should not be expected to remember these questions.

Follow-up

If children need consistent help in analyzing story problems, even after the problems are read to them, you might plan to work on story problems daily.

Before the arithmetic period each day, write a problem on the chalkboard. In the beginning, include only the needed facts and a simple question. Give the children a specific amount of time in which to do the work. Appoint one child for each row to collect and check the answers against the teacher's copy. During the regular arithmetic period, ask a volunteer to analyze the problem orally with your guidance. Encourage children who have other approaches to the solution to contribute to the class discussion. Gradually increase the difficulty of the problems by including unnecessary information or by requiring more than one step to solve the problem.

Teaching
Page h-61

Explain to the children that all of the exercises on this page involve subtraction and that they are to find the differences. Encourage them to use the shortcut method developed during this module; however, if children wish to use strips or sticks to help them with their regrouping, allow them to do so. Help any children with a reading difficulty to read the problems at the bottom of the page; however, encourage them to solve each problem independently.

Show you know
Find the differences.

52 − 17 35	71 − 45 26	63 − 36 27	74 − 29 45	81 − 56 25	34 − 17 17
41 − 14 27	82 − 25 57	94 − 65 29	62 − 38 24	53 − 14 39	72 − 24 48
72 − 68 4	94 − 55 39	63 − 9 54	61 − 43 18	95 − 28 67	77 − 64 13
81 − 33 48	74 − 36 38	52 − 7 45	83 − 37 46	56 − 24 32	35 − 15 20

Sandy had 55 cents. She spent 37 cents. How much does she have now? 18¢

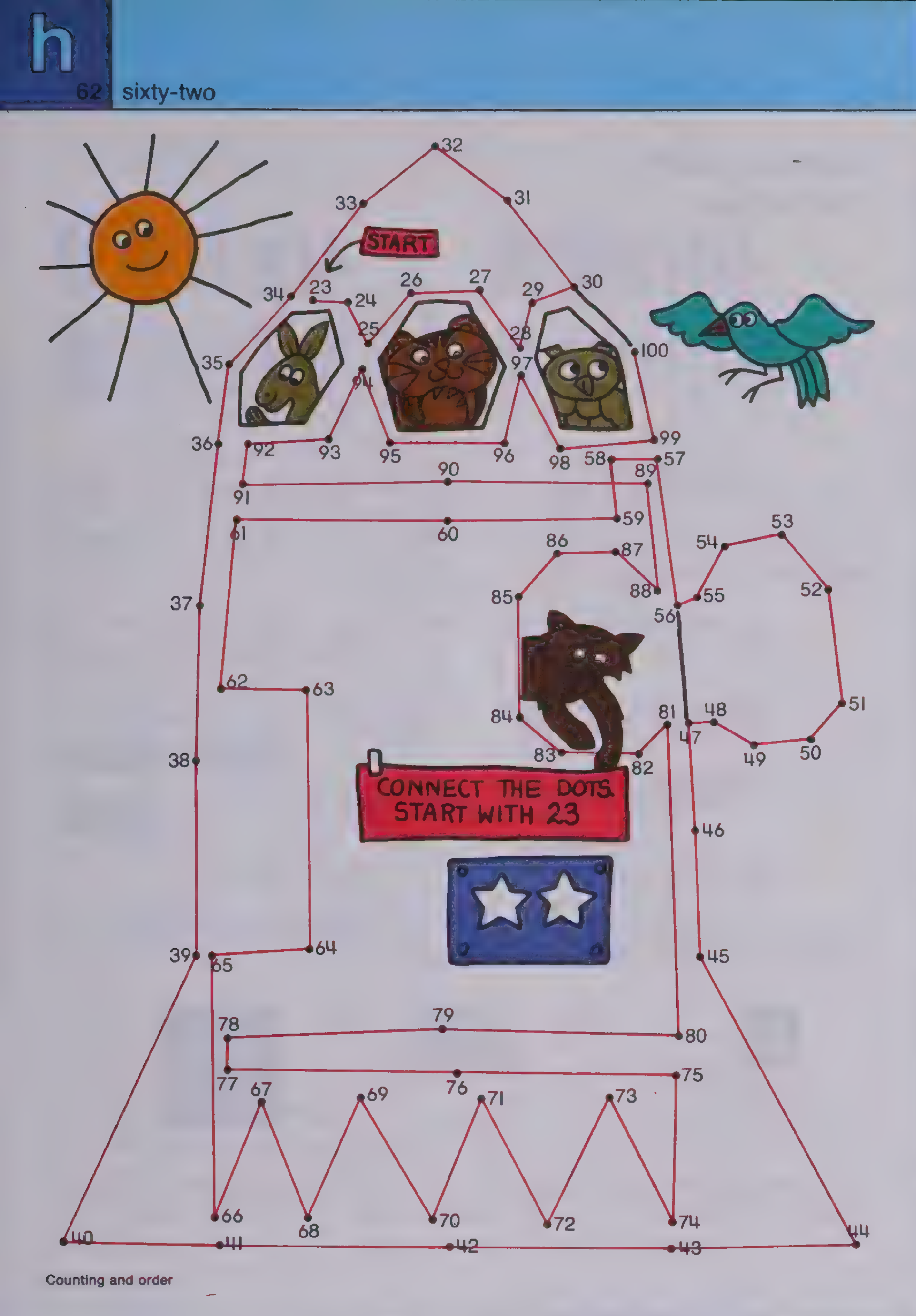
Ben had 24 baseball cards. He collected 18 more. How many does he have now? 42

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Conduct a drill on addition and subtraction with regrouping. Follow this by exhibiting reconstruction exercises similar to those at the bottom of text page h-59. Give all the children an opportunity to discuss these exercises. Some will find them too difficult to work independently, but they can benefit from participating in a group discussion in which others tell ways of finding the missing numbers. Some of the ideas necessary to an understanding of regrouping and subtraction will be brought out in the children's explanations.



Teaching
Page h-62

This is the last change of pace in the book. Since it is a dot-to-dot picture, children should need little guidance to complete it, as they have completed several other pictures like it in previous modules.

The children might find it interesting that all of the animals used for the illustrations at the top of the students' pages for this book are shown in the "rocket." So, in a sense, this is their farewell to the children for this year.

Follow-up

To give the children further practice, you might duplicate a cross number puzzle such as the following:

1) 3	2) 6		3) 6	0	4) 2		
	7				9		
		5) 9	6) 2			7) 8	
8) 3	9) 8		0			10) 5	6
	4		11) 6	12) 6			
	13) 1	14) 4		15) 3	16) 2		17) 7
		8			9		5

Across

1) 63 − 27
3) Six hundred two
5) 54 − 16
8) 54 − 16
10) 82 − 26
11) 17 + 49
13) 72 − 58
15) 17 + 15

Down

2) 48 + 19
4) 38 − 9
6) Two hundred six
7) 94 − 9
9) Eight hundred forty-one
12) 35 + 28
14) 39 + 9
16) 50 − 21
17) 26 + 49

Teaching
Page h-63

Read the title and remind the children that on this page they will be doing kinds of problems that they have had before. First direct the children to find the sums in the addition equations and the differences in the subtraction equations. Point out the exercises with fractions at the bottom of the page. Here children should study each region and decide which part of the whole region has been colored. Then they should ring the fraction which names this part.

Looking back

Find the sums.

$8 + 5 = \boxed{13}$ $\quad$ $6 + 8 = \boxed{14}$

$7 + 7 = \boxed{14}$ $\quad$ $5 + 7 = \boxed{12}$

$9 + 3 = \boxed{12}$ $\quad$ $8 + 9 = \boxed{17}$

$9 + 9 = \boxed{18}$ $\quad$ $8 + 4 = \boxed{12}$

Find the differences.

$12 - 8 = \boxed{4}$ $\quad$ $16 - 7 = \boxed{9}$

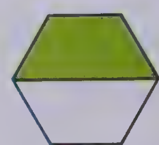
$15 - 6 = \boxed{9}$ $\quad$ $13 - 9 = \boxed{4}$

$17 - 8 = \boxed{9}$ $\quad$ $11 - 5 = \boxed{6}$

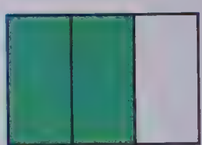
$14 - 7 = \boxed{7}$ $\quad$ $14 - 9 = \boxed{5}$

Ring the fraction for the colored part.

$\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{4}$ $\frac{1}{5}$



$\frac{1}{4}$ $\frac{1}{2}$ $\frac{3}{4}$ $\frac{1}{3}$



$\frac{1}{2}$ $\frac{1}{3}$ $\frac{2}{3}$ $\frac{2}{4}$

Cumulative review

Objective

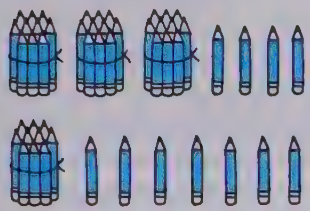
The child will demonstrate his ability to work with the concepts presented in Unit H.

Pre-Book Activity

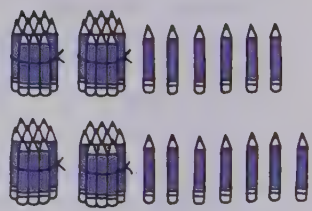
Review any concepts with which the children had difficulty. For example, you might wish to review concepts regarding halves, fourths, and thirds.

You might also review some skip counting, and stress odd and even numbers. Play a short oral game by asking the children to give the next larger odd or even number, or the next smaller odd or even number after you give a number. For example, say the number 9, and ask one child for the next larger even number; then ask another for the next smaller odd number. Next say 11, and ask a child to give the next larger odd number and then another to give the next smaller even number.

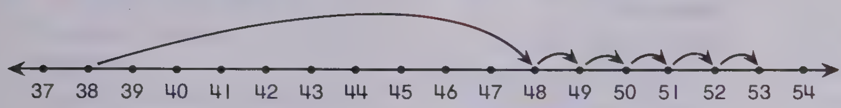
Find the sums.



34
+ 17
51

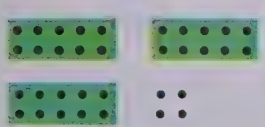


26
+ 27
53

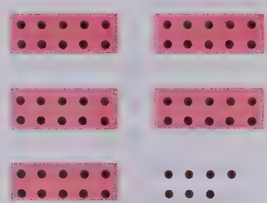


38 + 15 = 53

Find the differences.



34
− 18
16



57
− 29
28

74 − 15 = 59

John had 16 cents.
He earned 25 more cents.
How much does he have?
41¢

Tom read 23 pages in his library book. Mary read 17 pages. How many more pages did Tom read?
6

Cumulative review

Teaching
Page h-64

The exercises on this page are only for those children who studied the optional modules, pages h-39 to h-62. Ask these children to complete the page independently, if they can. You might want to read the story problems to any child still unable to read them. Although the concepts on this page are intended to be optional at this level, choose activities or develop exercises to clear up any misunderstandings which become evident from your evaluation of the children's work.

Follow-up

If your pre-book activity treated odd and even numbers, you may follow up this lesson by duplicating a table similar to the one in the next column.

+	0	2	4	6	8	36
1						
3						
5						
7						
9						
47						

The numbers 0, 2, 4, 6, 8, 36, are _____ numbers.
The numbers 1, 3, 5, 7, 9, 47 are _____ numbers.
Each number in the answer section of the table is an _____ number.
The sum of an odd and an even number is an _____ number.

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A Text for Teachers

Investigating Mathematics Learning

Phares G. O'Daffer

Robert E. Eicholz

Charles R. Fleenor

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James Sherrill

J. Norman C. Sharp

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INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

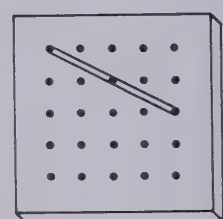


Figure 1

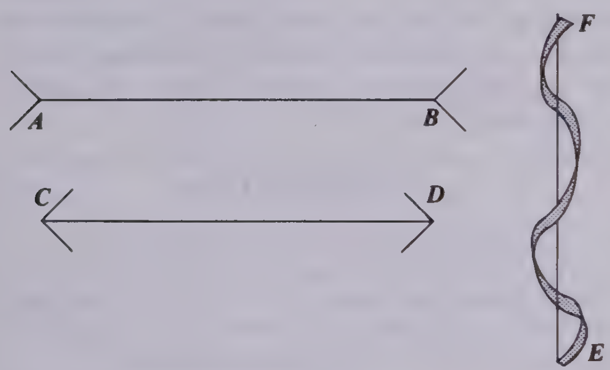
The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

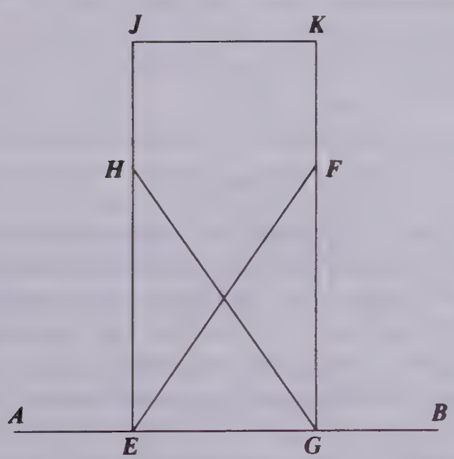
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.
How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

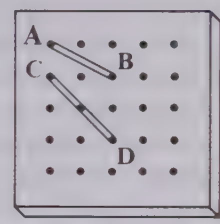


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill*, forming a *generalization*, learning a *fact*, or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

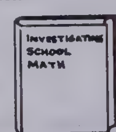


Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.

2 3 4

Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

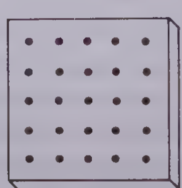


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

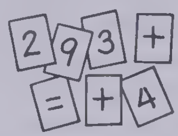
EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION

Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

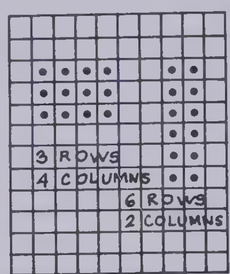


How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION

The graph paper shows two different ways to arrange 12 counters in a rectangular array.



How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

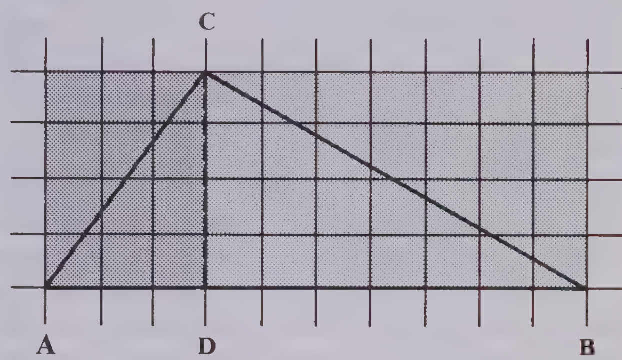
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41				

EXAMPLE B

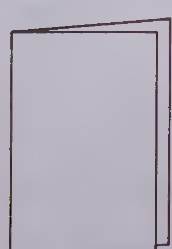


What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle *ADC*?
What is the area of triangle *BDC*?
What is the area of triangle *ABC*?
The area of triangle *ABC* is what part of the entire shaded region?

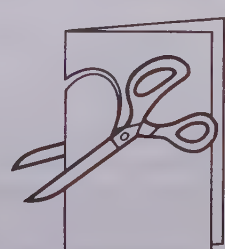
The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.



Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

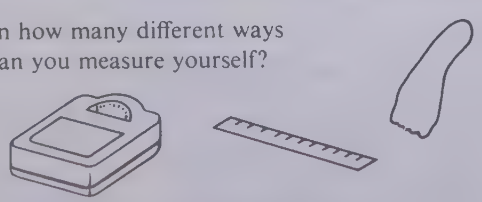
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?



Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

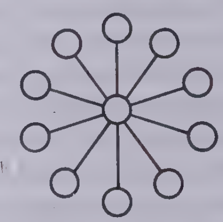
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?

27 ? 17

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
. . .					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

P F F
$72 \div 8 = n$ ← You find this quotient,
F F P
when you find this factor. → $n \times 8 = 72$

A *concept,* then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

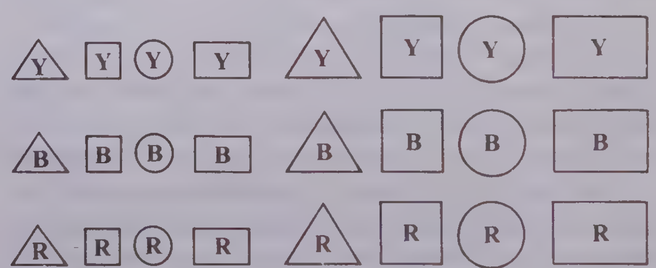


Figure 7

Now consider the Concept Card in Figure 8.

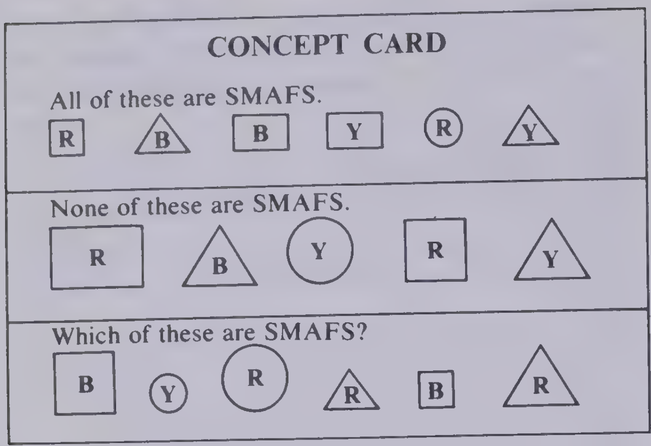


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

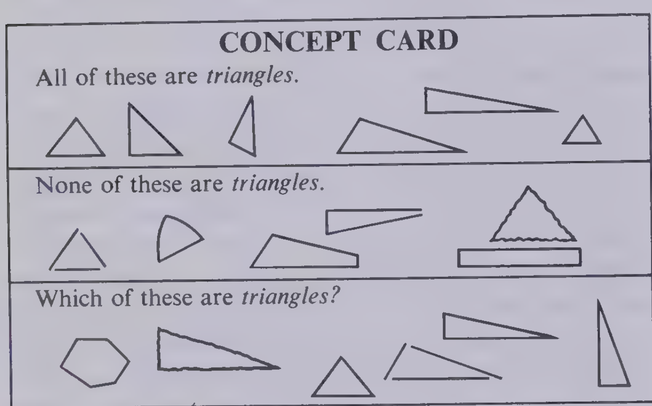


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   A quadrilateral
   B simple closed curve
   C odd number
   D greater than (the relation)
   E right triangle
   F is congruent to (the relation)
   G lowest-terms fraction
   H parallelogram
   I diagonal of a polygon
   J parallel lines
   K one half
   L isosceles triangle
   M equivalent sets
   N symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   A What are some examples of the concept you learned?
   B Give some characteristics of the concept you learned.
   C What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

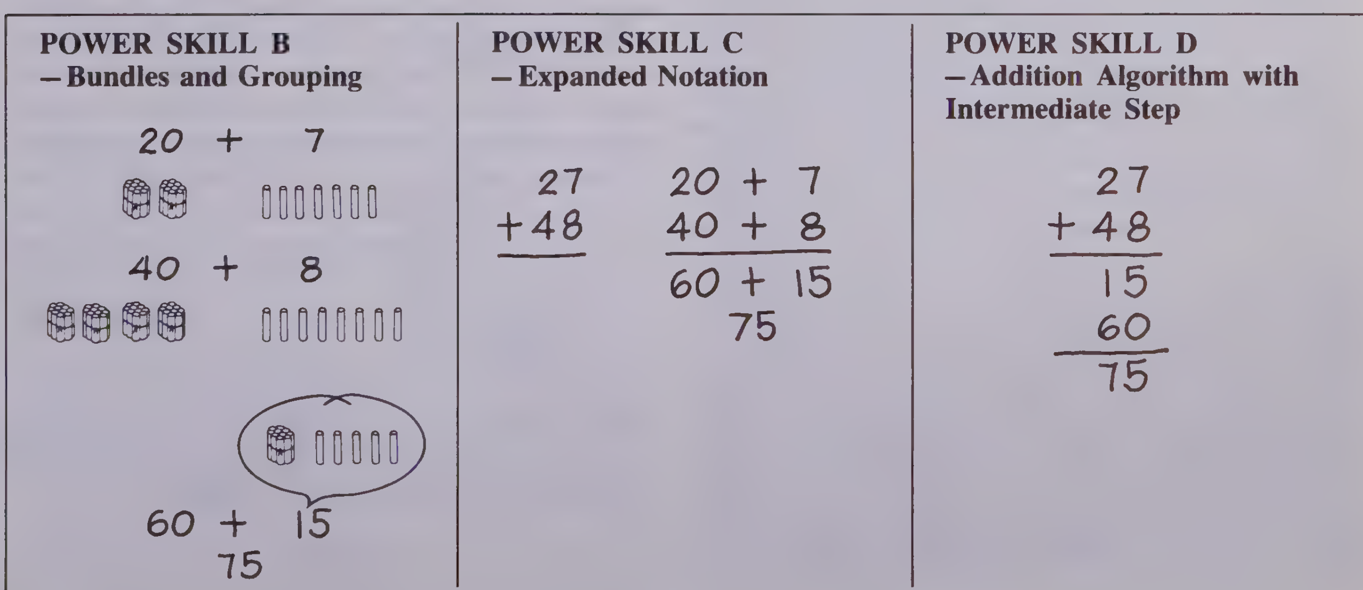


Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

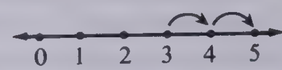
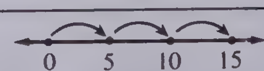
Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

	Specific Skill	Example
A	Using sets to find sums	 $3 + 2 = \square$
B	Using number line to find sums	 $3 + 2 = \square$
C	Counting fingers to find sums	 $3 + 2 = \square$
D	Memorizing that $3 + 2 = 5$	Think: $3, 2 \rightarrow 5$ $3 + 2 = 5$
E	Thinking about "take away" to find differences	 $5 - 2 = \square$
F	Using the inverse relation (missing addend) to find differences	Think: $? + 2 = 5$ $5 - 2 = \square$
G	Memorizing that $5 - 2 = 3$	Think: $5, 2 \rightarrow 3$ $5 - 2 = 3$
H	Using sets to find products	 $3 \times 6 = \square$
I	Using the number line to find products	 $3 \times 5 = \square$
J	Using logic (basic principles) to find products	Since $5 \times 5 = 25, 6 \times 5 = \square$ or Since $3 \times 5 = 15, 6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}^{\;13}$$

In what order should these be presented?

A
$$\begin{array}{r l} 7\overline{)91} & 10 \\ \underline{70} & \\ 21 & 3 \\ \underline{21} & \\ 0 & \end{array}$$

B Subtract 1 seven at a time.
$$\begin{array}{r} 91 \\ \underline{-\ 7} \\ 84 \\ \underline{-\ 7} \\ \vdots \end{array}$$

C
$$\begin{array}{r} 13 \\ 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \end{array}$$

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.

A Discuss the skill you learned and the way you learned in terms of power skills and speed skills.

B What part of the lesson helped you evolve a speed skill?

C What were your feelings about the lesson? How could it be improved?

GENERALIZATIONS

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery.* In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$
$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$
$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

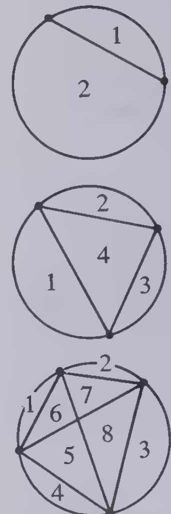
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, $6 \times 2 = 2 \times 6$ or $4 \times 3 = 3 \times 4$. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   A Fill in the table to show how many regions are formed if five points on a circle are connected.
   B Form a generalization about the right-hand column of the table.
   C Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   A The commutative principle for multiplication
   B The volume of a "box" is found by multiplying length times width times height.
   C In measuring length, the shorter the unit, the greater the measure.
   D Any angle inscribed in a semicircle is a right angle.
   E Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   A What generalization did you learn from the lesson?
   B How many specific examples did you consider before you understood the generalization?
   C In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as 4 + 3. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as 3 + 3 = 6. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

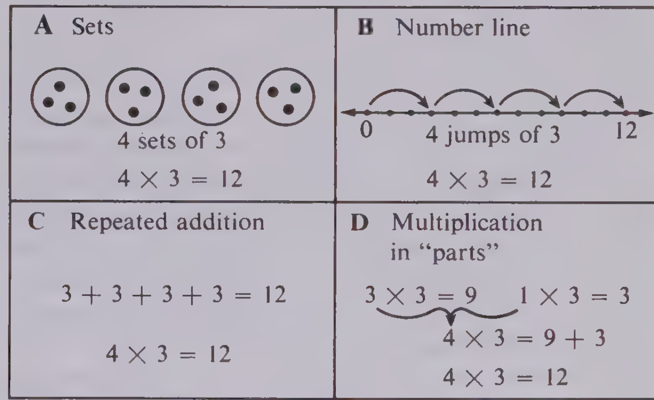
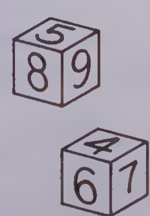


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game



×	4	5	6	7	8	9
4						
5						
6						
7						
8						
9						

Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   - **A** How many of the facts did you know?
   - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   - **C** Can you imagine some of the difficulties your children might have in learning facts?
   - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

Attitudes

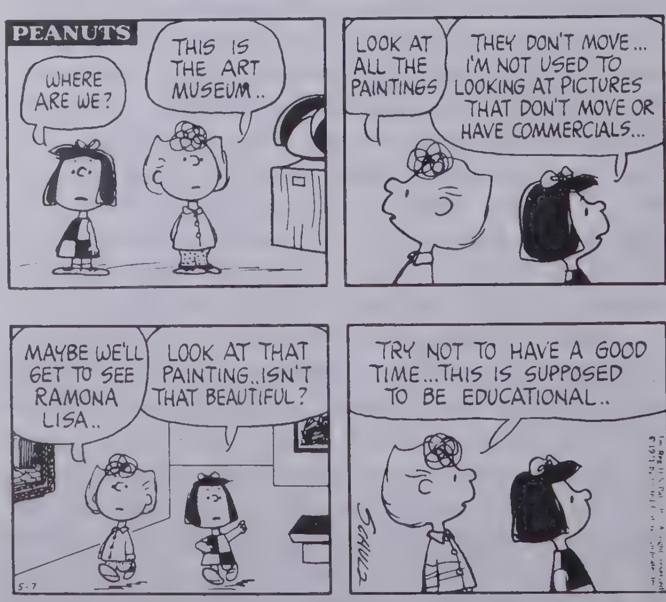
In his poem "Arithmetic," Carl Sandburg wrote:

> Arithmetic is numbers you squeeze from your head
>   to your hand to your pencil to your paper
>   till you get the answer.
> Arithmetic is where the answer is right
>   and everything is nice
>   and you can look out of the window
>   and see the blue sky –
>   or the answer is wrong
>   and you have to start all over and try again
>   and see how it comes out this time.
>
> . . . . .
>
> Arithmetic is where you have to multiply –
>   and you carry the multiplication table in your head
>   and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.



*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.

Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   A Was the lesson fun?
   B How did you feel when you had finished the lesson?
   C Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

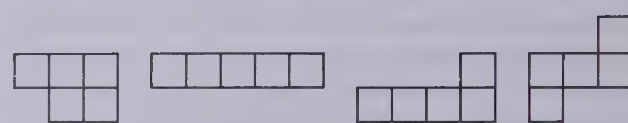
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

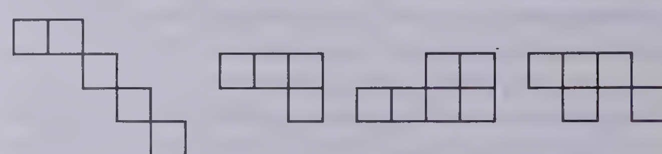
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



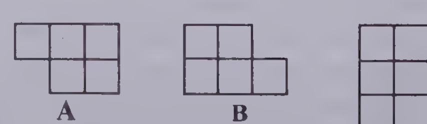
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



A B C

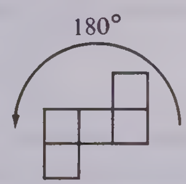
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

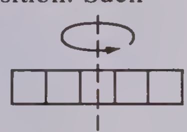
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



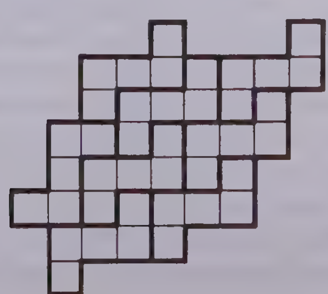
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.



Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.

Step 1	Step 2	Step 3
Think $4 \times 6 = 24$	Think $4 \times 3 = 12$ $7 \times 6 = 42$ 54 Add 2 2 56	Think $7 \times 3 = 21$ Add 5 5 26
Write 4 Remember 2	Write 6 Remember 5	Write 26
$\begin{array}{r} 36 \\ \times 74 \\ \hline 4 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 64 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 2664 \end{array}$

Can you use this method to write answers only for the products below? Check your answer using the "long" method.

$\begin{array}{r} 53 \\ \times 48 \\ \hline \end{array}$ $\begin{array}{r} 37 \\ \times 62 \\ \hline \end{array}$ $\begin{array}{r} 45 \\ \times 23 \\ \hline \end{array}$ $\begin{array}{r} 67 \\ \times 32 \\ \hline \end{array}$

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of __?__.
3. The 2 you remembered is really 2 __?__.
4. Explain what you are finding in Step 3.

Using the Ideas

Write answers only for each product.

1. $\begin{array}{r} 28 \\ \times 42 \\ \hline \end{array}$ 2. $\begin{array}{r} 46 \\ \times 33 \\ \hline \end{array}$ 3. $\begin{array}{r} 37 \\ \times 42 \\ \hline \end{array}$ 4. $\begin{array}{r} 82 \\ \times 56 \\ \hline \end{array}$ 5. $\begin{array}{r} 53 \\ \times 34 \\ \hline \end{array}$

6. $\begin{array}{r} 64 \\ \times 27 \\ \hline \end{array}$ 7. $\begin{array}{r} 29 \\ \times 63 \\ \hline \end{array}$ 8. $\begin{array}{r} 48 \\ \times 35 \\ \hline \end{array}$ 9. $\begin{array}{r} 53 \\ \times 53 \\ \hline \end{array}$ 10. $\begin{array}{r} 27 \\ \times 64 \\ \hline \end{array}$

Extension

1. Study the figures below for finding the product of two 3-digit numbers.

$\begin{array}{r} 352 \\ \times 436 \\ \hline 2 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 72 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 472 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 3472 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 153472 \end{array}$

2. Use the method shown in exercise 1 to find each product.

$$\begin{array}{r} 125 \\ \times 365 \\ \hline \end{array} \qquad \begin{array}{r} 757 \\ \times 426 \\ \hline \end{array} \qquad \begin{array}{r} 841 \\ \times 215 \\ \hline \end{array} \qquad \begin{array}{r} 525 \\ \times 525 \\ \hline \end{array}$$

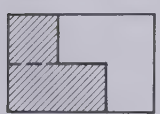
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

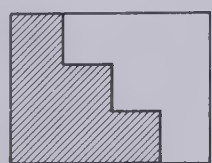
Can you find a pattern?

Investigating the Ideas

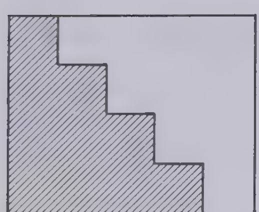
Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



A



B



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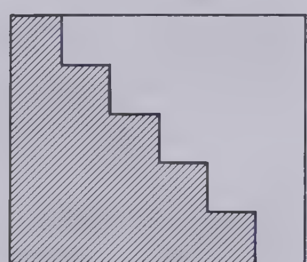
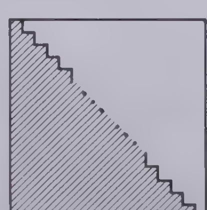


D

Discussing the Ideas

1. A Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. A Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n \cdot (n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

×	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. A How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. A What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$2^2 = 2 \times 2 = 4$$
$$2^3 = 2 \times 2 \times 2 = 8$$
$$2^4 = 2 \times 2 \times 2 \times 2 = 16$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

1. Give the next six powers of 2.

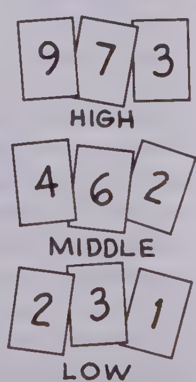
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

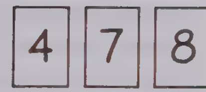
Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



Discussing the Ideas

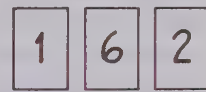
1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?

2. What is wrong with this arrangement for a middle hand?



3. If you were dealt these cards, would you try for a high or low? Why?

4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?



Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

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INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes –

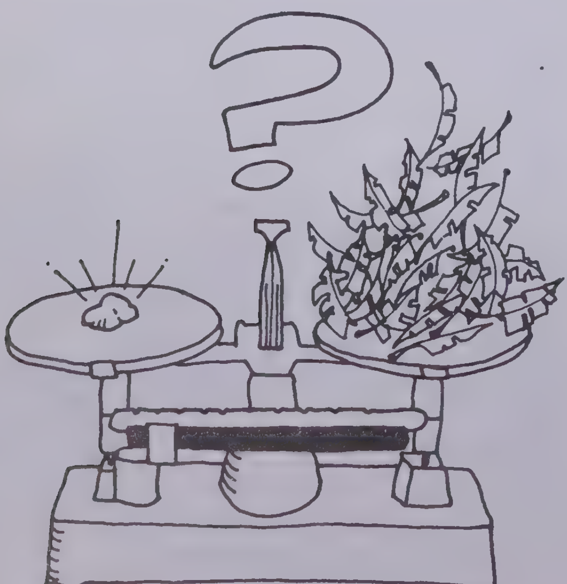
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object – himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound – 7 000 grains) and gold is weighed by the troy pound (1 troy pound – 5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron,* "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics – the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	Unit	Symbol
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	Metres
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted—

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts—

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized—

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.
2. Partition the object to be measured into units.
3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.

Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

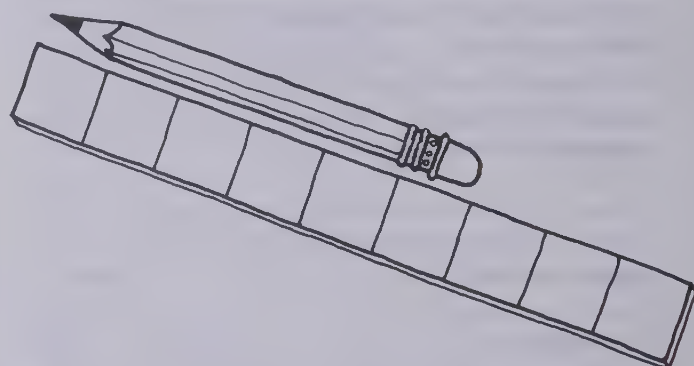
1. The first unit should be lined up with the "starting point" of the object.
2. The units should touch, but not overlap.
3. The "train" should be straight.
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

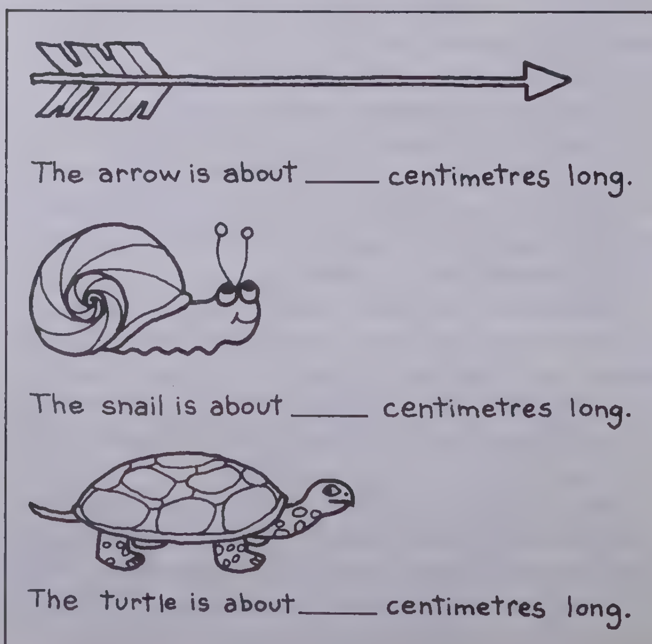
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



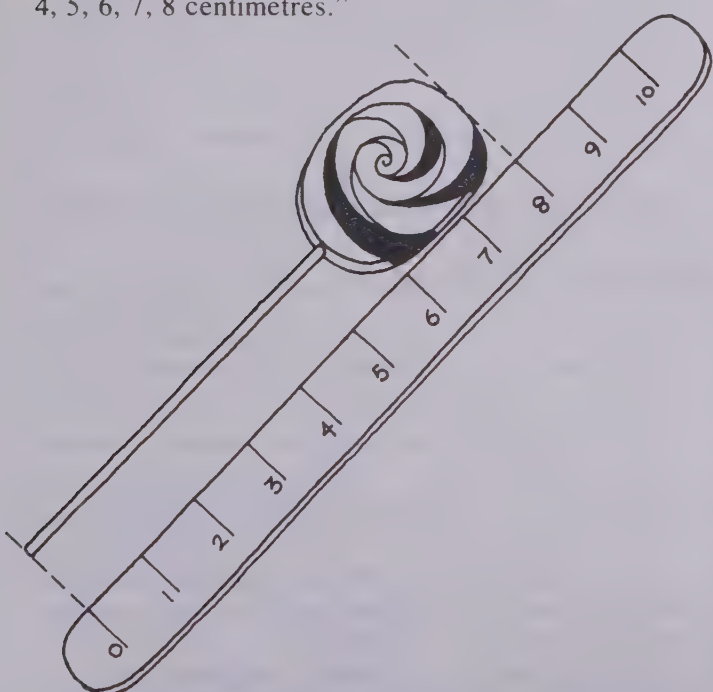
In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

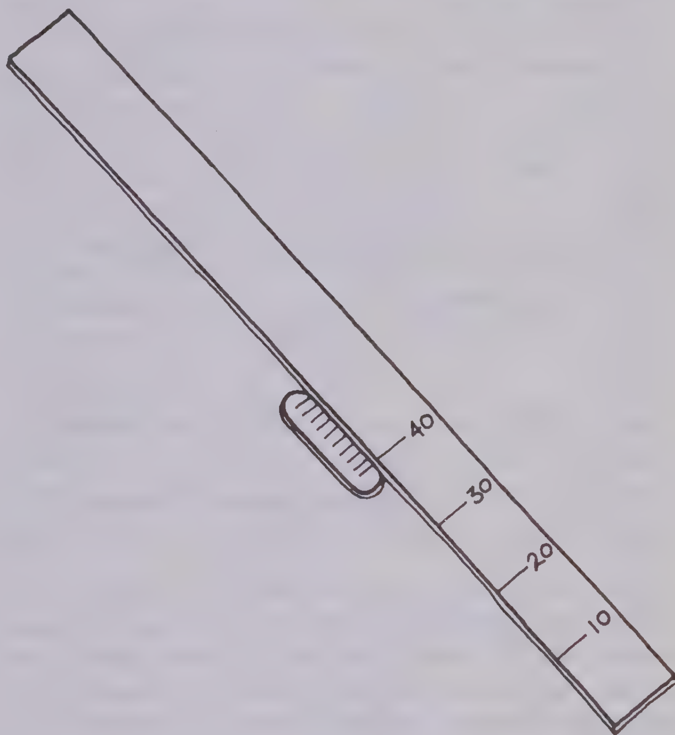


After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding



further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.

The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

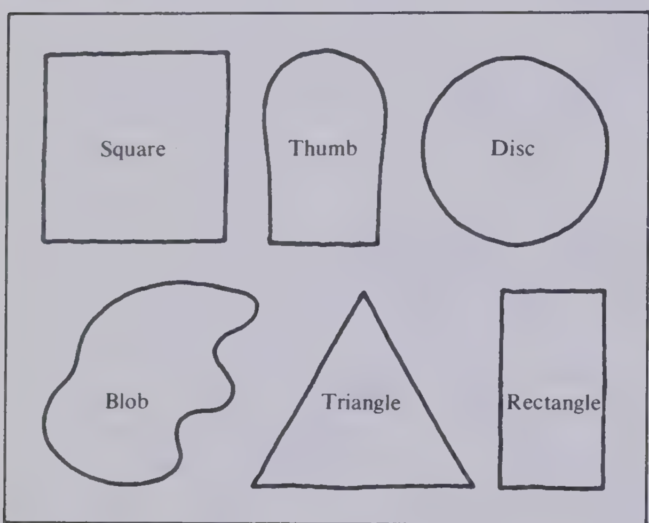
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as $1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation—it is not necessary!

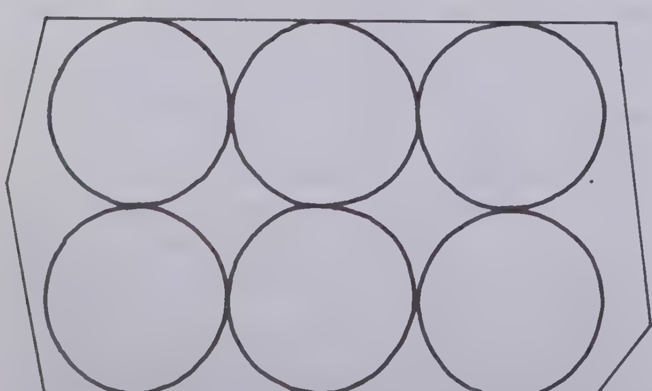
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

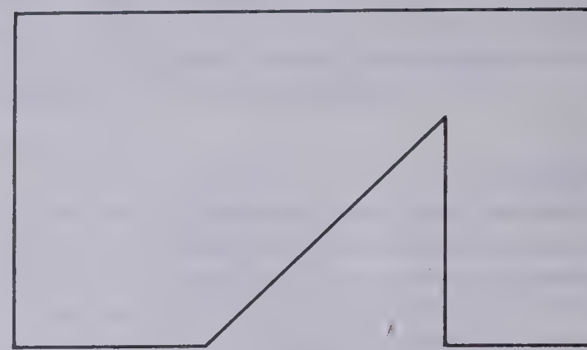


Figure X

Record the answers on the chalkboard in sentence form—

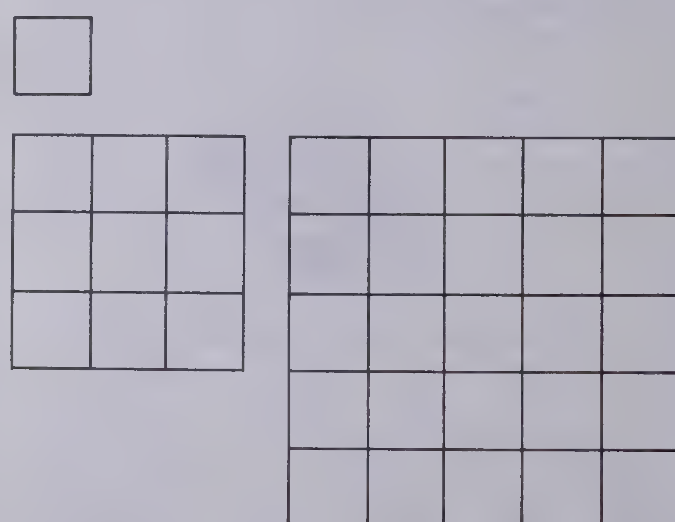
"The figure has an area of about ______ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are $1\,cm^2$, $9\,cm^2$, $25\,cm^2$, respectively.



You might point out that the square containing the $9\,cm^2$ has a side of 3 cm and the square containing the $25\,cm^2$ has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

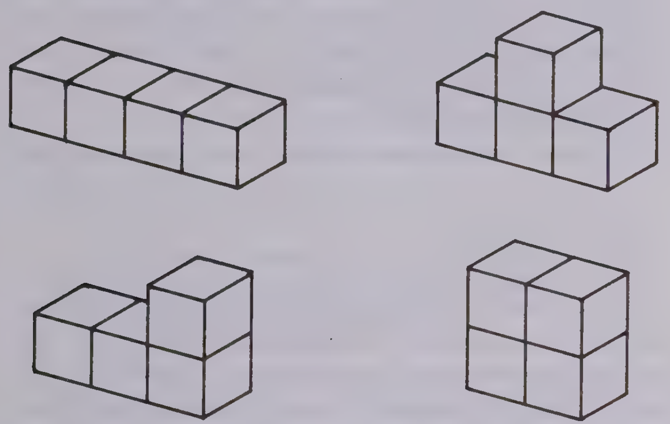
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.

a.	128 cm = ____ m	**e.**	1.06 m = ____ cm
b.	108 cm = ____ m	**f.**	10.01 m = ____ cm
c.	15 cm = ____ m	**g.**	23.86 m = ____ cm
d.	1010 cm = ____ m	**h.**	0.09 m = ____ cm

3. What would be the length of the sides in a square containing:
   a. 36 cm^2 → ____ cm?
   b. 25 cm^2 → ____ cm?
   c. 4 cm^2 → ____ cm?
   d. 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

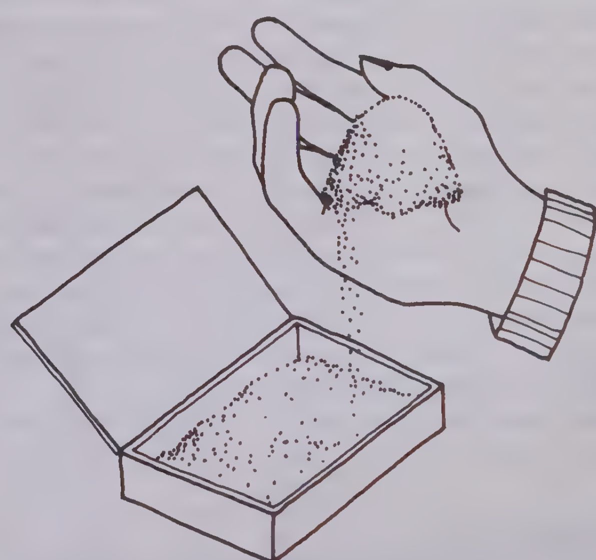
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds _____ handfuls of _____." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

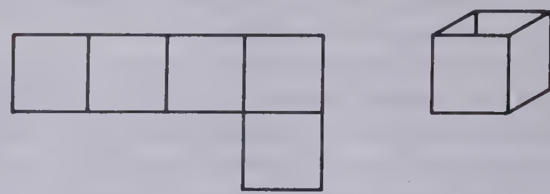
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about ______ ml.
It actually holds about ______ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

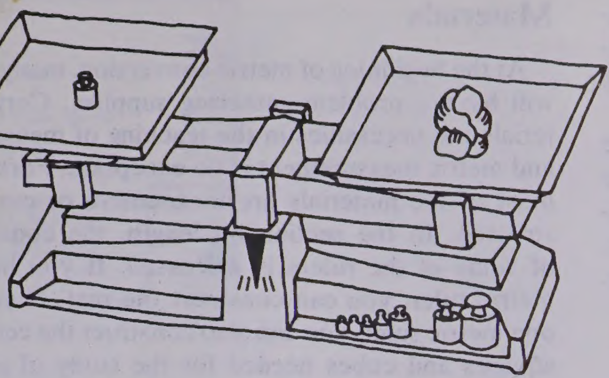
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about ______ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.
It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

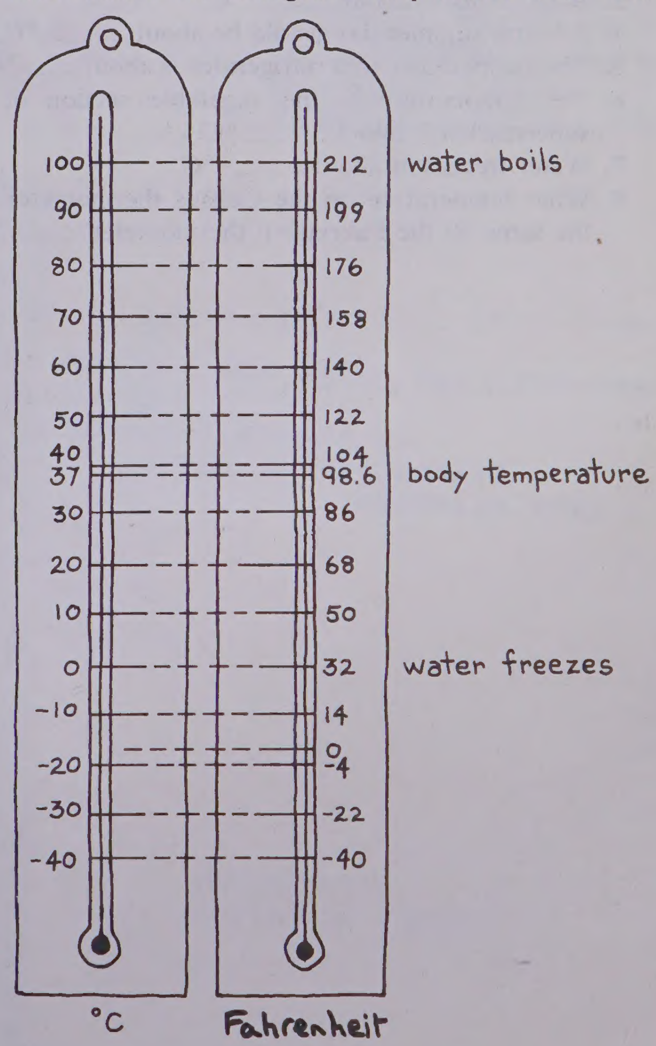
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
   - a. 28 ml of water has a mass of about ______ grams.
   - b. 170 ℓ is ______ ml.
   - c. 3.12 kg is ______ g and 438 g or ______ kg.
   - d. It would take ______ ml of water to balance 1 kg.
3. a. Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
   - b. Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
   - ★c. Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (° C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about ______ °C.
2. Normal room temperature is about ______ °C.
3. Water boils at about ______ °C.
4. A warm summer day would be about ______ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about ______ °C.
7. Water freezes at about ______ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materials

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd.—Don Mills, Ontario
Buntin Gillies & Co. Ltd.—Ottawa, Ontario
Cameron Products—Bramalea, Ontario
Canadian Metric Association—(P.O. Box 35)—Fonthill, Ontario
Contrasts 20—Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd.—Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd.—Stratford, Ontario
Lufkin Rule Co. of Canada Ltd.—Don Mills, Ontario
Lily Cups Ltd.—Scarborough, Ontario
MacLean-Hunter Learning Materials Co.—Toronto 101, Ontario
Metric-Aids Ltd.—Toronto, Ontario
Moyer-Vico Ltd.—Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics—1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd.—Weston, Ontario
Spectrum Education Ltd.—Toronto, Ontario
Spicars International Ltd.—Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

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